

# **Industrial Mathematics: One Canadian Perspective Part 1**

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Canada-China Workshop in Industrial Math,  
BIRS, August 2007

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# Range of Projects with industrial collaborators

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- **Property & Casualty Insurance Compensation Corporation (started 2006, ongoing)**
- Princess Margaret Hospital (started 2005, ongoing)
- Department of National Defence (Navy) (started 2005, ongoing)
- Environment Canada (started 2007, ongoing)
- IBM Toronto software Lab (started 2004, ongoing)
- Bank of Canada (started 2006, ongoing)
- Ontario Power Generation (2000-2002)
- Dydex Ltd (2003)
- Canadian Energy Wholesalers Inc (Jan-Feb 2007)
- Waterloo Maple Inc (2006)

## Property & Casualty Insurance

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- Collaboration with Dr. Sharon Wang , Dr. Lindsay Anderson & Mr. Darrell Leadbetter (PACICC)
- Project supported by grant from PACICC supplemented by a MITACS internship for Dr. Wang

## General Insurance

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- Insurance is a form of risk management to hedge against potential future financial loss;
- Policyholders substitute a small, defined payment (premium) for a large, uncertain loss;
- The insurers pool the premiums to pay for the losses;
- Insurers: collect premium, pay claims (risk pooling);
- Premiums paid are invested until required to provide for claims and operating expenses;
- Insurers' revenues are generated from premiums and investment income
- Types of Insurance:
  - Life;
  - health;
  - property & casualty (other than life and health)

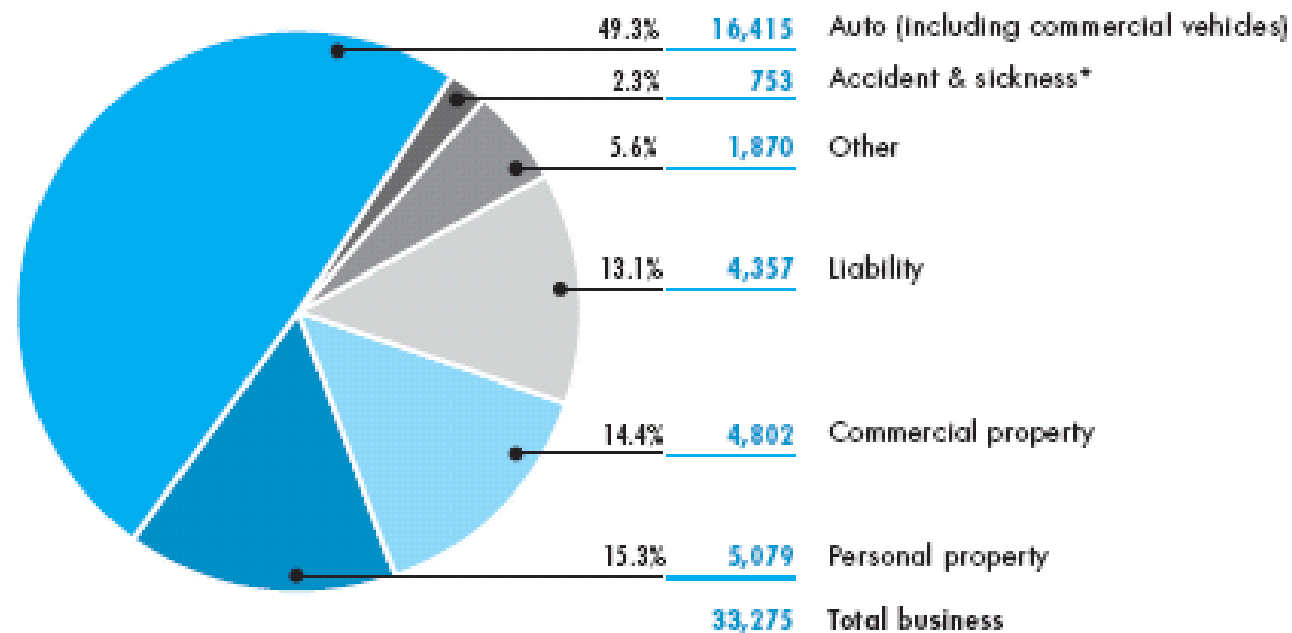
## Property and Casualty Insurance in Canada

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- Assets: \$99 billion;
- Net premium: \$33 billion;
- Number of insurers: 217;
- Unlike banking and life insurance,
  - Significant degree of foreign ownership (64%);
  - More fragmented:
    - No one has more than 10% of market;
    - 10 companies control 60%;
    - Competitive market
- Reinsurance: \$1.9 billion (8% of total industry)
- Profitability: underwriting (loss)+investments (gain)
- Policyholder protection: PACICC

