Probabilistic Perspectives on Climate Dynamics

Adam Monahan

monahana@uvic.ca

School of Earth and Ocean Sciences
University of Victoria

Probability and Climate

- “Climate is what you expect,

Probability and Climate

- "Climate is what you expect, weather is what you get."
  - Robert A. Heinlein

UVic

Probability and Climate

- "Climate is what you expect, weather is what you get."
  - Robert A. Heinlein

$\Rightarrow$ "expectation" lies at heart of notion of climate

Probability and Climate

- "Climate is what you expect, weather is what you get."
  - Robert A. Heinlein

$\Rightarrow$ "expectation" lies at heart of notion of climate

$\Rightarrow$ this is a fundamentally probabilistic perspective

Probability and Climate

- “Climate is what you expect, weather is what you get.”
  - Robert A. Heinlein

$\Rightarrow$ “expectation” lies at heart of notion of climate

$\Rightarrow$ this is a fundamentally probabilistic perspective

- The ultimate goal of climate physics is the “measure” of the climate system, with emphasis on

UVic

Probability and Climate

- “Climate is what you expect, weather is what you get.”
  - Robert A. Heinlein

$\Rightarrow$ “expectation” lies at heart of notion of climate

$\Rightarrow$ this is a fundamentally probabilistic perspective

- The ultimate goal of climate physics is the “measure” of the climate system, with emphasis on
  - characterisation

UVic

Probability and Climate

- “Climate is what you expect, weather is what you get.”
  - Robert A. Heinlein

⇒ “expectation” lies at heart of notion of climate

⇒ this is a fundamentally probabilistic perspective

- The ultimate goal of climate physics is the “measure” of the climate system, with emphasis on
  - characterisation
  - physical understanding

UVic

Probability and Climate

- "Climate is what you expect, weather is what you get."
  - Robert A. Heinlein

$\Rightarrow$ "expectation" lies at heart of notion of climate

$\Rightarrow$ this is a fundamentally probabilistic perspective

- The ultimate goal of climate physics is the "measure" of the climate system, with emphasis on
  - characterisation
  - physical understanding
  - predictability
    (evolution of trajectory **and** response of measure to forcing)

UVic

Probability and Climate

- Toward this end, a natural language of climate physics is

Probability and Climate

- Toward this end, a natural language of climate physics is
  - dynamical systems

Probability and Climate

- Toward this end, a natural language of climate physics is
  - dynamical systems
  - stochastic processes

Probability and Climate

- Toward this end, a natural language of climate physics is
  - dynamical systems
  - stochastic processes
  - random dynamical systems (where they meet)

UVic

Probability and Climate

- Toward this end, a natural language of climate physics is
  - dynamical systems
  - stochastic processes
  - random dynamical systems (where they meet)
- Triple goals of probabilistic climate dynamics:

UVic

Probability and Climate

- Toward this end, a natural language of climate physics is
  - dynamical systems
  - stochastic processes
  - random dynamical systems (where they meet)
- Triple goals of probabilistic climate dynamics:
  1. characterise distributions of climate states

UVic

Probability and Climate

- Toward this end, a natural language of climate physics is
  - dynamical systems
  - stochastic processes
  - random dynamical systems (where they meet)
- Triple goals of probabilistic climate dynamics:
  1. characterise distributions of climate states
  2. understand how these distributions arise from the underlying physics

UVic

Probability and Climate

- Toward this end, a natural language of climate physics is
  - dynamical systems
  - stochastic processes
  - random dynamical systems (where they meet)
- Triple goals of probabilistic climate dynamics:
  1. characterise distributions of climate states
  2. understand how these distributions arise from the underlying physics
  3. use pdfs to improve forecasts/predictions

UVic

Overview

- Why is climate complex?

Overview

- Why is climate complex?
- Origins of stochasticity in the climate system.

Overview

- Why is climate complex?
- Origins of stochasticity in the climate system.
- Case Study I: Stochastic dynamics of sea-surface winds.

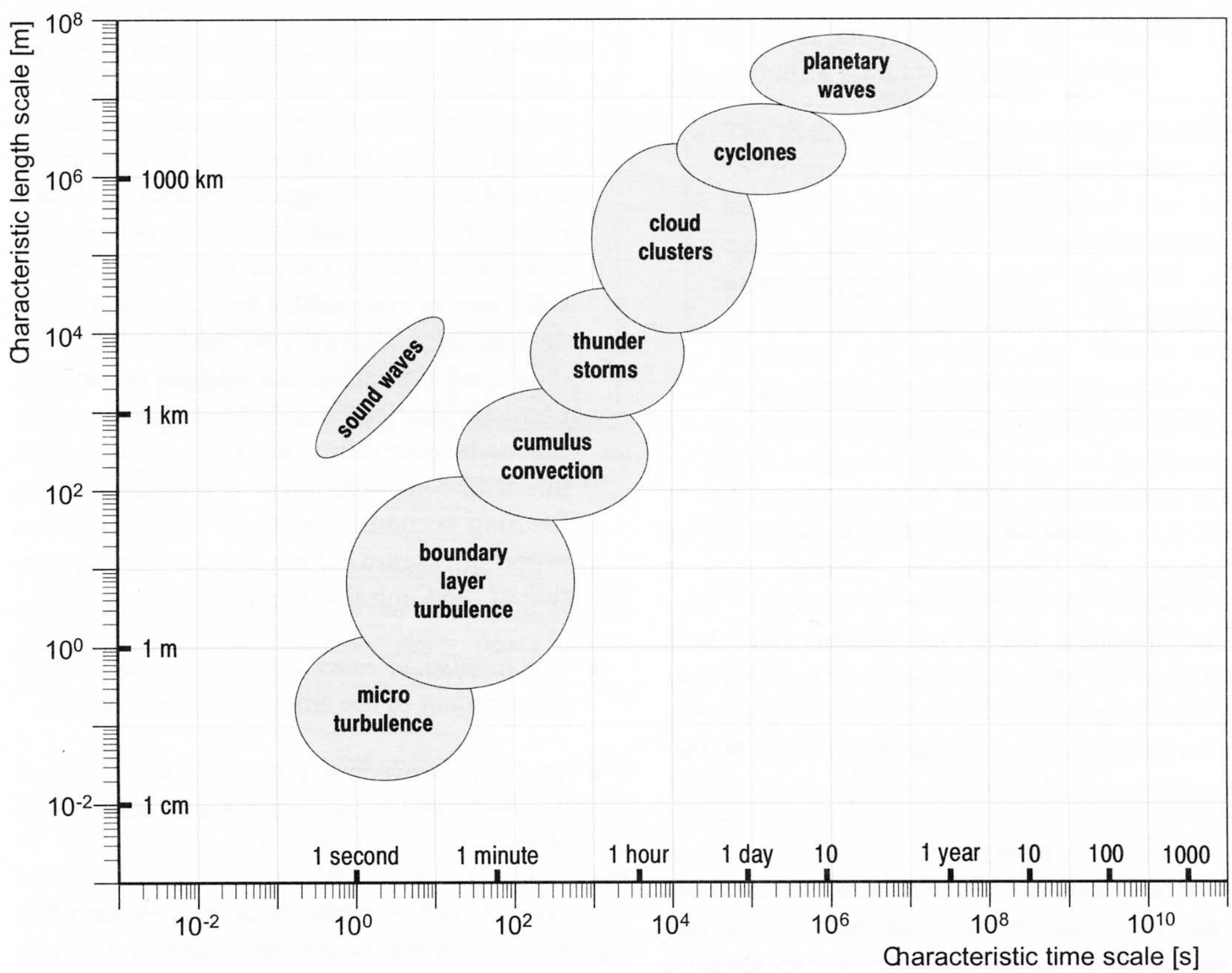
UVic

Overview

- Why is climate complex?
- Origins of stochasticity in the climate system.
- Case Study I: Stochastic dynamics of sea-surface winds.
- Case Study II: Stochastic dynamics of El Niño/Southern Oscillation.

