



Banff International Research Station

for Mathematical Innovation and Discovery

Low-dimensional Topology and Number Theory

October 21–26, 2007

MEALS

*Breakfast (Buffet): 7:00–9:00 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations.

SCHEDULE

Sunday

- 16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
Lecture rooms available after 16:00 (if desired)
- 17:30–19:30** Buffet Dinner, Sally Borden Building
- 20:00** Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and small assortment of snacks available on a cash honour-system.

Monday

- 7:00–8:45** Breakfast
- 8:45–9:00** Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
- 9:00–10:00** F. Rodriguez Villegas (Texas), *Explicit examples of Beilinson's construction for elliptic curves*
- 10:00–10:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11:00** C. Sinclair (Boulder), *The geometry of polynomials with all roots on the unit circle*
- 11:10–11:40** K. Petersen (Queens), *Cusps and Congruence Subgroups*
- 11:40–13:00** Lunch
- 13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
- 14:00–15:00** T. Chinburg (UPenn), *Length spectra and hyperbolic manifolds*
- 15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 15:30–16:00** J. Lewis (Alberta), *Regulator currents on Milnor complexes*
- 16:10–17:10** S. Williams (U. South Alabama), *Knot invariants from solenoid automorphisms*
- 17:30–19:30** Dinner

Tuesday

- 7:00–9:00** Breakfast
9:00–10:00 N. Boston (Wisconsin), *Random Pro-p Groups and Random Galois Groups*
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:30 A. Reid (Texas), *The geometry and topology of arithmetic hyperbolic 3-manifolds*
11:30 Group photo; meet on the front steps of Corbett Hall
11:30–13:30 Lunch
14:00–15:00 T. Le (Georgia Tech), *On integrality of quantum invariants*
15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:00 E. Hironaka (Florida State), *Cyclotomic factors of Coxeter polynomials*
16:15–16:45 M. Culler (UIC), *A numerical method for computing A-polynomials*
17:00–17:30 M. Lalin (Alberta), *Mahler measures under variations of the base group*
17:30–19:30 Dinner

Wednesday

- 7:00–9:00** Breakfast
9:00–10:00 S. Gukov (Caltech), *A state sum model for $SL_2(\mathbb{C})$ Chern-Simons theory*
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:00 M. Macasieb (UBC), *Derived Arithmetic Fuchsian Groups of Genus Two*
11:10–11:40 F. Hajir (UMass Amherst), *Tame Galois Groups, A Survey*
11:30–13:30 Lunch
Free Afternoon
17:30–19:30 Dinner

Thursday

- 7:00–9:00** Breakfast
9:00–10:00 F. Calegari (Northwestern), *Rational homology growth in certain towers of hyperbolic 3-manifolds.*
10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
10:30–11:00 A. Sikora (SUNY Buffalo), *Idele theory for 3-manifolds*
11:10–11:40 M. Trifkovic (Fordham/Victoria), *Elliptic Curves over Imaginary Quadratic Fields, Homology of 3-Manifolds, and p-adic Constructions of Rational Points*
11:30–13:30 Lunch
14:00–15:00 D. Zagier (MPIM Bonn/College de France), *Modularity and three-manifolds*
15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:00 T. Mattman (CSU Chico), *Trace fields of two families of knot complements*
16:10–17:10 M. Morishita (Kyushu), *Chern-Simons theory and Hida-Mazur variation*
17:30–19:30 Dinner
19:30–20:30 C. Zickert (Columbia), *Complex volume and the extended Bloch group*

Friday

- 7:00–9:00** Breakfast
9:00–9:30 I. Kofman (CUNY Staten Island), *Mahler measure of Jones polynomials*
9:40–10:10 A. Champanerkar (U. South Alabama), *On links with cyclotomic Jones polynomials*
10:20–10:50 Coffee Break, 2nd floor lounge, Corbett Hall
10:50–11:30 Informal Discussion
11:30–13:30 Lunch
Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **



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Name of 2007 5-day Workshop

Date of 2007 5-day Workshop

ABSTRACTS

Speaker: **Nigel Boston** (U. of Wisconsin)

Title: *Random Pro- p Groups and Random Galois Groups*

Abstract: Dunfield and Thurston studied how the distribution of finite quotients of a random g -generator g -relator abstract group compares with that of the fundamental group of a random 3-manifold obtained from a genus- g Heegard splitting. We consider analogous questions for random g -generator g -relator pro- p groups and for Galois groups of maximal pro- p extensions unramified away from a finite set S of primes with $|S| = g$.

Speaker: **Frank Calegari** (Northwestern U.)

Title: *Rational Homology Growth in certain towers of hyperbolic 3-manifolds.*

Abstract: We discuss the question of obtaining upper and lower bounds for certain (exhaustive) covers of hyperbolic three manifolds using techniques from automorphic forms and non-commutative Iwasawa theory.