## Products of P&C Insurance

- Provides coverage of all risks other than life and health;



*Consumers' concerns:*

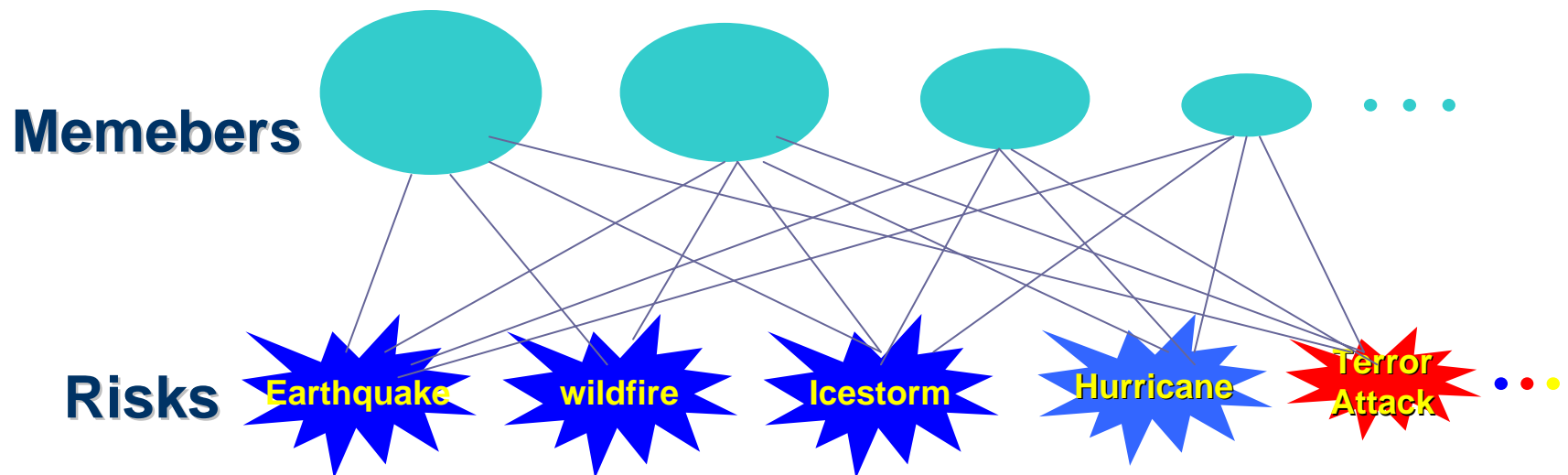
*Will insurance contracts be fulfilled and eligible claims be paid?*

## About PACICC

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- Property and Casualty Insurance Compensation Corporation
- help maintain public confidence in P/C insurance industry.
- monitor all the members' insolvency risks;
- assess surviving members when insolvency happens
- Compensation claims, protect policy holders
- Member's risks:
  - Earthquake (Vancouver, Montreal)
  - storm,
  - ice storm,
  - hurricane,
  - industry disaster
  - wild fire

## Understanding PACICC Members' Risks



- Risks to Members' financial reserves (correlated, idiosyncratic)
- Model natural disaster risk using extreme value theory
- Dependence structure of individual members on a given natural disaster
- Incorporating the correlation structure of individual risks
- Model of overall PACICC risk



# Why Do Insurers Fail?

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- Inadequate pricing
- Deficient loss reserves
- Specific risk exposure

Dibra, S. and Leadbetter, D. (2007). *Why Insurers Fail*. PACICC

## Our Model

- Solvency test:

$$E_i(t+1) = E_i(t) + P_i(t) - C_i(t) + I_i(t) - D_i(t)$$

↙
↘
↓
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Equity
Premium written
Claim Loss
Disaster Loss
Investment Income

$$\Rightarrow D_i(t) = \gamma \frac{C_i(t)}{\sum C_i(t)} CAT = \gamma \frac{C_i(t)}{\sum C_i(t)} \sum_{k=1}^{CN} CS_k$$

$$E_i(t+1) \geq \alpha A_i(t) \rightarrow \text{Assets}$$

- Contagion effect:

$$E_i(t+1) = E_i(t+1) - \bar{L}(t+1) \times \frac{P_i(t+1)}{\sum_{k=1}^N P_k(t+1)}$$

Liabilities of failed companies

- Two parameters:

$\alpha$  - solvency level;