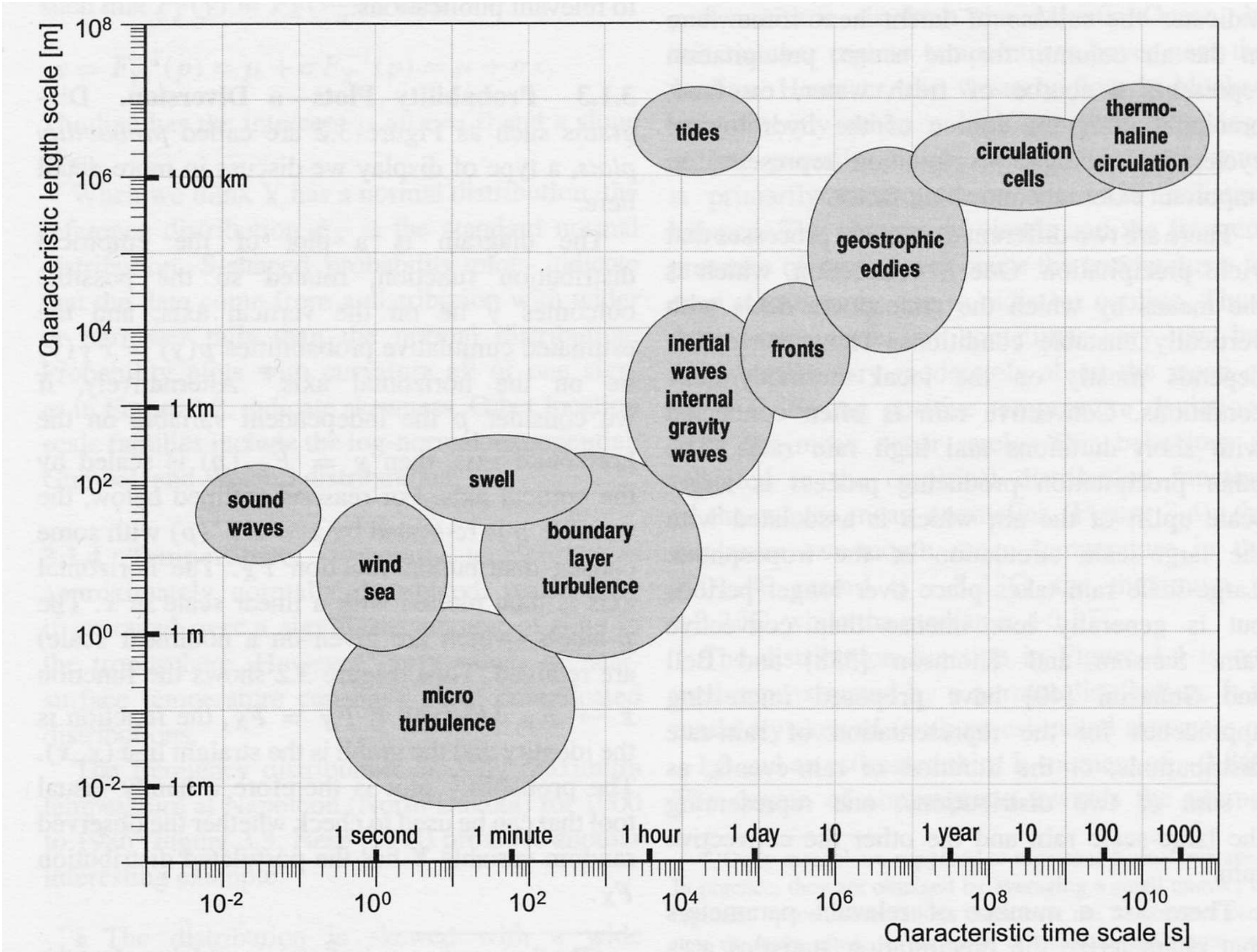
UVic

Why is Climate Complex? Coupling Across Scales



From von Storch and Zwiers, 1999

Why is Climate Complex? Coupling Across Scales



From von Storch and Zwiers, 1999

UVic

Why is Climate Complex? Coupling Across Scales

- Generic dynamical equation for climate state

Why is Climate Complex? Coupling Across Scales

- Generic dynamical equation for climate state

$$\frac{d\mathbf{z}}{dt} = L\mathbf{z} + \mathbf{N}(\mathbf{z}, \mathbf{z}) + \mathbf{F}$$

UVic

Why is Climate Complex? Coupling Across Scales

- Generic dynamical equation for climate state

$$\frac{d\mathbf{z}}{dt} = L\mathbf{z} + \mathbf{N}(\mathbf{z}, \mathbf{z}) + \mathbf{F}$$

- Decompose state into "slow" and "fast" variables $\mathbf{x}$ and $\mathbf{y}$

UVic

Why is Climate Complex? Coupling Across Scales

- Generic dynamical equation for climate state

$$\frac{d\mathbf{z}}{dt} = L\mathbf{z} + \mathbf{N}(\mathbf{z},\mathbf{z}) + \mathbf{F}$$

- Decompose state into "slow" and "fast" variables $\mathbf{x}$ and $\mathbf{y}$

$\Rightarrow$ coupled dynamics

UVic

Why is Climate Complex? Coupling Across Scales

- Generic dynamical equation for climate state

$$\frac{d\mathbf{z}}{dt} = L\mathbf{z} + \mathbf{N}(\mathbf{z},\mathbf{z}) + \mathbf{F}$$

- Decompose state into "slow" and "fast" variables $\mathbf{x}$ and $\mathbf{y}$

$\Rightarrow$ coupled dynamics

$$\begin{aligned}
\frac{d\mathbf{x}}{dt} &= L_{xx}\mathbf{x} + L_{xy}\mathbf{y} + N^{(x)}_{xx}(\mathbf{x},\mathbf{x}) + N^{(x)}_{xy}(\mathbf{x},\mathbf{y}) + N^{(x)}_{yy}(\mathbf{y},\mathbf{y}) + F_x \\
\frac{d\mathbf{y}}{dt} &= L_{yx}\mathbf{x} + L_{yy}\mathbf{y} + N^{(y)}_{xx}(\mathbf{x},\mathbf{x}) + N^{(y)}_{xy}(\mathbf{x},\mathbf{y}) + N^{(y)}_{yy}(\mathbf{y},\mathbf{y}) + F_y
\end{aligned}$$

UVic

Why is Climate Complex? Coupling Across Scales

- Generic dynamical equation for climate state

$$\frac{d\mathbf{z}}{dt} = L\mathbf{z} + \mathbf{N}(\mathbf{z},\mathbf{z}) + \mathbf{F}$$

- Decompose state into "slow" and "fast" variables $\mathbf{x}$ and $\mathbf{y}$

$\Rightarrow$ coupled dynamics

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= L_{xx}\mathbf{x} + L_{xy}\mathbf{y} + N_{xx}^{(x)}(\mathbf{x},\mathbf{x}) + N_{xy}^{(x)}(\mathbf{x},\mathbf{y}) + N_{yy}^{(x)}(\mathbf{y},\mathbf{y}) + F_x \\ \frac{d\mathbf{y}}{dt} &= L_{yx}\mathbf{x} + L_{yy}\mathbf{y} + N_{xx}^{(y)}(\mathbf{x},\mathbf{x}) + N_{xy}^{(y)}(\mathbf{x},\mathbf{y}) + N_{yy}^{(y)}(\mathbf{y},\mathbf{y}) + F_y\end{aligned}$$

- Averaging to project on "slow" dynamics retains "upscale" influence of eddies on resolved flow ("closure problem")

Why is Climate Complex? Coupling Across Scales

- Generic dynamical equation for climate state

$$\frac{d\mathbf{z}}{dt} = L\mathbf{z} + \mathbf{N}(\mathbf{z},\mathbf{z}) + \mathbf{F}$$

- Decompose state into "slow" and "fast" variables $\mathbf{x}$ and $\mathbf{y}$

$\Rightarrow$ coupled dynamics

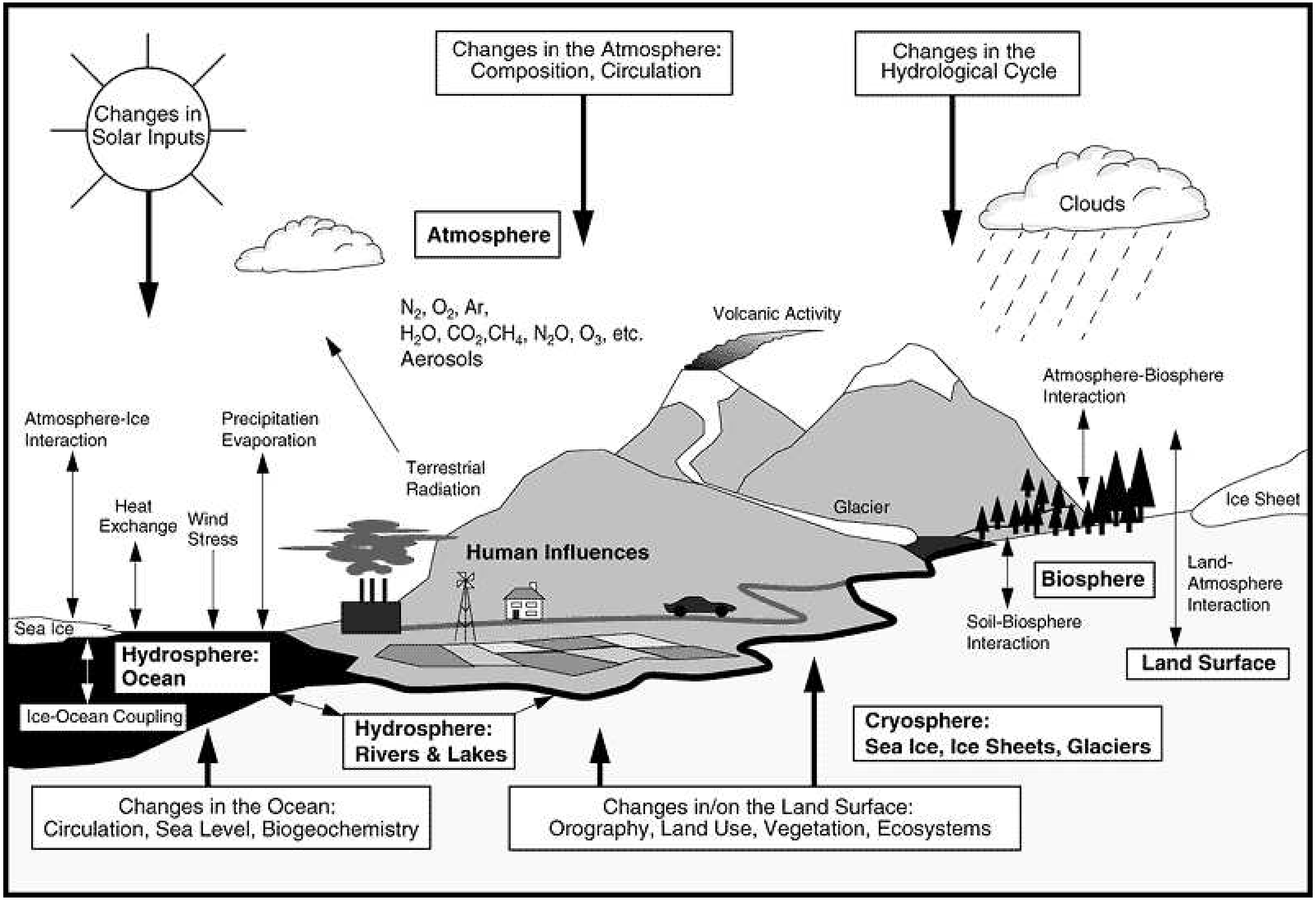
$$\begin{aligned}
\frac{d\mathbf{x}}{dt} &= L_{xx}\mathbf{x} + L_{xy}\mathbf{y} + N^{(x)}_{xx}(\mathbf{x},\mathbf{x}) + N^{(x)}_{xy}(\mathbf{x},\mathbf{y}) + N^{(x)}_{yy}(\mathbf{y},\mathbf{y}) + F_x \\
\frac{d\mathbf{y}}{dt} &= L_{yx}\mathbf{x} + L_{yy}\mathbf{y} + N^{(y)}_{xx}(\mathbf{x},\mathbf{x}) + N^{(y)}_{xy}(\mathbf{x},\mathbf{y}) + N^{(y)}_{yy}(\mathbf{y},\mathbf{y}) + F_y
\end{aligned}$$

