Industrial Mathematics: One Canadian Perspective

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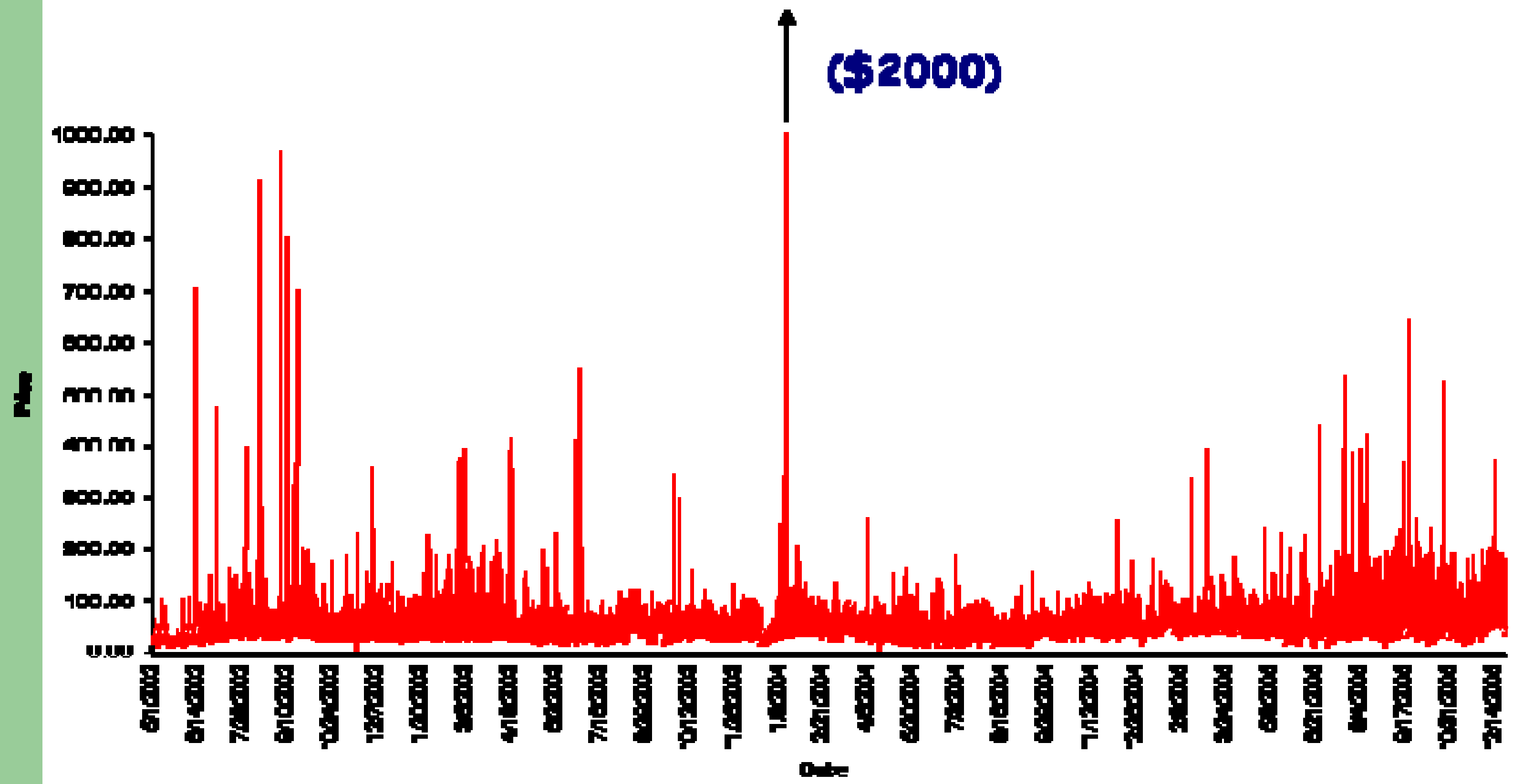
Range of Projects with industrial collaborators

- Property & Casualty Insurance Compensation Corporation (started 2006, ongoing)
- Princess Margaret Hospital (started 2005, ongoing)
- Department of National Defence (Navy) (started 2005, ongoing)
- Environment Canada (started 2007, ongoing)
- IBM Toronto software Lab (started 2004, ongoing)
- Bank of Canada (started 2006, ongoing)
- **Ontario Power Generation (2000-2002)**
- **Dydex Ltd (2003)**
- **Canadian Energy Wholesalers Inc (Jan-Feb 2007**)
- Waterloo Maple Inc (2006)