$\gamma$  - severeness of disasters

# Risk Management

## VI. Natural disasters – major multiple-payment occurrences

Date and place	Event	PROPERTY			AUTOMOBILE			TOTAL		
		# of claims	Loss (\$000)	Average paid (\$)	# of claims	Loss (\$000)	Average paid (\$)	# of claims	Loss (\$000)	Adjusted for inflation 2005 \$000
1992 July 31 Calgary	Hailstorm	4,285	12,098	2,823	6,324	9,980	1,578	10,609	22,078	28,305
1992 July 31 Toronto	Flooding	993	4,596	4,628	341	302	885	1,334	4,898	6,279
1992 Aug. 28 Alberta	Hailstorm	1,060	3,594	3,390	905	1,669	1,844	1,965	5,263	6,747
1992 Aug. 28 Elmira, Aurora, Ontario	Flooding	1,137	4,292	3,776	92	56	609	1,229	4,348	5,574
1992 Sept. 1 Alberta	Hailstorm	1,457	4,611	3,165	1,628	2,810	1,726	3,085	7,421	9,514
1992 Oct. 6-7 Avalon, NL	Wind	3,549	7,487	2,109	641	729	1,137	4,190	8,216	10,533
1992 Nov. 12-13 southern Ontario	Wind	18,259	35,209	1,928	1,048	1,228	1,172	19,307	36,437	46,714
1992 Nov. 12-13 Quebec	Wind	5,624	10,106	1,797	1,412	1,950	1,381	7,036	12,056	15,456
1993 March 13-14 Quebec	Storm	6,280	11,814	1,881	3,440	6,633	1,928	9,720	18,447	23,351
1993 July 25-Aug. 14 Winnipeg	Flooding	21,264	184,837	8,692	----	not available	----	----	184,837	233,971
1993 July 29-30 Alberta	Hailstorm	759	7,078	9,322	673	1,038	1,542	1,432	8,116	10,273
1993 July 29 Saskatchewan	Flooding	2,741	5,383	1,964	----	not available	----	----	5,383	6,814
1993 July 29-30 Quebec	Flooding	1,366	7,624	5,581	----	not available	----	----	7,624	9,651
1994 Jan. 16-17 southern Ontario	Flooding	3,289	11,759	3,576	1,042	1,386	1,330	4,331	13,145	16,431
1994 Jan. 28 southern Ontario	Storms	1,781	5,470	3,072	579	780	1,346	2,360	6,250	7,813
1994 May 18 southern Manitoba	Storms	2,141	8,260	3,859	----	not available	----	----	8,260	10,325
1994 May 22 Saskatchewan	Storms	5,048	8,666	1,717	----	not available	----	----	8,666	10,833
1994 June 18 southern Alberta	Hailstorm	1,653	4,284	2,592	2,032	3,979	1,958	3,685	8,263	10,329
1994 Aug. 4 Salmon Arm, BC	Storm	2,026	10,225	5,048	----	not available	----	----	10,225	12,781
1994 Aug. 4 Aylmer, Quebec	Tornado	484	6,730	13,904	81	181	2,235	565	6,911	8,639
1994 Aug. 27 southern Manitoba	Hailstorm	1,908	4,845	2,540	1,559	3,267	2,096	3,467	8,112	10,140
1994 Aug. 28 southern Ontario	Storms	1,236	6,772	5,479	366	448	1,223	1,602	7,219	9,024
1995 June 6-9 Calgary	Flooding	1,596	20,292	12,714	298	472	1,584	1,894	20,764	25,322
1995 July 4 Edmonton	Hailstorm	1,785	14,083	7,890	424	615	1,450	2,209	14,698	17,924
1995 July 10 southern Alberta	Hailstorm	3,093	17,997	5,819	3,634	8,392	2,309	6,727	26,389	32,182
1995 July 13-15 southern Ontario	Storms	12,762	36,448	2,856	11,074	16,991	1,534	23,836	53,439	65,170
1995 July 17 Calgary	Hailstorm	9,843	32,887	3,341	8,996	19,417	2,158	18,839	52,304	63,785
1995 July 30 southern Manitoba	Storm	1,582	4,971	3,142	1,645	3,497	2,126	3,227	8,468	10,327
1995 Aug. 26 Regina	Storm	3,309	12,294	3,715	----	not available	----	----	12,294	14,993
1995 Oct. 5-6 Hamilton, Ontario	Storm	5,141	15,916	3,096	262	409	1,560	5,403	16,325	19,909
1996 July 16 Winnipeg	Flood/hailstorm	21,027	94,250	4,482	24,444	52,575	2,151	45,471	146,825	176,898
1996 July 16-18 Calgary	Hailstorm	15,845	91,981	5,805	10,778	27,110	2,515	26,623	119,091	143,483
1996 July 24-25 Calgary	Hailstorm	15,742	71,400	4,536	6,005	13,822	2,302	21,747	85,222	102,677
1996 July 19-20 Saguenay, Quebec	Flooding	5,289	203,579*	[*]	1,172	3,580	3,054	6,461	207,159	249,589
1996 July 23 Outaouais, Quebec	Wind/hailstorm	330	1,257	3,809	192	314	1,639	522	1,571	1,893
1996 Aug. 8 Ottawa	Flooding	2,341	19,705	8,417	246	552	2,243	2,587	20,257	24,406
1996 Aug. 8 Outaouais, Estrie, Quebec	Flooding	1,459	7,729	5,297	65	153	2,353	3,207	7,882	9,496
1996 Nov. 9 Montreal and Quebec City	Flooding	9,813	75,684	7,713	131	356	2,721	9,944	76,040	91,614
1997 Feb. 27 Niagara Peninsula, Ontario	Wind	13,080	22,130	1,692	1,194	1,646	1,378	14,274	23,776	28,305
1997 April 6-7 Sudbury, Ontario	Flooding	2,553	20,426	8,000	65	132	2,042	2,618	20,558	24,474
1997 July 14-15 Chambly, Quebec	Flooding	3,118	29,865	9,579	----	not available	----	----	29,865	35,554