- Averaging to project on "slow" dynamics retains "upscale" influence of eddies on resolved flow ("closure problem")

$$\frac{d\overline{\mathbf{x}}}{dt} = L_{xx}\overline{\mathbf{x}} + N^{(x)}_{xx}(\overline{\mathbf{x}},\overline{\mathbf{x}}) + \overline{N^{(x)}_{yy}(\mathbf{y},\mathbf{y})} + \overline{F_x}$$

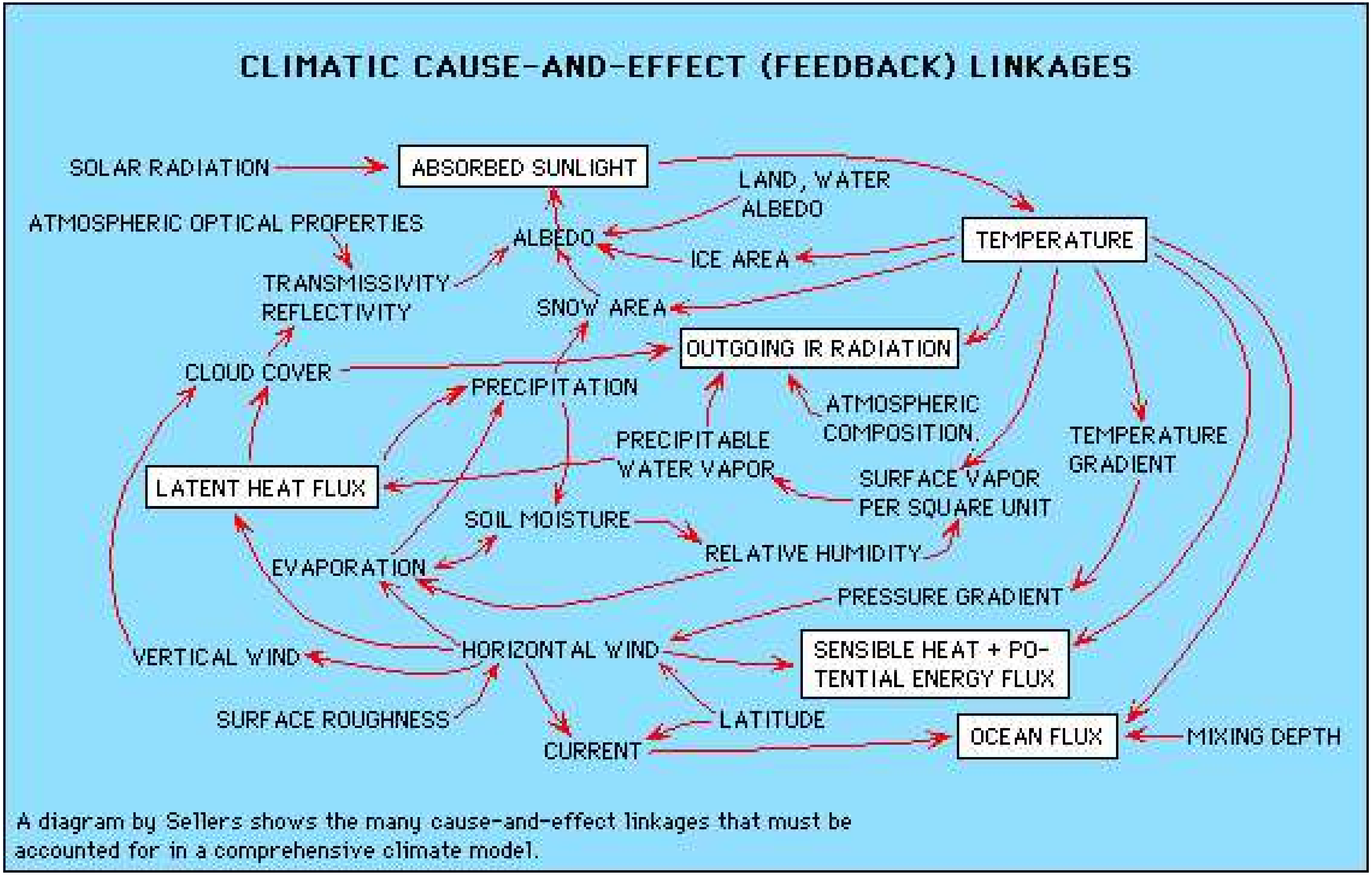
UVic

Why is Climate Complex? Coupling Across Systems



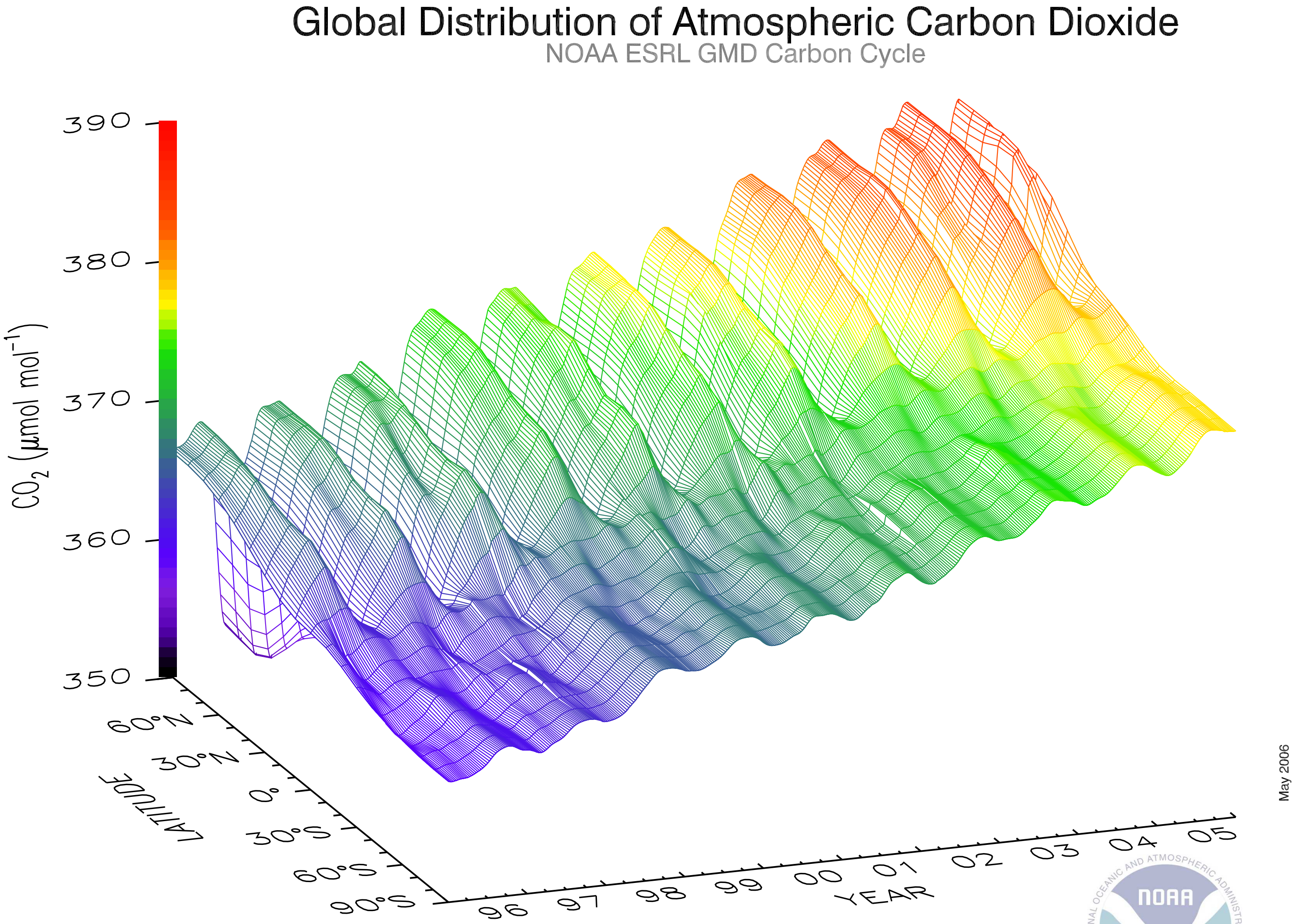
From IPCC Third Assessment Report

Why is Climate Complex? Feedback Loops



From http://eesc.columbia.edu/courses/ees/slides/climate/

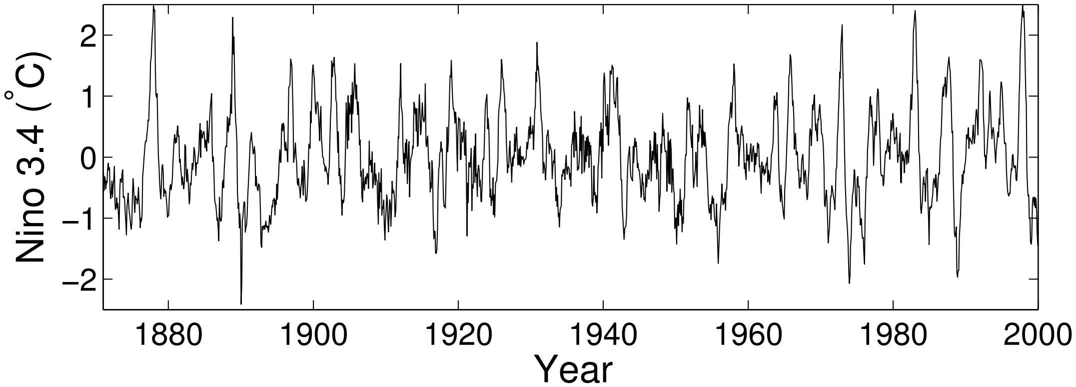
Why is Climate Complex? Non-Stationarity