Speaker: **Abhijit Champanerkar** (U. South Alabama)

Title: *On links with cyclotomic Jones polynomials*

Abstract: We show that any infinite sequence of distinct prime alternating links with cyclotomic Jones polynomials must have unbounded hyperbolic volume. This is joint work with Ilya Kofman.

Speaker: **Ted Chinburg** (U. of Pennsylvania)

Title: *Length spectra and hyperbolic manifolds*

Abstract: This talk will survey some recent work on the question of when the length spectrum of a hyperbolic manifold determines and is determined by its commensurability class. Two particular cases are an affirmative result for compact arithmetic hyperbolic 3-manifolds (joint with E. Hamilton, D. Long and A. Reid) and a negative result for hyperbolic 5-manifolds by Prasad and Rapinchuk. Eventually I will discuss quotients of products of hyperbolic upper half planes and half spaces. These provide a test case for combining techniques from the theory of algebraic groups and algebraic number theory.

Speaker: **Marc Culler** (UIC)

Title: *A numerical method for computing A -polynomials.*

Abstract: I will describe a heuristic method for computing A -polynomials of hyperbolic knot manifolds. While a rigorous proof of correctness would require other techniques, this method does have the features that it is almost completely automatic and produces plausible results for a large class of examples, including all 8 crossing knot complements and all of the hyperbolic knot complements that can be triangulated with no more than 7 tetrahedra, as tabulated by Callahan-Dean-Weeks and Champanerkar-Kofman-Patterson.

The computations are done in floating point arithmetic, using polynomial homotopy continuation and discrete Fourier transforms.

Speaker: **S. Gukov** (Caltech)

Title: "A State Sum Model for $SL(2, \mathbb{C})$ Chern-Simons Theory"

Abstract: TBA

Speaker: **Farshid Hajir** (UMass Amherst)

Title: *Trace fields of two families of knot complements*

Abstract: Let p be a prime number, K a finite extension of \mathbb{Q} , and S a finite set of primes of K . Let K_S be the compositum of all finite Galois extensions of K of p -power order which are unramified outside S . The Galois group $G_{K,S} = \text{Gal}(K_S/K)$ is quite mysterious if S contains no primes above p . A deep conjecture of Fontain and Mazur sheds some light on their structure. I will give a brief survey of recent advances in the study of $G_{K,S}$ most notably by Labute.

Speaker: **Ilya Kofman** (CUNY Staten Island)

Title: *Mahler measure of Jones polynomials*

Abstract: For any link, the Mahler measure of the Jones polynomial converges under twisting to that of a certain multivariable polynomial which depends on the number of strands we twist at each site. This is consistent with the convergence of hyperbolic volume under Dehn surgery. This is joint work with Abhijit Champanerkar.

Speaker: **Matilde Lalin** (Alberta)

Title: *Mahler measures under variations of the base group*

Abstract: The Mahler measure of an n -variable polynomial P is the integral of $\log|P|$ over the n -dimensional unit torus T^n with the Haar measure. For one-variable polynomials, this is a natural quantity that appears in different problems such as Lehmer's question.

We consider a generalization of the Mahler measure to elements in group rings, in terms of the Lueck-Fuglede-Kadison determinant. We study the variation of the Mahler measure when the base group changes. In particular, we discuss the Mahler measure over infinite groups as limit of Mahler measures over finite groups. This is joint work with O. Dasbach.

Speaker: **Thang Le** (Georgia Tech)

Title: *On integrality of quantum invariants*

Abstract: We discuss our joint work with Habiro on the strong integrality of quantum invariants (for general Lie algebras) of integral homology 3-spheres. We show that the quantum invariants have a very strong integrality: they take values in an interesting rings (the Habiro ring) which can be considered as a ring of "analytic functions on the set of roots of unity" (p -adic convergence of Taylor series is obvious in the ring). It follows that the quantum invariants are always algebraic integers, and much more. An extension to rational homology sphere is also discussed.

Speaker: **James Lewis** (Alberta)

Title: *Regulator currents on Milnor complexes*

Abstract: Let X be a projective algebraic manifold. For integers $k, m \geq 0$ we consider a cycle group $CH_M^k(X, m)$ defined in terms of the Zariski cohomology of the sheaf of Milnor K -groups on X , and a corresponding twisted variant $CH_{TM}^k(X, m)$. We construct real logarithmic type maps ("real regulators") on $CH_{(TM)}^k(X, m)$ with values in Hodge cohomology, and investigate their properties.

Speaker: **Melissa Macasieb** (UCB)

Title: *Derived Arithmetic Fuchsian Groups of Genus Two*

Abstract: A hyperbolic 2- or 3-orbifold M is called arithmetic if $M = H^2/G$ or H^3/G where G is an arithmetic Fuchsian or Kleinian group, respectively. Considerable work has been done in the last two decades classifying these groups. I will explain how arithmetic Fuchsian and Kleinian groups can be described in terms of quaternion algebras over number fields and discuss special properties of these groups.

Finally, I will outline the classification of derived arithmetic Fuchsian groups of genus 2 and end with some open questions.