# Risk Management

natural disasters – major multiple-payment occurrences ... continued

Date and place	Event	PROPERTY			AUTOMOBILE			TOTAL		
		# of claims	Loss (\$000)	Average paid (\$)	# of claims	Loss (\$000)	Average paid (\$)	# of claims	Loss (\$000)	Adjusted for inflation 2005 \$000
1998 Jan. southern Quebec	Icestorm	658,575	1,376,945	2,090	71,594	99,101	1,384	730,169	1,476,046	1,757,198
1998 Jan. eastern Ontario	Icestorm	59,351	161,558	2,722	2,994	4,004	1,337	62,345	165,562	197,098
1998 July 4-9 Calgary	Hailstorm	10,582	46,645	4,408	7,999	23,097	2,887	18,581	69,742	83,026
1998 Sept. 26-27 Niagara Peninsula, Ontario	Wind	7,294	26,184	3,590	19,013	37,219	1,958	26,307	63,403	75,480
1999 Jan. southern Ontario	Snowstorms	28,608	99,821	3,489	10,736	20,200	1,881	39,344	120,021	141,201
1999 June 5 Drummondville, Quebec	Hailstorm	2,315	14,225	6,145	2,921	6,330	2,167	5,236	20,555	24,182
1999 July 5-6 Quebec	Wind	20,680	38,289	1,851	2,726	5,032	1,846	23,406	43,321	50,966
1999 July 28 Atlantic provinces	Flooding	1,661	15,251	9,181	858	505	589	2,519	15,756	18,537
1999 Sept. 22 Atlantic provinces	Flooding	1,912	14,391	7,527	1,566	1,257	803	3,478	15,648	18,409
2000 May 12 southern Ontario	Storm	14,653	123,773	8,447	1,676	4,348	2,594	16,329	128,121	143,956
2000 July 7 southern Manitoba	Storm	2,389	18,559	7,768	-----	not available	-----	2,389	18,559	20,853
2000 July 14 Pine Lake, Alberta	Tornado	1,907	12,617	6,616	1,248	5,299	4,246	3,155	17,916	20,130
2000 Aug. 9 Calgary	Storm	4,624	21,229	4,591	2,681	6,829	2,547	7,305	28,058	31,526
2000 Oct. 30 Sydney, Nova Scotia	Flooding	346	3,909	11,303	86	101	1,174	432	4,010	4,505
2000 Dec. 17 Atlantic provinces	Windstorm	5,478	18,149	11,267	1,773	1,607	906	7,251	19,756	22,198
2001 Feb. 1 Atlantic provinces	Snowstorm	2,584	12,321	9,432	2,140	1,425	666	4,724	13,746	15,274
2001 Feb. 8 southern Ontario	Storm	12,606	52,135	4,136	1,865	1,943	1,042	14,471	54,078	60,086
2001 Feb. 8 Quebec	Storm	19,077	53,843	2,822	-----	not available	-----	19,077	53,843	59,826
2001 July 13 Alberta	Storm	5,000	16,964	3,393	2,582	8,549	3,311	7,582	25,513	28,348
2001 July 28 Edmonton	Storm	2,424	22,068	9,104	606	1,834	3,026	3,030	23,902	26,557
2001 Sept. 19 Atlantic provinces	Flooding	701	6,201	8,848	64	161	2,523	765	6,362	7,069
2001 Dec. 14 southwestern British Columbia	Windstorm	4,658	27,035	5,804	-----	not available	-----	4,658	27,035	30,039
2002 Jan. 31 southern Ontario	Windstorm	6,837	31,972	4,676	1,842	2,536	1,377	8,679	34,508	37,921
2002 March 9 Ontario	Windstorm	26,466	107,774	4,072	2,078	3,215	1,547	28,544	110,989	121,966
2002 June 8 southern Alberta	Flooding	3,502	42,828	12,229	-----	not available	-----	3,502	42,828	47,064
2002 June 10 southern Ontario	Storm	5,311	53,377	10,049	467	566	1,211	5,778	53,943	59,278
2002 July 26 southwestern Ontario	Storm	4,253	59,220	13,925	733	839	1,145	4,986	60,060	65,999
2003 March 30-April 1 New Brunswick	Flooding	663	4,695	7,083	-----	not available	-----	663	4,695	4,995
2003 March 30-April 1 Newfoundland	Flooding	118	711	6,021	-----	not available	-----	118	711	757
2003 March 30-April 1 Prince Edward Island	Flooding	81	628	7,713	-----	not available	-----	81	628	668
2003 March 30-April 1 Nova Scotia	Flooding	1,865	18,557	9,952	-----	not available	-----	1,865	18,557	19,742
2003 Aug. 11-12 Alberta	Wind/hailstorm	4,013	24,180	6,026	2,401	9,385	3,909	6,414	33,565	35,708
2003 Aug. 11-12 Saskatchewan	Wind/hailstorm	5,702	29,055	5,095	-----	not available	-----	5,702	29,055	30,910
2003 Summer British Columbia	Forest fires	3,385	200,000	59,084	-----	not available	-----	3,385	200,000	212,766
2003 Sept. 28-29 Prince Edward Island	Hurricane	1,251	6,665	5,327	-----	not available	-----	1,251	6,665	7,090
2003 Sept. 28-29 Nova Scotia	Hurricane	23,077	132,671	5,749	-----	not available	-----	23,077	132,671	141,139
2004 July 2-11 Edmonton	Hailstorm	12,955	166,000	12,814	-----	not available	-----	12,955	166,000	171,134
2004 July 15 Calgary	Hailstorm	4,200	21,500	5,119	-----	not available	-----	4,200	21,500	22,165
2004 July 15 Peterborough, Ontario	Flood	5,154	87,303	16,939	-----	not available	-----	5,154	87,303	90,003
2004 Sept. 9 eastern Ontario	Rainstorm	5,587	57,600	10,310	-----	not available	-----	5,587	57,600	59,381
2005 June 6-8 & June 17-19 Alberta	Flooding	-----	300,000	-----	-----	not available	-----	-----	not available	300,000
2005 June 20-30 & July 1-2 Manitoba	Flooding	-----	60,000	-----	-----	not available	-----	-----	not available	60,000
2005 July 5 & Sept. 26 Quebec	Rainstorms	-----	57,000	-----	-----	not available	-----	-----	not available	57,000
2005 Aug. 19 Ontario	Wind/rainstorm	-----	500,000	-----	-----	not available	-----	-----	not available	500,000