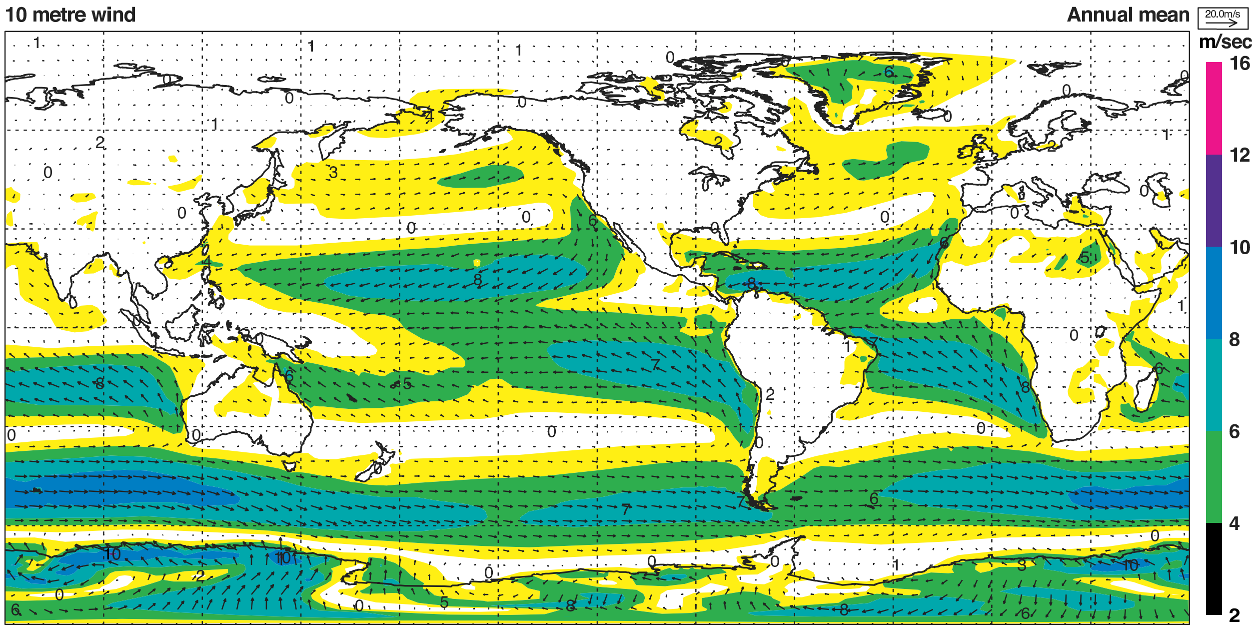


Three dimensional representation of the latitudinal distribution of atmospheric carbon dioxide in the marine boundary layer. Data from the GMD cooperative air sampling network were used. The surface represents data smoothed in time and latitude. Contact: Dr. Pieter Tans and Thomas Conway, NOAA ESRL GMD Carbon Cycle, Boulder, Colorado, (303) 497-6678 (pieter.tans@noaa.gov, http://www.cmdl.noaa.gov/ccgg).

UVic

Why is Climate Complex? Data Surfeit & Paucity

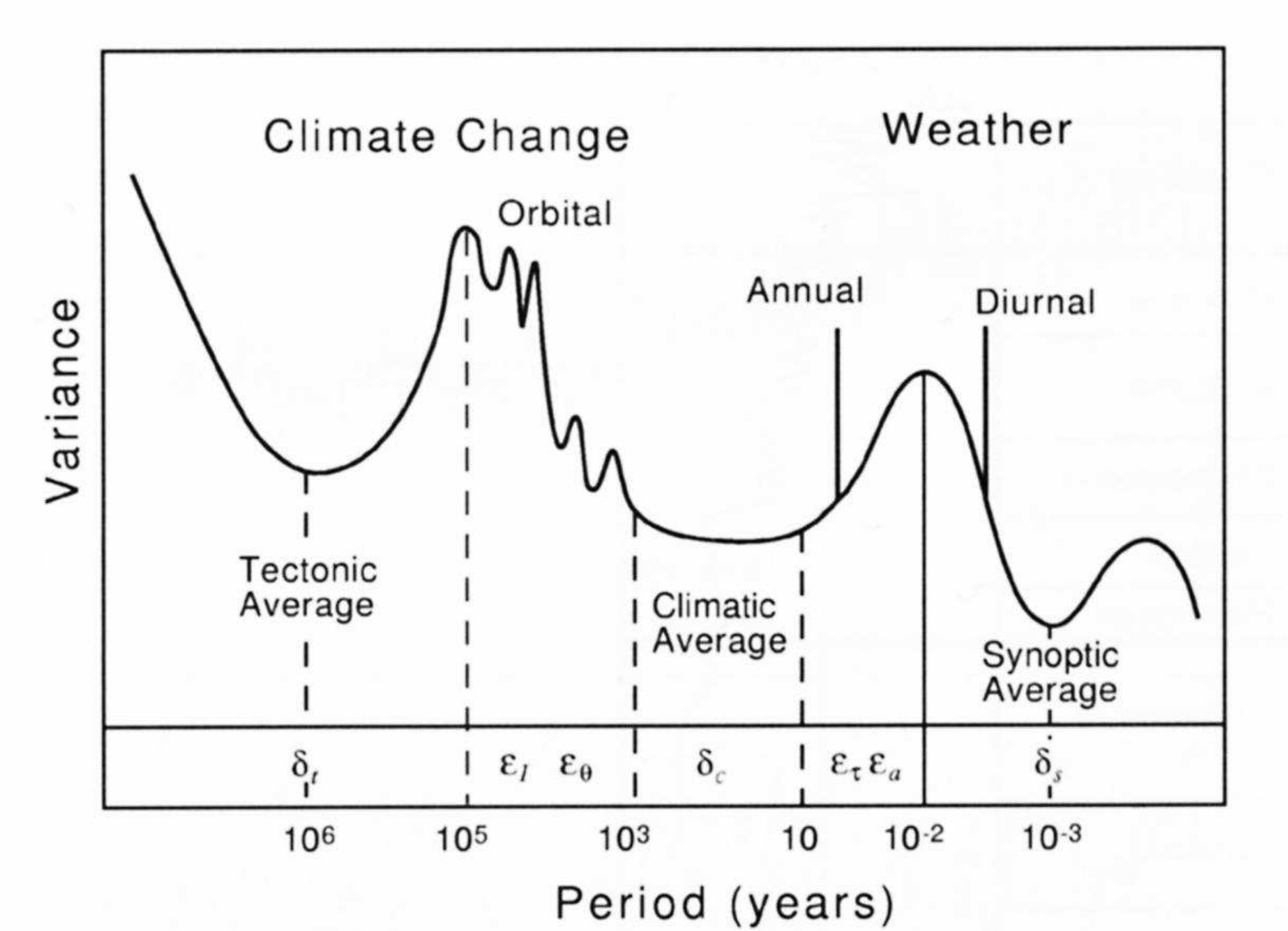




From ERA-40 Project Report Series 19

Origins of Stochasticity: Multiscale Dynamics

- Climate system displays variability over broad range of space and time scales



From Saltzman, 2002

Origins of Stochasticity: Multiscale Dynamics

- When modelling slower components of system, don't want (need?) to explicitly simulate faster components

UVic

Origins of Stochasticity: Multiscale Dynamics

- When modelling slower components of system, don't want (need?) to explicitly simulate faster components

$\Rightarrow$ "subgrid-scale parameterisations" (closure)

Origins of Stochasticity: Multiscale Dynamics

- When modelling slower components of system, don't want (need?) to explicitly simulate faster components

$\Rightarrow$ "subgrid-scale parameterisations" (closure)

- Analogy with statistical mechanics; can talk about "pressure" and "temperature" of gas without accounting for state of each molecule

Origins of Stochasticity: Multiscale Dynamics

- When modelling slower components of system, don't want (need?) to explicitly simulate faster components

$\Rightarrow$ "subgrid-scale parameterisations" (closure)

- Analogy with statistical mechanics; can talk about "pressure" and "temperature" of gas without accounting for state of each molecule
- Can also consider separation of scales in space; parameterisation problem arises because of finite gridsize of operational models

UVic

Origins of Stochasticity: Multiscale Dynamics

- When modelling slower components of system, don't want (need?) to explicitly simulate faster components

$\Rightarrow$ "subgrid-scale parameterisations" (closure)

- Analogy with statistical mechanics; can talk about "pressure" and "temperature" of gas without accounting for state of each molecule
- Can also consider separation of scales in space; parameterisation problem arises because of finite gridsize of operational models
- Nonlinear dynamics $\Rightarrow$ fast dynamics has upscale effect on slow

UVic

Origins of Stochasticity: Multiscale Dynamics

- When modelling slower components of system, don't want (need?) to explicitly simulate faster components

$\Rightarrow$ "subgrid-scale parameterisations" (closure)

- Analogy with statistical mechanics; can talk about "pressure" and "temperature" of gas without accounting for state of each molecule
- Can also consider separation of scales in space; parameterisation problem arises because of finite gridsize of operational models
- Nonlinear dynamics $\Rightarrow$ fast dynamics has upscale effect on slow
- Coarse-graining results not only in unresolved *scales*, but also unresolved *processes* (e.g. internal gravity waves, convection, cloud mircophysics)

UVic

Origins of Stochasticity: Multiscale Dynamics

- State of unresolved processes/scales conditioned on resolved scales not unique: adopt distributional perspective on subgrid-scale