Collaborators

- Former PhD students (**Lindsay Anderson**, now dept of Environmental Eng, Cornell, **Matt Thompson**, now Faculty of Business, Queen's University).
- Former M.Sc. students (**Abu Bah**, **Rizwan Mukadam**, **Karen Anderson**)
- Current research team (**Guangzhi Zhao**, **Sharon Wang**)
- Private sector (**Peter Vincent**, OPG,, **Ligong Kang**, Transalta, **Peter Stabins**, Dydex and Canadian Energy Wholesalers Inc)

Ontario Open Market Price

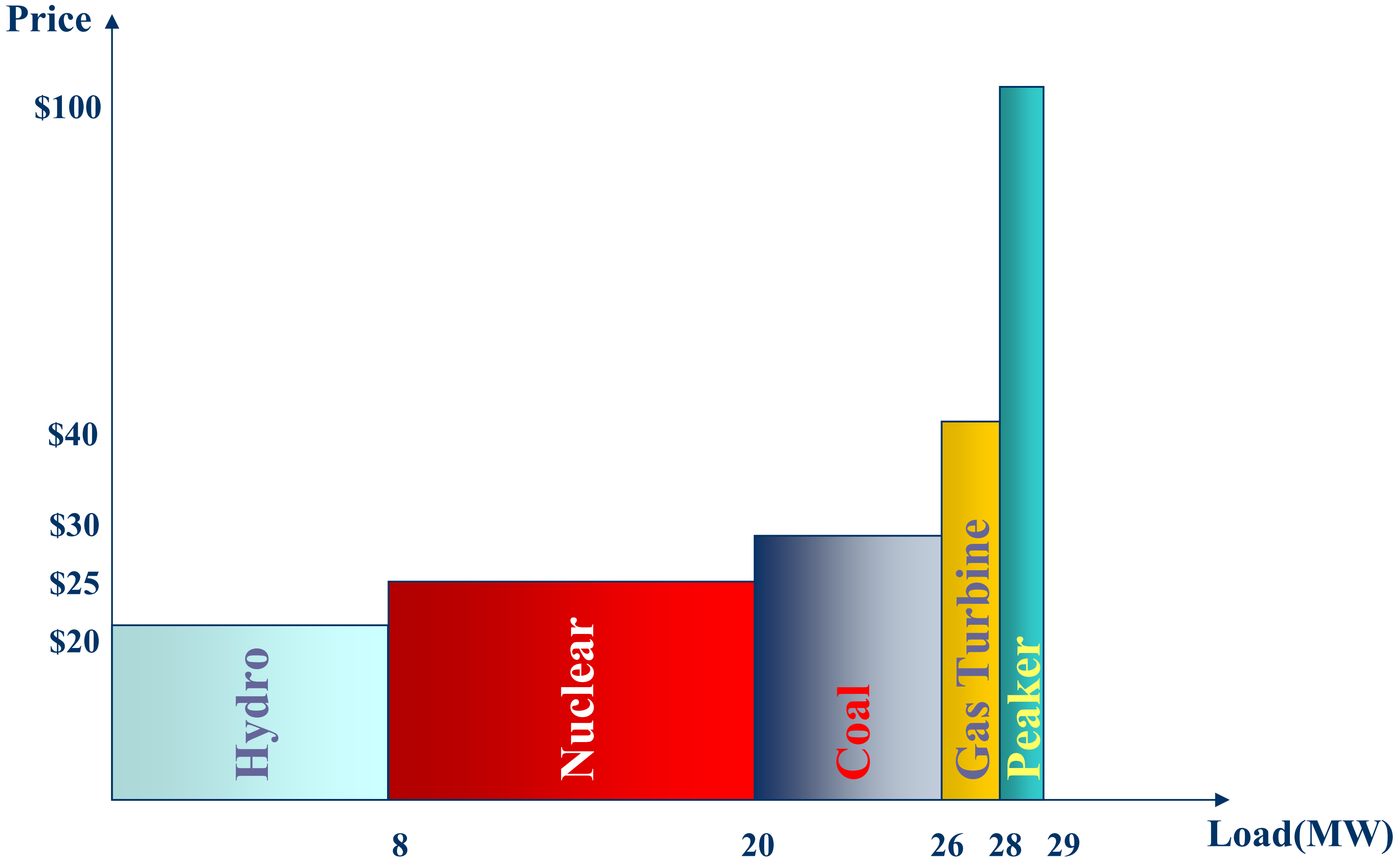


Why Is Electricity Different?