\*One company reported three commercial property claims totaling \$108 million.

# Risk Management

## Modelling catastrophes

Catastrophe losses:

$$CAT = \sum_{k=1}^{CN} CS_k$$

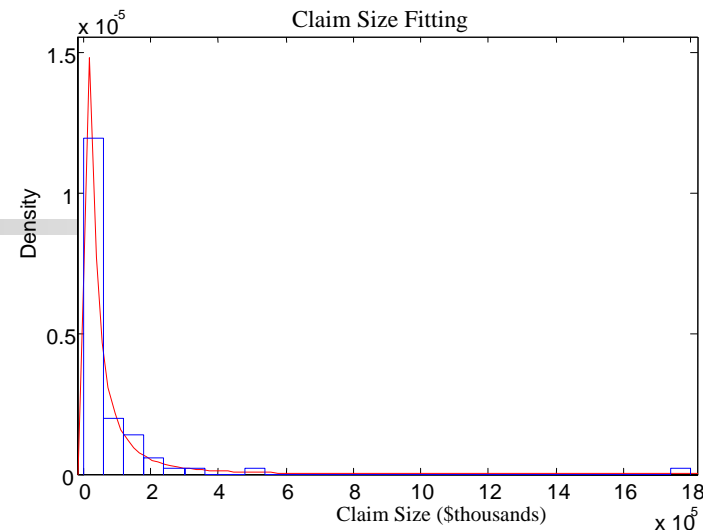
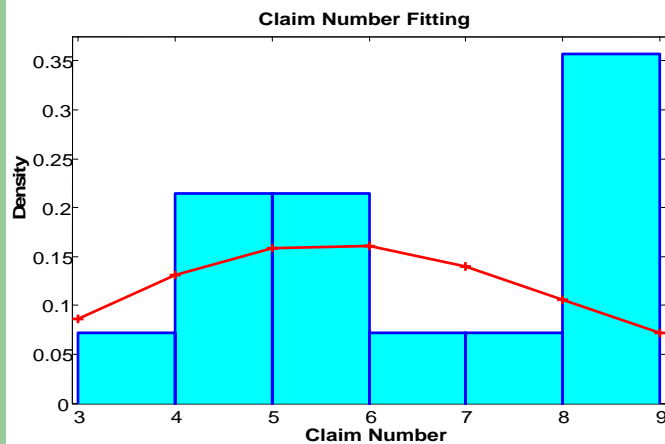
CN – Claim Number