UVic

Origins of Stochasticity: Multiscale Dynamics

- State of unresolved processes/scales conditioned on resolved scales not unique: adopt distributional perspective on subgrid-scale
- Formally decompose climate state $\mathbf{z}$ into slow and fast variables $\mathbf{x}$ and $\mathbf{y}$ (resp. "climate" and "weather"):

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{f}(\mathbf{x}, \mathbf{y}, t) \\ \frac{d\mathbf{y}}{dt} &= \frac{1}{\epsilon}\mathbf{g}(\mathbf{x}, \mathbf{y}, t) \end{aligned}$$

UVic

Origins of Stochasticity: Multiscale Dynamics

- State of unresolved processes/scales conditioned on resolved scales not unique: adopt distributional perspective on subgrid-scale
- Formally decompose climate state $\mathbf{z}$ into slow and fast variables $\mathbf{x}$ and $\mathbf{y}$ (resp. "climate" and "weather"):

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= \mathbf{f}(\mathbf{x}, \mathbf{y}, t)\\ \frac{d\mathbf{y}}{dt} &= \frac{1}{\epsilon}\mathbf{g}(\mathbf{x}, \mathbf{y}, t)\end{aligned}$$

- Obtain effective stochastic dynamics for "climate" as $\epsilon \rightarrow 0$

UVic

Origins of Stochasticity: Multiscale Dynamics

- State of unresolved processes/scales conditioned on resolved scales not unique: adopt distributional perspective on subgrid-scale
- Formally decompose climate state $\mathbf{z}$ into slow and fast variables $\mathbf{x}$ and $\mathbf{y}$ (resp. "climate" and "weather"):

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{f}(\mathbf{x}, \mathbf{y}, t) \\ \frac{d\mathbf{y}}{dt} &= \frac{1}{\epsilon}\mathbf{g}(\mathbf{x}, \mathbf{y}, t) \end{aligned}$$

- Obtain effective stochastic dynamics for "climate" as $\epsilon \longrightarrow 0$
- In many (most?) climate applications, ϵ is not small

UVic

Origins of Stochasticity: Multiscale Dynamics

- State of unresolved processes/scales conditioned on resolved scales not unique: adopt distributional perspective on subgrid-scale
- Formally decompose climate state $\mathbf{z}$ into slow and fast variables $\mathbf{x}$ and $\mathbf{y}$ (resp. “climate” and “weather”):

$$
\begin{aligned}
\frac{d\mathbf{x}}{dt} &= \mathbf{f}(\mathbf{x}, \mathbf{y}, t) \\
\frac{d\mathbf{y}}{dt} &= \frac{1}{\epsilon}\mathbf{g}(\mathbf{x}, \mathbf{y}, t)
\end{aligned}
$$

- Obtain effective stochastic dynamics for “climate” as $\epsilon \longrightarrow 0$
- In many (most?) climate applications, ϵ is not small
- Fast/slow decomposition not unique;
  “one person’s noise is another person’s signal”

UVic

Origins of Stochasticity: Model Uncertainty

- All models of climate (or subsystems) contain both

Origins of Stochasticity: Model Uncertainty

- All models of climate (or subsystems) contain both
  - model error, and

Origins of Stochasticity: Model Uncertainty

- All models of climate (or subsystems) contain both
  - model error, and
  - poorly constrained (sometimes unphysical) parameters

UVic

Origins of Stochasticity: Model Uncertainty

- All models of climate (or subsystems) contain both
  - model error, and
  - poorly constrained (sometimes unphysical) parameters
- Ideally: given pdf of parameters and model structure, obtain pdf of climate state

Origins of Stochasticity: Model Uncertainty

- All models of climate (or subsystems) contain both
  - model error, and
  - poorly constrained (sometimes unphysical) parameters
- Ideally: given pdf of parameters and model structure, obtain pdf of climate state
- Reality:

UVic

Origins of Stochasticity: Model Uncertainty

- All models of climate (or subsystems) contain both
  - model error, and
  - poorly constrained (sometimes unphysical) parameters
- Ideally: given pdf of parameters and model structure, obtain pdf of climate state
- Reality:
  - full climate state pdf cannot be computed; must adopt Monte-Carlo or **ensemble** forecast approach in which pdf is sampled; "curse of dimensionality"

UVic

Origins of Stochasticity: Model Uncertainty

- All models of climate (or subsystems) contain both
  - model error, and
  - poorly constrained (sometimes unphysical) parameters
- Ideally: given pdf of parameters and model structure, obtain pdf of climate state
- Reality:
  - full climate state pdf cannot be computed; must adopt Monte-Carlo or **ensemble** forecast approach in which pdf is sampled; "curse of dimensionality"
  - parameter pdfs generally not well known *a priori*

UVic

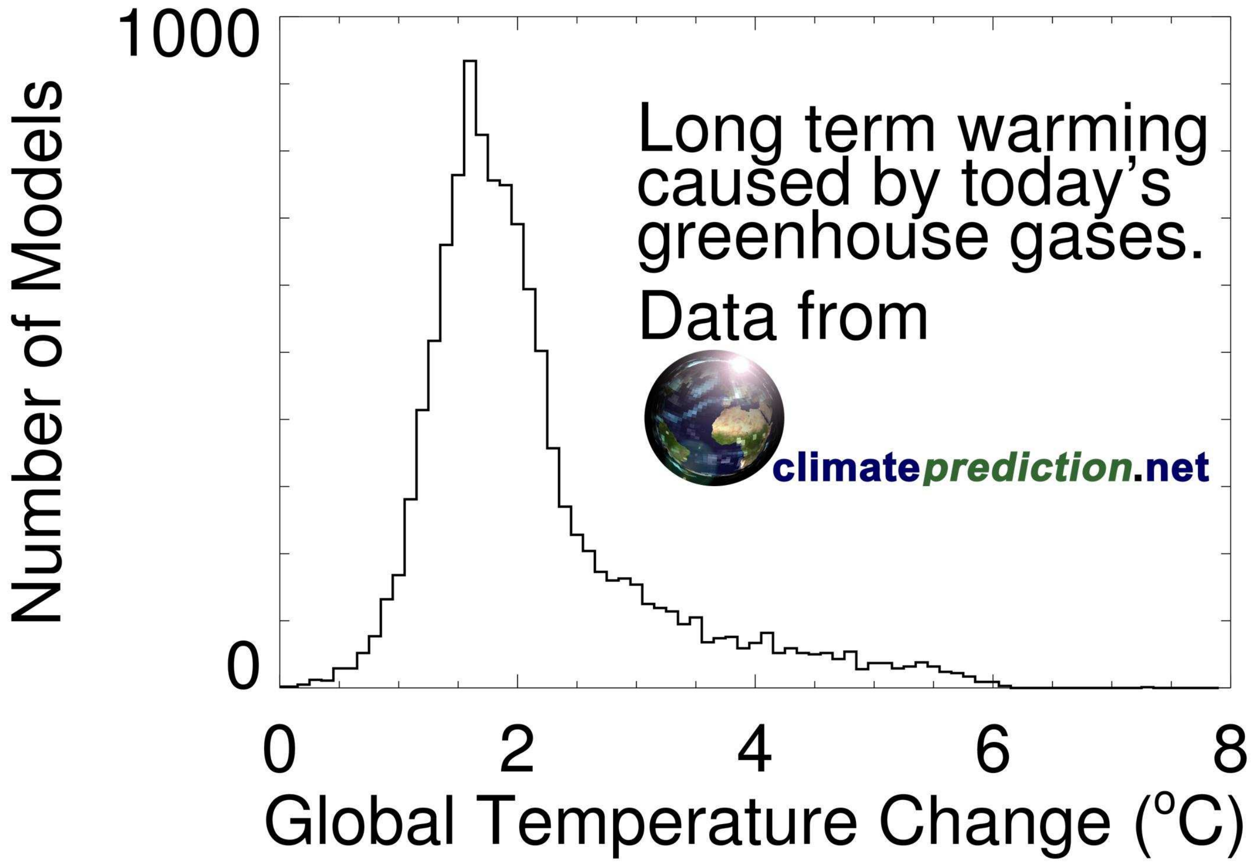
Origins of Stochasticity: Model Uncertainty

- All models of climate (or subsystems) contain both
  - model error, and
  - poorly constrained (sometimes unphysical) parameters
- Ideally: given pdf of parameters and model structure, obtain pdf of climate state
- Reality:
  - full climate state pdf cannot be computed; must adopt Monte-Carlo or **ensemble** forecast approach in which pdf is sampled; "curse of dimensionality"
  - parameter pdfs generally not well known *a priori*
  - Building large ensembles computationally expensive