- Electricity cannot be stored
- Demand for electricity is inelastic
- Electricity produced must dispatched

What appears to be a complication can be a modeling advantage..

Stack-Based Pricing



What Would We Want in a Price Model

- What do we want to use the model for?
- Price spikes
- Two distinct price regimes
- Prices don’t drift indefinitely
- Seasonal pattern of price spikes

A two-regime switching model can incorporate these characteristics

For a discussion of the modelling philosophy and early implementation, see MD, L. Anderson et al. IEEE Transactions on Power Systems 17(2): 257-264 2002

A Two-Regime Switching Model

- Switching variable controls the process
- What controls the switching variable?
  - When do spikes typically occur?
  - Seasonal (summer, winter)
  - Some spiking in shoulder months as well

The α-Ratio and Probability of A Spike

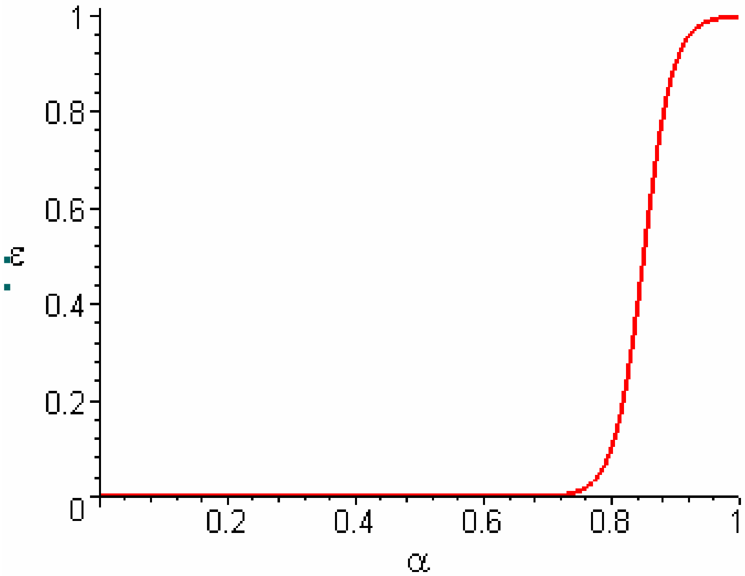
- The primary driver of the switching variable is

$$\alpha(t) = \frac{\mathrm{Load}(t)}{\mathrm{Capacity}(t)} = \frac{\mathrm{Demand}(t)}{\mathrm{Supply}(t)}$$

- The following should be true:

$$\lim_{\alpha \to 0} \Pr(\mathit{price\ spike}) = 0$$

$$\lim_{\alpha \to 1} \Pr(\mathit{price\ spike}) = 1$$

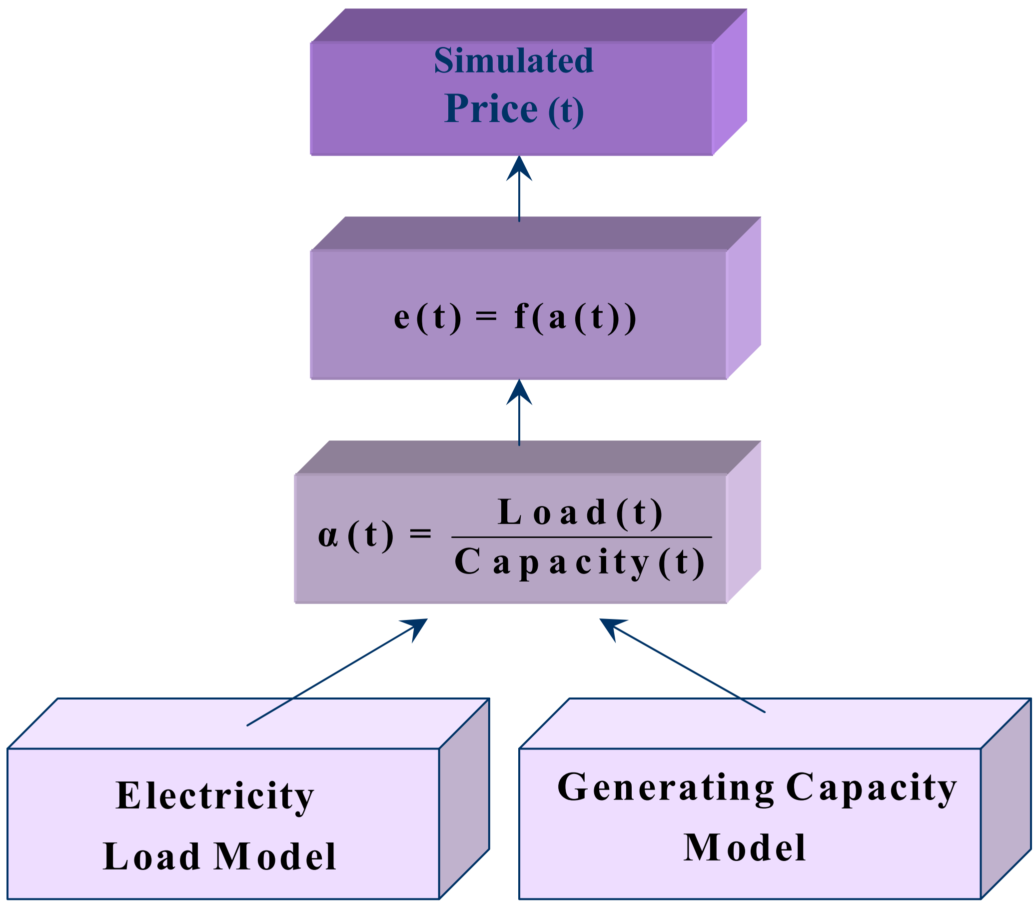


Pr(High price) vs. α

- The probability of a spike increases rapidly near

$$\alpha = 0.85$$

A Hybrid Model



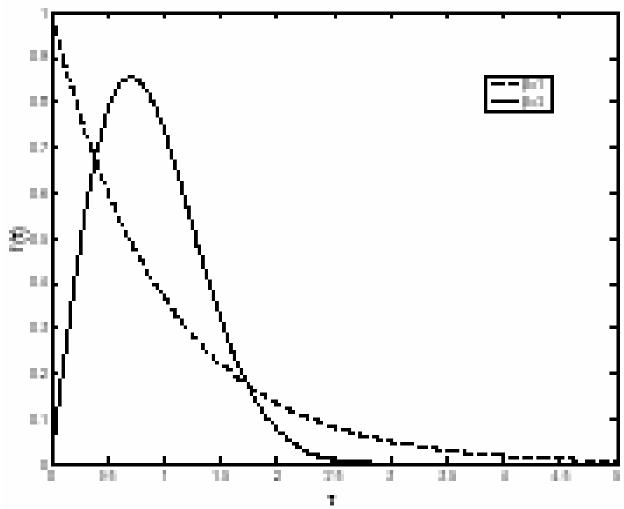
Modelling Generating Capacity

- Generating system has fixed maximum capacity
- Available operating capacity is the maximum, less;
  - Planned (maintenance) outages
  - Unplanned (forced) outages
- Build a probabilistic model of system-wide capacity
  - Aggregate exponential
  - Sequential simulation
  - Aggregate Weibull

Modelling Unplanned Outages

- Each generating unit has Weibull distributed TTF and TTR
- Weibull CDF is given by:

$$\Pr(t > D) = 1 - e^{-\left(\frac{D}{\eta_i}\right)^{\beta_i}}$$



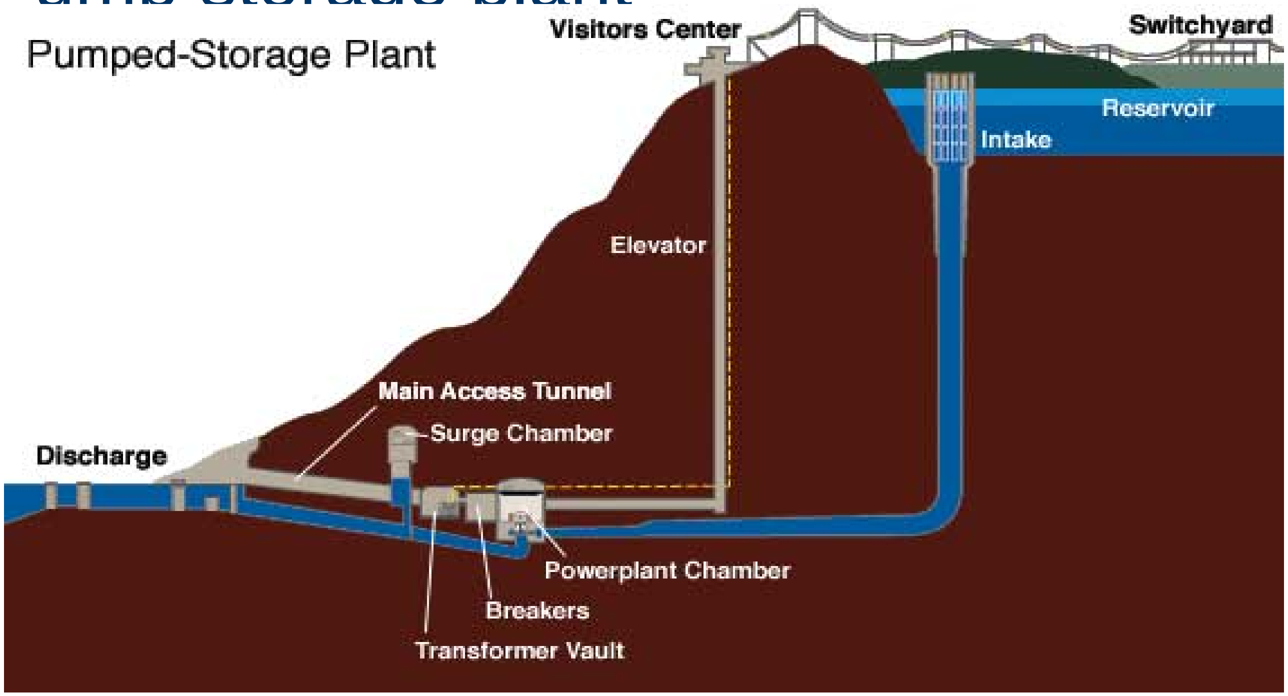
Weibull Probability Distributions

Pump Storage Facilities

- Conversion of mechanical to electrical energy is efficient
- Can get 80% round trip efficiency from electricity → running water → electricity
- So pump water when power price is low
- Use water to run turbine when power price is high
- What is the best way of doing this?

Pump Storage

- Pump storage plant



Stochastic Optimal Control

- Valuation and Optimal Operation of electric power plants in competitive markets
- Continuous time model for power prices including Poisson jumps
- Price dynamics

$$dP = \mu_1(P,t)dt + \sigma_1(P,t)dX_1 + \sum_{k=1}^{N} \gamma_k(P,t,J_k)dq_k,$$

where μ, σ and the γ_k can be any arbitrary functions of price and/or time.

For detailed discussion, see M. Thompson, MD & H. Rasmussen (2004), *Operations Research* **52**, 546-562.

The PIDE

- Merton-style portfolio optimization problem
- Plus lots of engineering fluid mechanics
- Leads to PIDE with initial and boundary conditions:

$$V_t + \frac{1}{2}\sigma(P)V_{pp} + \mu(P,t)V_p - \frac{3600c}{20000}V_h - (r + \lambda_{up}(P) + \lambda_{down}(P))V + H(c,h)P$$

$$+\lambda_{up}(P)\int_{-\infty}^{\infty} V(J_1,h,t)\frac{1}{100\sqrt{2\pi}}\exp\left(\frac{-(S-700)^2}{2(10)^2}\right)dJ_1$$