CS – Claim Size



Poisson Distribution

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \lambda \text{ is the mean}$$



Lognormal distribution:

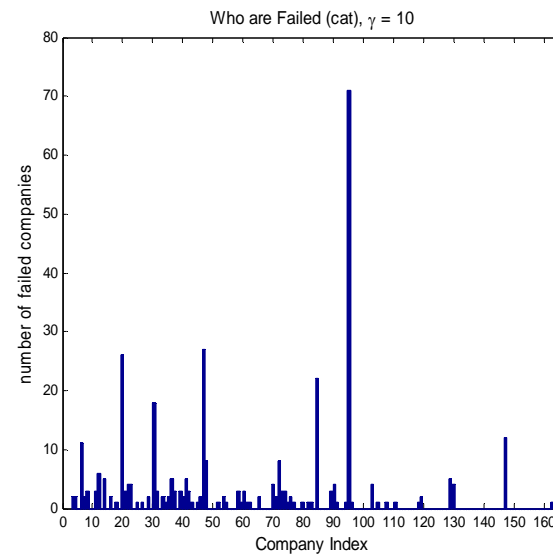
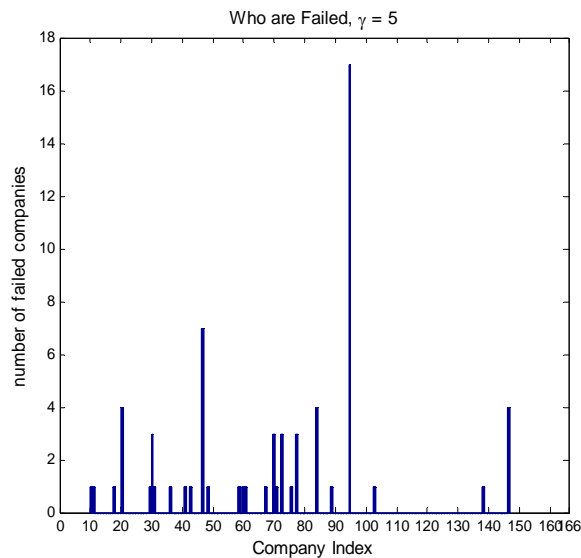
$$l(x) = \frac{\exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right)}{x\sigma\sqrt{2\pi}}$$

$$\mu = 10.263, \sigma = 1.34508.$$

$$\begin{aligned} \text{Mean} &= e^{\left(\mu + \frac{1}{2}\sigma^2\right)} \\ &= \$70,798,600 \end{aligned}$$

# Simulation Results

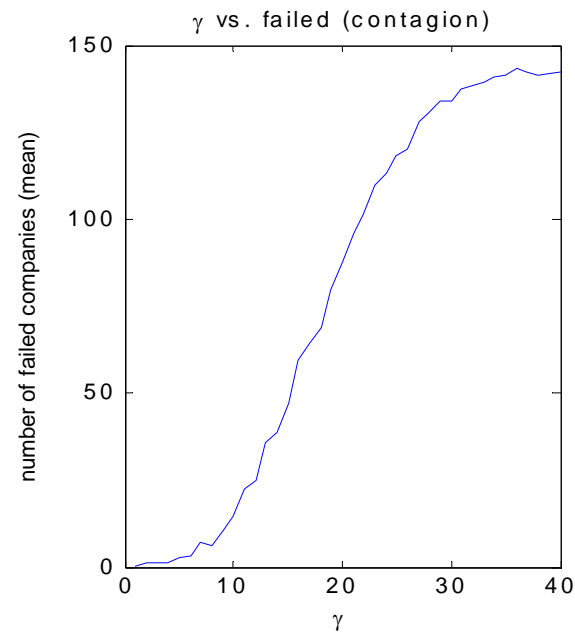
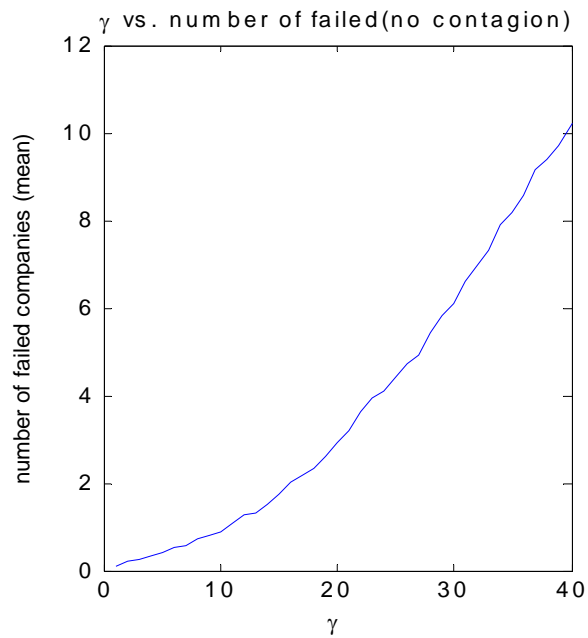
- Identify insurers' financial weakness (1000 simulation runs)