UVic

Parameter Uncertainty: Ensemble Prediction

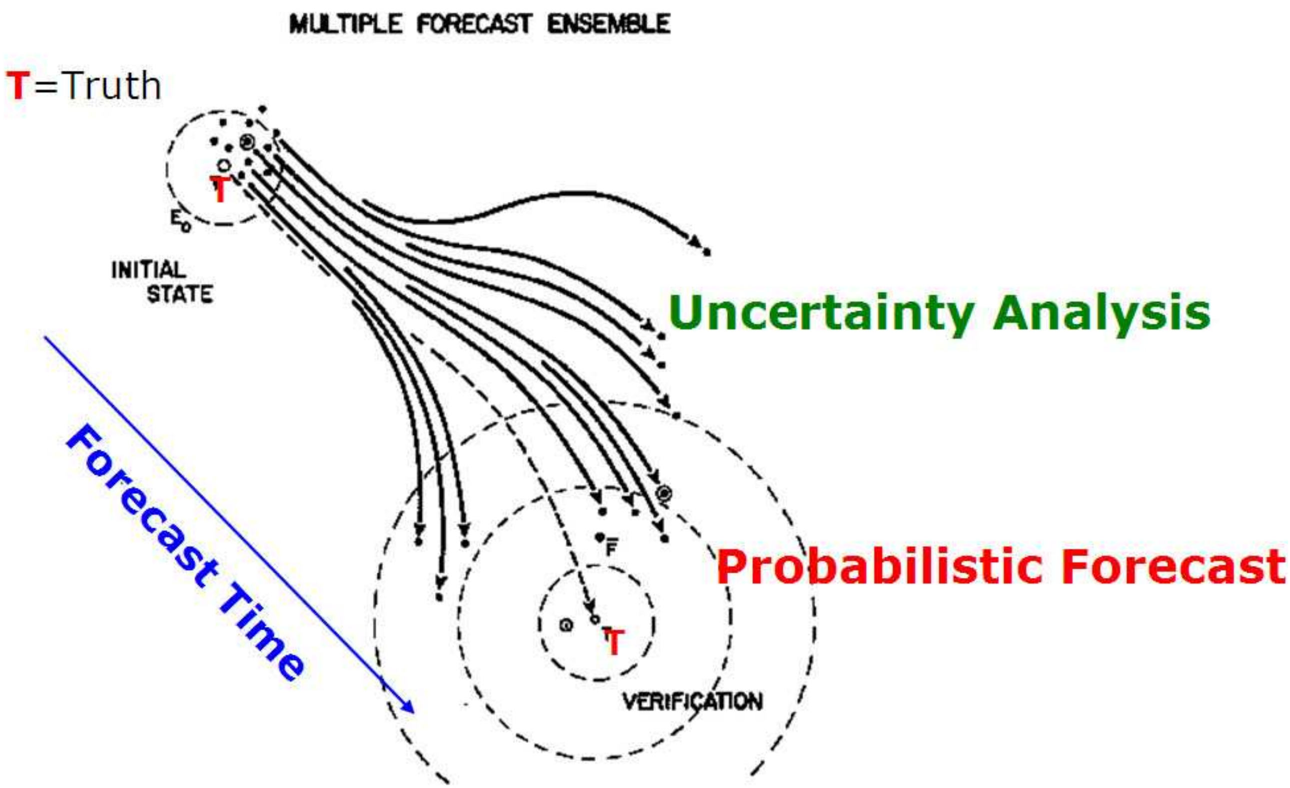
- climateprediction.net uses idle private CPUs to integrate ensembles with different parameter settings



http://www.climateprediction.net

Initial Condition Uncertainty: Ensemble Forecasting

- Model uncertainties can also include initial conditions



http://chrs.web.uci.edu/images/ensemble_large_atmo.jpg

UVic

Stochastic Climate Models: Case Studies

- Two distinct end-member approaches to modelling pdfs of climate variables:

UVic

Stochastic Climate Models: Case Studies

- Two distinct end-member approaches to modelling pdfs of climate variables:
  - running fully complex general circulation models

Stochastic Climate Models: Case Studies

- Two distinct end-member approaches to modelling pdfs of climate variables:
  - running fully complex general circulation models
  - considering physically-motivated idealised models

UVic

Stochastic Climate Models: Case Studies

- Two distinct end-member approaches to modelling pdfs of climate variables:
  - running fully complex general circulation models
  - considering physically-motivated idealised models
- First approach has benefit of being more realistic, but is also much more complex; mechanisms are not always clear

Stochastic Climate Models: Case Studies

- Two distinct end-member approaches to modelling pdfs of climate variables:
  - running fully complex general circulation models
  - considering physically-motivated idealised models
- First approach has benefit of being more realistic, but is also much more complex; mechanisms are not always clear
- Second approach not always quantitatively accurate, but important for developing understanding and elucidating mechanism

UVic

Stochastic Climate Models: Case Studies

- Two distinct end-member approaches to modelling pdfs of climate variables:
  - running fully complex general circulation models
  - considering physically-motivated idealised models
- First approach has benefit of being more realistic, but is also much more complex; mechanisms are not always clear
- Second approach not always quantitatively accurate, but important for developing understanding and elucidating mechanism
- Will now consider two “idealised” stochastic models for:

UVic

Stochastic Climate Models: Case Studies

- Two distinct end-member approaches to modelling pdfs of climate variables:
  - running fully complex general circulation models
  - considering physically-motivated idealised models
- First approach has benefit of being more realistic, but is also much more complex; mechanisms are not always clear
- Second approach not always quantitatively accurate, but important for developing understanding and elucidating mechanism
- Will now consider two "idealised" stochastic models for:
  - stochastic dynamics of sea-surface winds

UVic

Stochastic Climate Models: Case Studies

- Two distinct end-member approaches to modelling pdfs of climate variables:
  - running fully complex general circulation models
  - considering physically-motivated idealised models
- First approach has benefit of being more realistic, but is also much more complex; mechanisms are not always clear
- Second approach not always quantitatively accurate, but important for developing understanding and elucidating mechanism
- Will now consider two “idealised” stochastic models for:
  - stochastic dynamics of sea-surface winds
  - stochastic dynamics of El Niño-Southern Oscillation

UVic

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases
  - fluxes depend on surface winds, in general nonlinearly

UVic

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases
  - fluxes depend on surface winds, in general nonlinearly
  - ocean currents largely driven by surface winds

UVic

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases
  - fluxes depend on surface winds, in general nonlinearly
  - ocean currents largely driven by surface winds
- *Sea State*

UVic

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases
  - fluxes depend on surface winds, in general nonlinearly
  - ocean currents largely driven by surface winds
- *Sea State*
  - sea state important for shipping, recreation

UVic

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases
  - fluxes depend on surface winds, in general nonlinearly
  - ocean currents largely driven by surface winds
- *Sea State*
  - sea state important for shipping, recreation
  - determined by both local and remote winds

UVic

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases
  - fluxes depend on surface winds, in general nonlinearly
  - ocean currents largely driven by surface winds
- *Sea State*
  - sea state important for shipping, recreation
  - determined by both local and remote winds
- *Power Generation*

UVic

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases
  - fluxes depend on surface winds, in general nonlinearly
  - ocean currents largely driven by surface winds
- *Sea State*
  - sea state important for shipping, recreation
  - determined by both local and remote winds
- *Power Generation*
  - wind power potentially significant source of energy

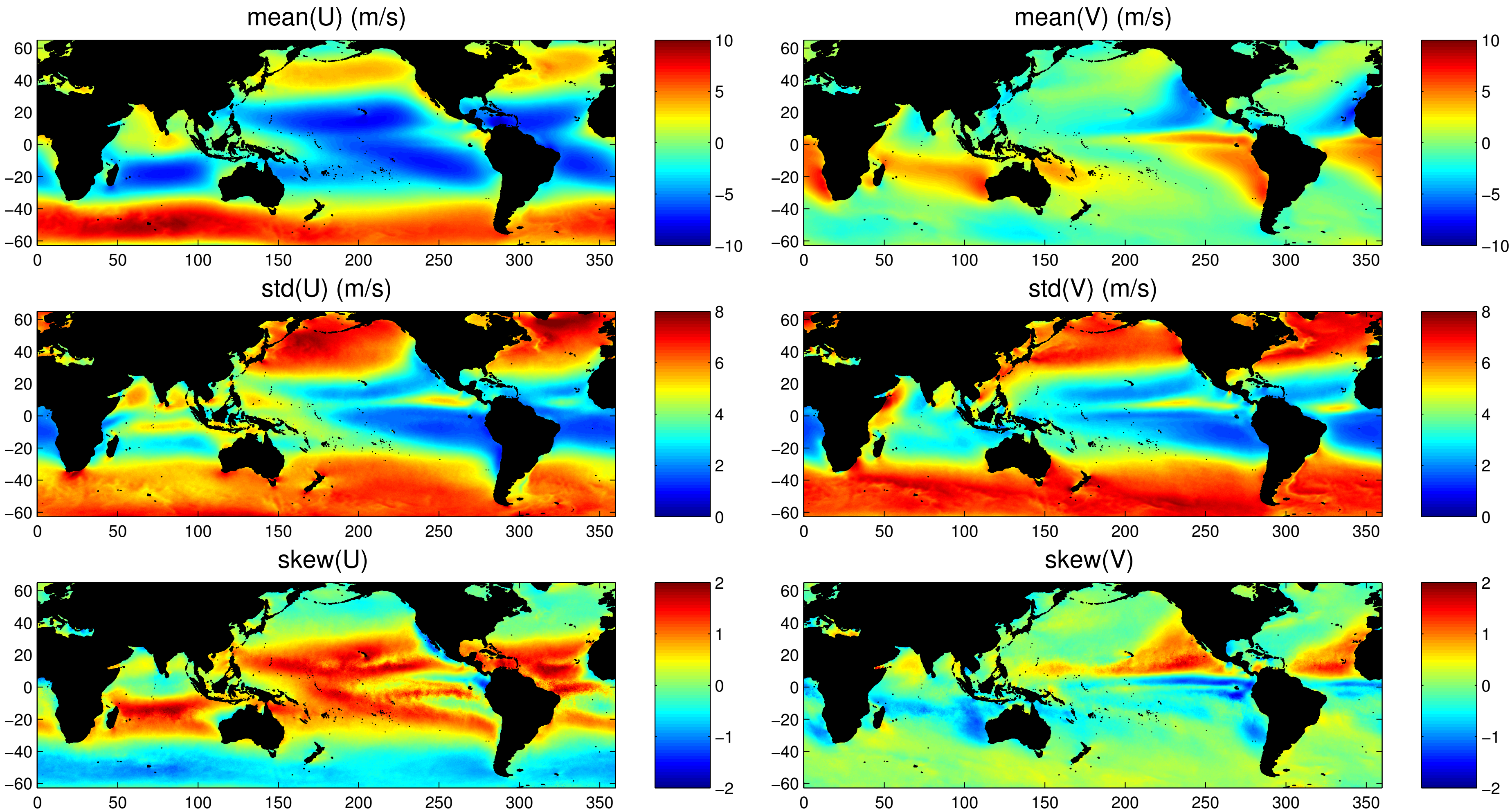
UVic

Sea Surface Winds: Why Should We Care?