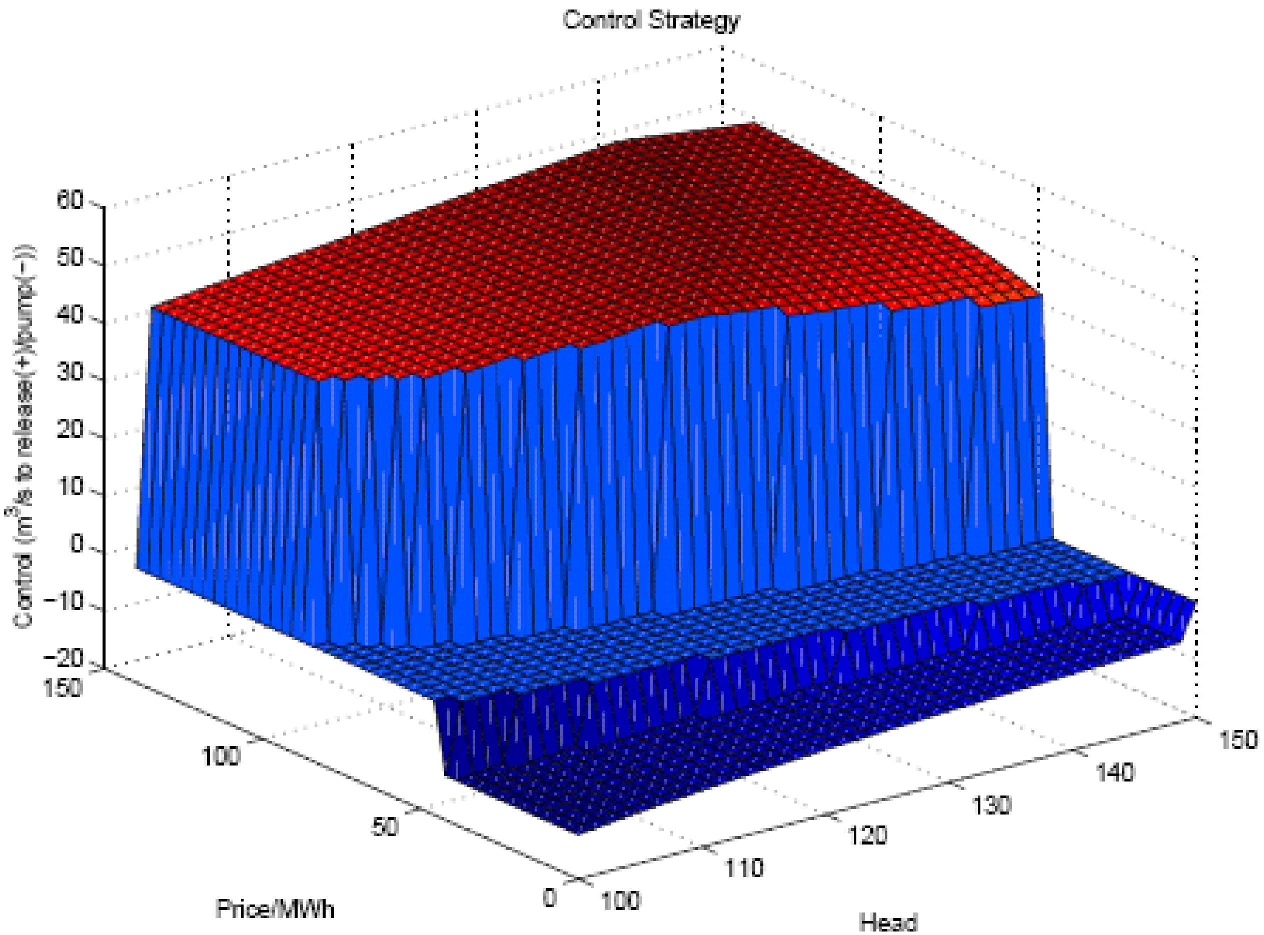
$$+\lambda_{down}(P)\int_{-\infty}^{\infty} V(J_2,h,t)\frac{1}{10\sqrt{2\pi}}\exp\left(\frac{-(S-100)^2}{2(10)^2}\right)dJ_2 = 0,$$

Initial condition: $V(P,h,T) = 0$,

Boundary conditions: $V_{PP} \to 0$ (for P large),

$$V_{PP} \to 0 \ (as\ P \to 0).$$

The control surface



Lessons Learned

- Practitioners know a lot of details and the modelling process of leaving details out to get to the essentials MUST include them not only to tap this knowledge but also to improve buy in.
- Best to talk to people at the “right” level in a company (even better if this is supported by senior leaders)
- Despite years of hiring quants, “Business” organizations are still typically less technical than “Technology” organizations and the relationship must be managed accordingly
- Best to have a single person who “owns” the problem
- Need to “pay dues”
- Need to expand definition of academic project success: (Publication can sometimes be a challenge, placing students is not)