Companies # 95, 47, 20, 84 have weaker financial reserves.

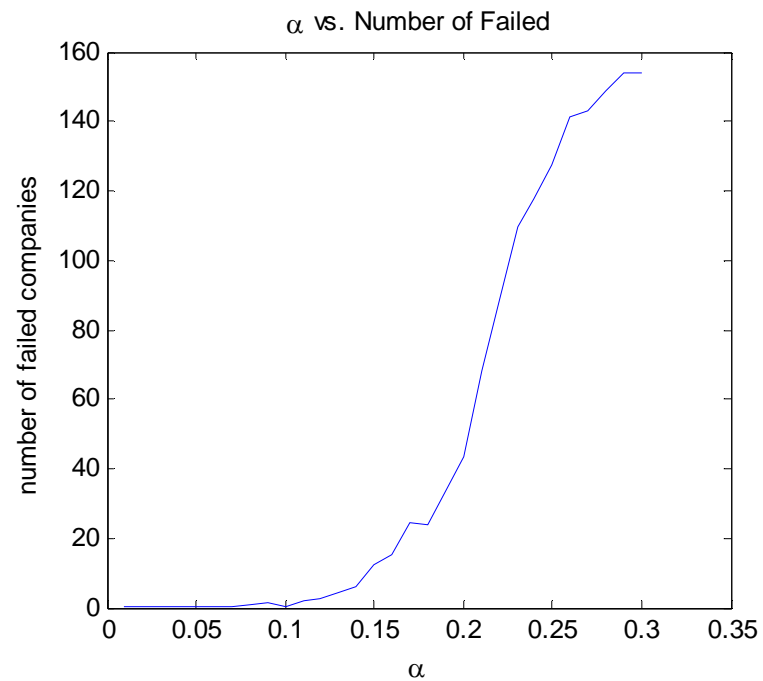
# Contagion Phenomena

Number of failed companies (with and without contagion)



# Solvency Level

Company fails when  $E_i(t+1) \leq \alpha A_i(t)$



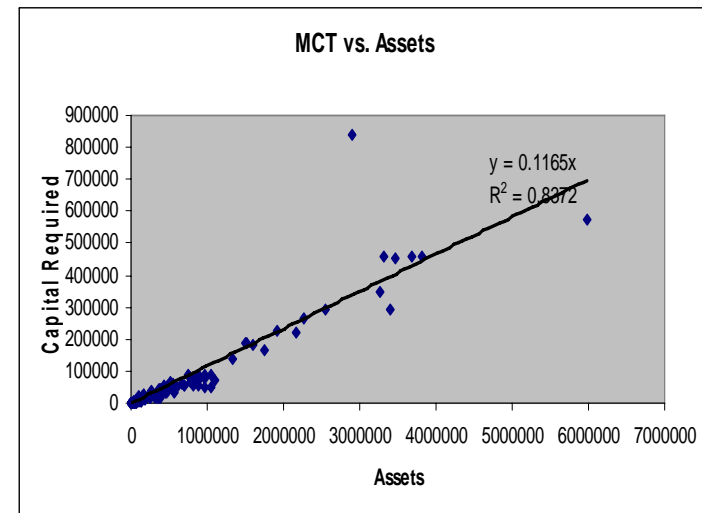
Failure start to show when  $\alpha > 0.1$



# Match Federal Regulations

- Capital requirement by federal regulators (Minimum Capital Test (MCT))

<b>Capital Available</b>		
Equity .....	01	973225
Subordinated Indebtedness and Redeemable Preferred Shares .....	03	30000
Investments - Adjustment to Market .....	05	71154
Less: Assets with a Capital Requirement of 100% .....	07	45599
<b>Total Capital Available .....</b>	<b>09</b>	<b>1028780</b>
<b>Capital Required</b>		
Balance Sheet Assets .....	20	271240
Unearned Premiums/Unpaid Claims .....	22	270968
Catastrophes .....	24	194
Reinsurance Ceded to Unregistered Insurers .....	26	2492
Off-Balance Sheet Exposures .....	28	1758
<b>Minimum Capital Required .....</b>	<b>29</b>	<b>546652</b>
<b>Excess Capital Available over Capital Required</b>		
(line 09 minus line 29) .....	89	482128
<b>Line 09 as a % of line 29 .....</b>	<b>90</b>	<b>188.20</b>