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases
  - fluxes depend on surface winds, in general nonlinearly
  - ocean currents largely driven by surface winds
- *Sea State*
  - sea state important for shipping, recreation
  - determined by both local and remote winds
- *Power Generation*
  - wind power potentially significant source of energy
  - generation rate scales as cube of wind speed; extreme events important

UVic

Vector Wind Moments

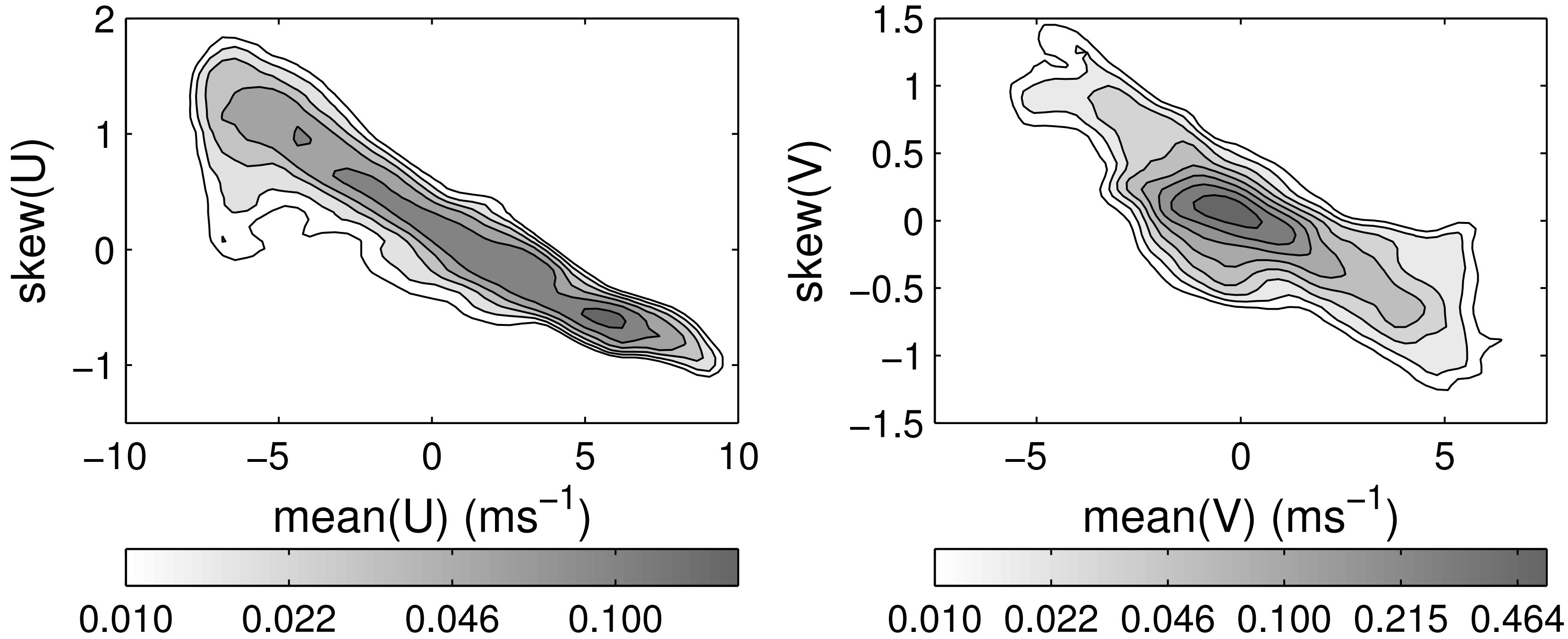


Zonal Wind

Meridional Wind

Mean and Skewness of Vector Wind

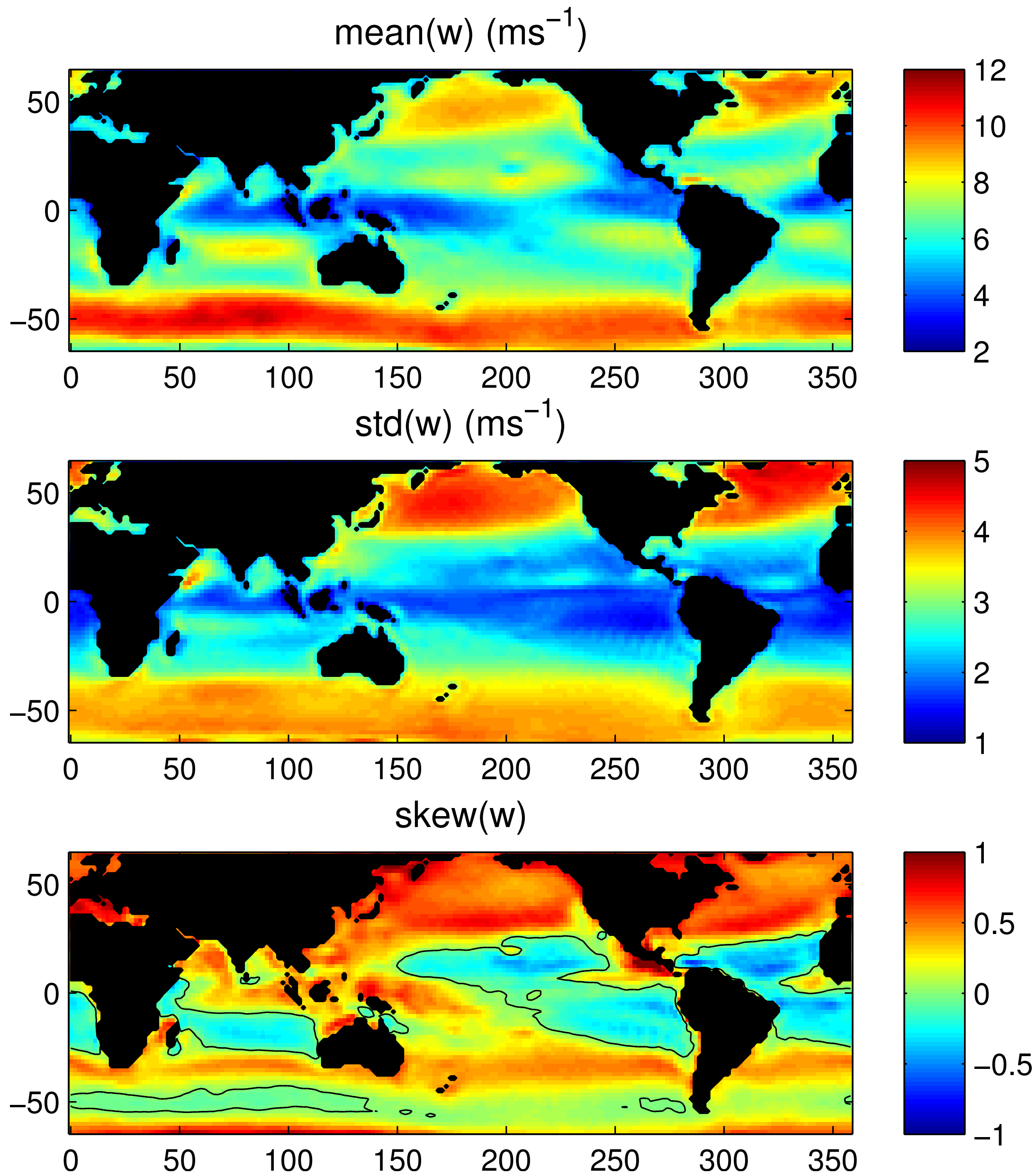
- Joint pdfs of mean and skew for zonal and meridional winds



(note logarithmic contour scale)

UVic

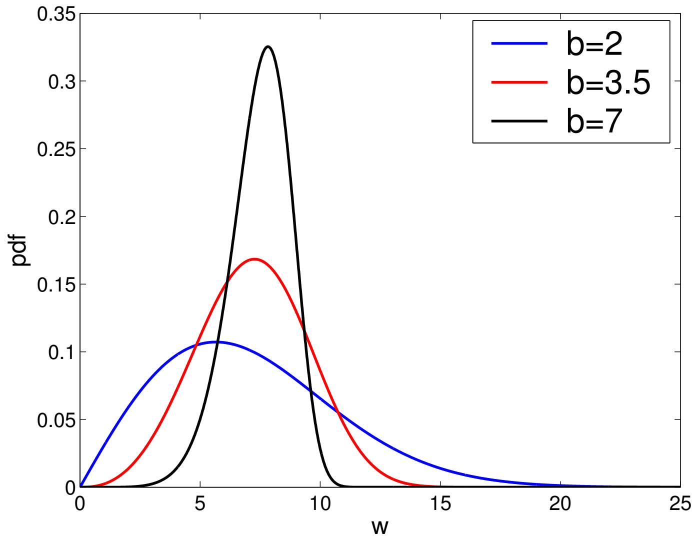
Wind Speed Moments



UVic

Wind Speed pdf: Weibull distribution

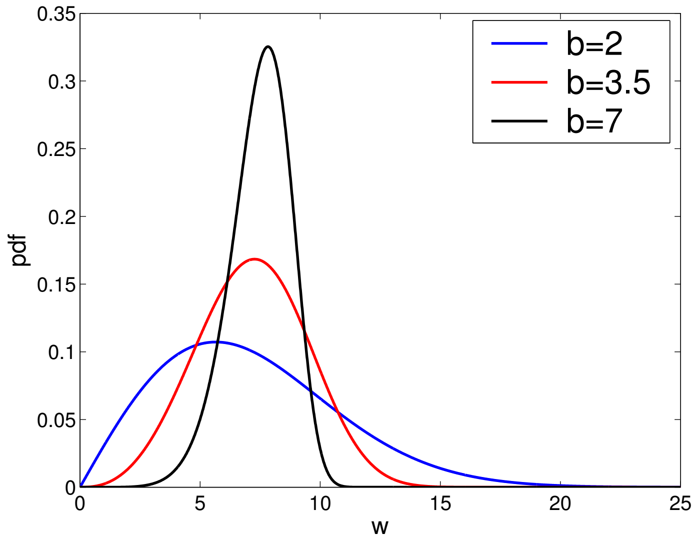
- The pdf of wind speed w has traditionally (and empirically) been represented by 2-parameter Weibull distribution:



$$p(w) = \frac{b}{a}\left(\frac{w}{a}\right)^{b-1} \exp\left[-\left(\frac{w}{a}\right)^{b}\right]$$