Speaker: **Thomas Mattman** (CSU Chico)

Title: *Trace fields of two families of knot complements*

Abstract: We give a very preliminary overview of projects that aim to characterise the trace fields of two families of knot complements. The first (joint with Macasieb) follows the approach given by Reid and Walsh for two-bridge knots to investigate the trace fields of pretzel knots. The second (joint with Indurskis) generalises work of Hoste and Shanahan on Twist knots to other fillings of the Whitehead Link.

Speaker: **Masanori Morishita** (Kyushu)

Title: *Chern-Simons variation and Hida-Mazur theory*

Abstract: Based on the analogy between knots and primes, I will discuss the variation of Chern-Simons invariants over the deformation space of hyperbolic structures on a knot complement from the view point of the analogy with Hida-Mazur theory on the deformation of Galois representations and p -adic modular L -functions. One of our results is to describe the variation of Chern-Simons invariants in terms of Deligne cohomology of the deformation curve.

Speaker: **Kathleen Petersen** (Queens)

Title: *Cusps and Congruence Subgroups*

Abstract: It is a classical result that the existence of non-congruence subgroups of $PSL(2, \mathbb{Z})$ can be seen topologically, by counting cusps. We discuss generalizations of this to $PSL(2, A)$ (and $PGL(2, A)$) where A is an integer ring of a global field.

Speaker: **Alan Reid** (U. of Texas)

Title: *The geometry and topology of arithmetic hyperbolic 3-manifolds*

Abstract: This talk will survey some recent work on understanding the geometry and topology of arithmetic hyperbolic 3-manifolds. This is motivated in part by the virtual Haken and positive first Betti number conjectures.

Speaker: **Fernando Rodriguez Villegas** (Texas)

Title: *Explicit examples of Beilinson's construction for elliptic curves*

Abstract: I will discuss how one can make explicit the construction of elements of K_2 of a (modular) elliptic curve E over \mathbb{Q} due to Beilinson. I will use the curve $E = X_1(11)$ as the main example. As part of the outcome we obtain infinitely many two-variable polynomials P with rational coefficients for which we can prove $m(P) = BL'(X_1(11), 0)$ for some $B \in \mathbb{Q}$, where $m(P)$ is the Mahler measure of P . The constant B can be intrinsically computed in advance, up to an integer factor c . This constant c is (appropriately for this workshop) a topological invariant. It can be bounded a priori but I don't know a closed expression for it.

Speaker: **Adam S. Sikora** (SUNY Buffalo)

Title: *Idele theory for 3-manifolds*

Abstract: *We discuss the analogies between number fields and 3-manifolds postulated by Arithmetic Topology with particular attention to group actions. We discuss the progress and the difficulties in constructing the theory ideles for 3-manifolds.*

Speaker: **Christopher Sinclair** (Boulder)

Title: *The geometry of polynomials with all roots on the unit circle*

Abstract: The set of monic polynomials with all roots on the unit circle demonstrates a surprising amount of structure. We will demonstrate some of this structure using low-dimensional examples and show a connection with random matrix theory. (Parts of this work are joint with Kathleen Petersen and Jeff Vaaler).

Speaker: **Mak Trifkovic** (Fordham/Victoria)

Title: *Elliptic Curves over Imaginary Quadratic Fields, Homology of 3-Manifolds and p -adic Constructions of Rational Points*

Abstract: We present a conjectural p -adic construction of rational points on elliptic curves over imaginary quadratic fields, along with some numerical evidence. The main ingredient in the construction are classes in (relative) homology of certain arithmetic quotients of the upper half-space. The hope is that hyperbolic geometry can to some extent mitigate the absence of the usual tools of algebraic geometry which makes the field mostly conjectural.

Speaker: **Susan Williams** (U. South Alabama)

Title: *Knot invariants from solenoid automorphisms*

Abstract: The Pontryagin dual of a finitely generated $\mathbb{Z}[t, t^{-1}]$ -module is a solenoid, with an automorphism dual to multiplication by t . We study classical and twisted Alexander invariants of knots in this dual universe, giving topological interpretations of periodic points and topological entropy of these algebraic dynamical systems. The leading coefficient of the (twisted) Alexander polynomial accounts for p -adic components of entropy. We are particularly interested in twisting by representations into $SL_2(\mathbb{Q})$ derived from faithful $SL_2(\mathbb{C})$ representations for hyperbolic knots.

Speaker: **Christian Zickert** (Columbia U.)

Title: *Complex volume and the extended Bloch group*

Abstract: A hyperbolic manifold defines a fundamental class in the extended Bloch group. Evaluating a suitable extension of Rogers dilogarithm on this class gives the complex volume (Vol+iCS) of the manifold. We describe a new way of constructing the fundamental class, giving rise to a very fast algorithm for computing complex volume.

Speaker: **Don Zagier** (MPIM Bonn/College de France)

Title: *Modularity and three-manifolds*

Abstract: TBA.