- Define  $\alpha$  by MCT (Linear regression)

$$\alpha = 0.1165$$

## Risk Pooling

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- Potential Counterparties
  - Life Insurance
  - Credit Union (correlation: interest rate level won't matter, volatility matters)
  - Mutual funds
  - Other guaranteed funds

## Risk Management

# Canada Deposit Insurance Corporation (CDIC)

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- **Members:** Banks, trust companies and loan companies
- **Number of members:** 81(34 banks+43 Trust and loan co + 4 Provincial institutions)
- **Capital:** \$1.3 Billion
- **2004 Revenues:** \$124M (\$93M premium +\$31M interest, etc.)
- **The last member failed:** in 1996
- **Total failed since 1967:** 43
- **Premium:** differential premiums (1/6 ~ 1/48) of 1%
- **Total Insured Deposits:** \$375.6 billion
- **Risks:** Higher interest rates, higher energy prices, real estate market and financial market uncertainty, strengthening Canadian dollar, terror attack, consumer debt defaults, fraud issues (money, credit cards)

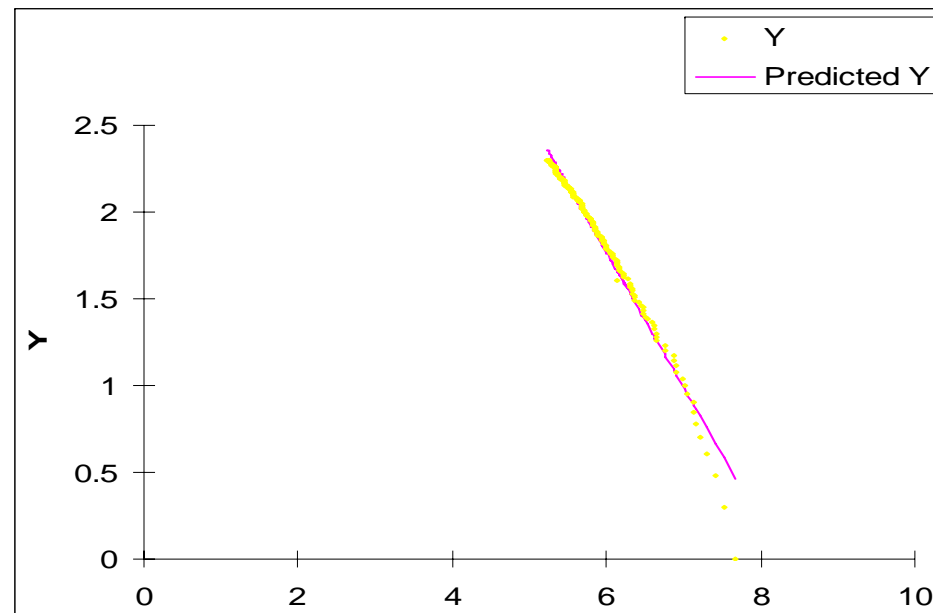
## Credit Union Central of Canada (CUCC)

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- Provincially regulated
- Members are protected in each provinces
  - British Columbia: Financial Institutions Commission
  - Alberta: Credit Union Deposit Guarantee
  - Saskatchewan: Credit Union Deposit Guarantee Co.
  - Manitoba: Credit Union Deposit Guarantee Co.
  - Ontario: Deposit Insurance Corporation of Ontario
  - Quebec: Quebec Deposit Insurance Board (QDIB)
  - New Brunswick: Credit Union Stabilization Fund
  - Nova Scotia: Credit Union Deposite Insurance Corporation
  - PEI: Credit Union Central of PEI
- Size: Each one of above comparable to PACICC

## Something from USA P/C Insurance

- Approximately 3000 insurers
- Top 200 insurers worth 94% (exclude insurers < 200?)
- Evidence for power law from US data



# Lessons Learned

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- Practitioners know a lot of details and the modelling process of leaving details out to get to the essentials MUST include them not only to tap this knowledge but also to improve buy in.
- Best to talk to people at the “right” level in a company (even better if this is supported by senior leaders)
- Despite years of hiring quants, “Business” organizations are still typically less technical than “Technology” organizations and the relationship must be managed accordingly
- Best to have a single person who “owns” the problem
- Need to “pay dues”
- Need to expand definition of academic project success: (Publication can sometimes be a challenge, placing students is not)