Wind Speed pdf: Weibull distribution

- The pdf of wind speed w has traditionally (and empirically) been represented by 2-parameter Weibull distribution:

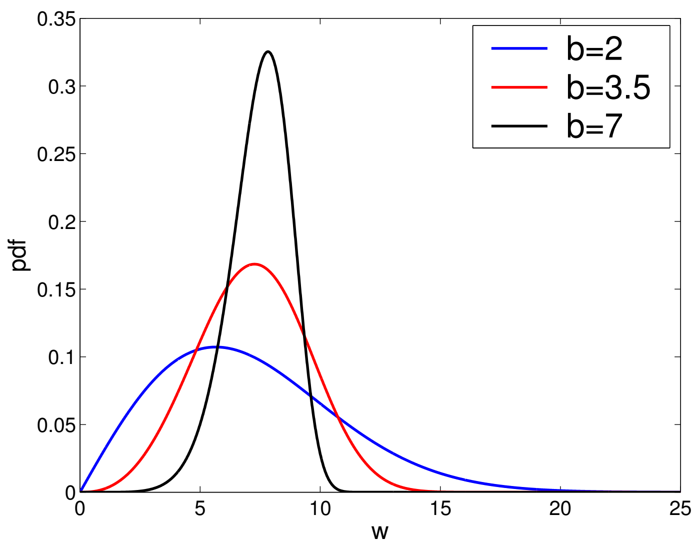


$$p(w) = \frac{b}{a}\left(\frac{w}{a}\right)^{b-1} \exp\left[-\left(\frac{w}{a}\right)^{b}\right]$$

- a is the scale parameter (pdf centre)

Wind Speed pdf: Weibull distribution

- The pdf of wind speed w has traditionally (and empirically) been represented by 2-parameter Weibull distribution:



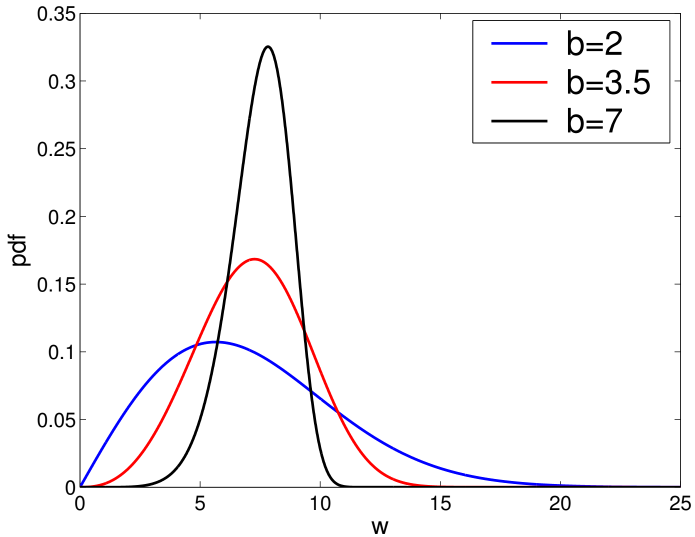
$$p(w) = \frac{b}{a}\left(\frac{w}{a}\right)^{b-1} \exp\left[-\left(\frac{w}{a}\right)^{b}\right]$$

- a is the <u>scale</u> parameter (pdf centre)
- b is the <u>shape</u> parameter (pdf tilt)

UVic

Wind Speed pdf: Weibull distribution

- The pdf of wind speed w has traditionally (and empirically) been represented by 2-parameter Weibull distribution:



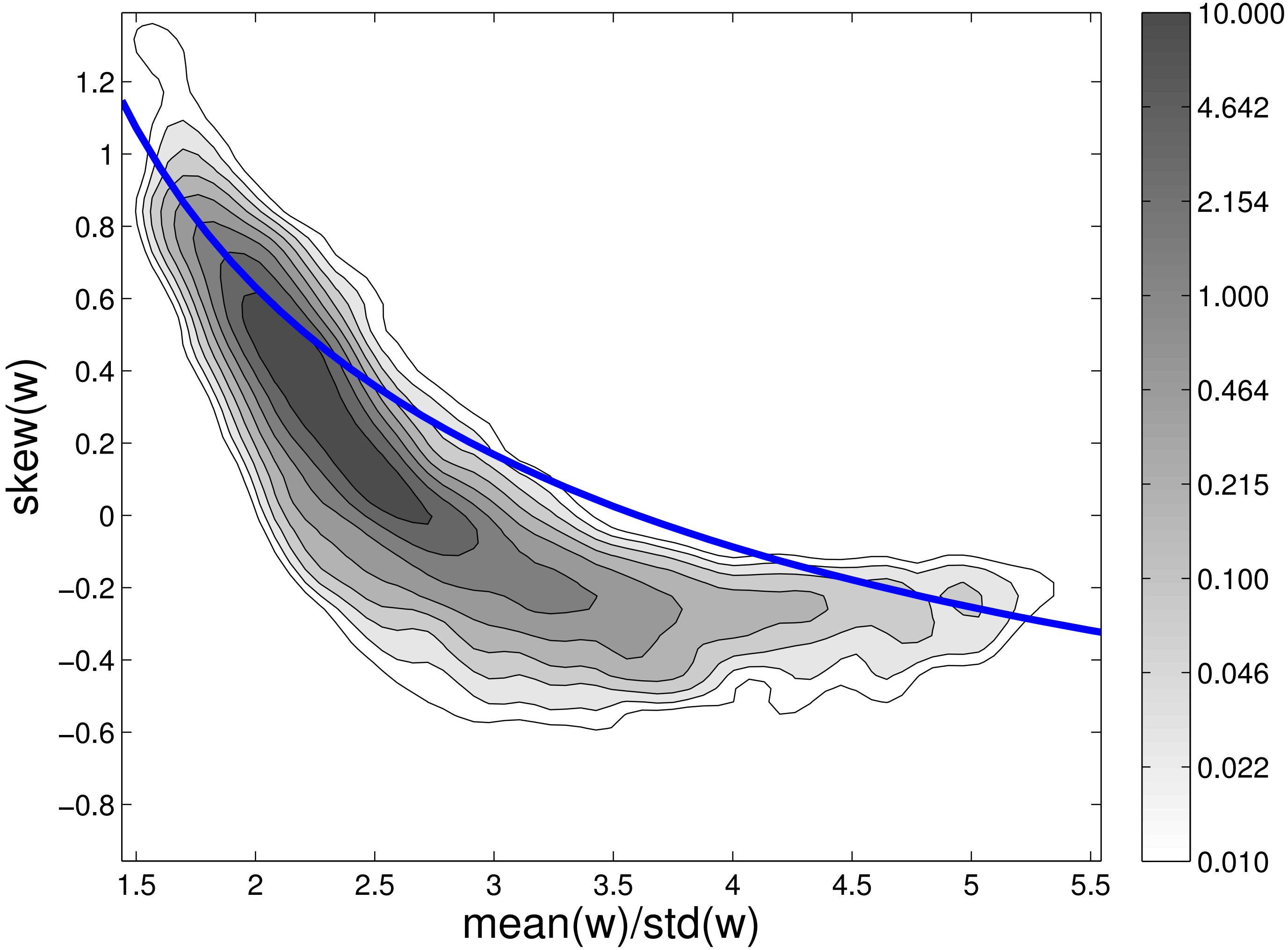
$$p(w) = \frac{b}{a}\left(\frac{w}{a}\right)^{b-1} \exp\left[-\left(\frac{w}{a}\right)^{b}\right]$$

- a is the <u>scale</u> parameter (pdf centre)
- b is the <u>shape</u> parameter (pdf tilt)
- $p_w(w)$ is unimodal

UVic

Wind Speed pdfs: Observed

- Observed speed moments fall around Weibull curve



UVic

Boundary Layer Dynamics

- Horizontal momentum equations:

Boundary Layer Dynamics

- Horizontal momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p - f \hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho} \frac{\partial (\rho \overline{\mathbf{u}' u_3'})}{\partial z}$$

Boundary Layer Dynamics

- Horizontal momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p - f \hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho} \frac{\partial (\rho \overline{\mathbf{u}' u_3'})}{\partial z}$$

- Momentum tendency due to:

UVic

Boundary Layer Dynamics

- Horizontal momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho}\frac{\partial(\rho\overline{\mathbf{u}' u_3'})}{\partial z}$$

- Momentum tendency due to:
  - advection (transport by flow; secondary importance on daily timescales)

UVic

Boundary Layer Dynamics

- Horizontal momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p - f \hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho} \frac{\partial (\rho \overline{\mathbf{u}' u_3'})}{\partial z}$$

- Momentum tendency due to:
  - advection (transport by flow; secondary importance on daily timescales)
  - pressure gradient force

UVic

Boundary Layer Dynamics

- Horizontal momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho}\frac{\partial(\rho\overline{\mathbf{u}' u_3'})}{\partial z}$$

- Momentum tendency due to:
  - advection (transport by flow; secondary importance on daily timescales)
  - pressure gradient force
  - Coriolis force

UVic

Boundary Layer Dynamics

- Horizontal momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho}\frac{\partial(\rho\overline{\mathbf{u}' u_3'})}{\partial z}$$

- Momentum tendency due to:
  - advection (transport by flow; secondary importance on daily timescales)
  - pressure gradient force
  - Coriolis force
  - turbulent momentum flux (in vertical)

UVic

Boundary Layer Dynamics

- Horizontal momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho}\frac{\partial(\rho\overline{\mathbf{u}' u_3'})}{\partial z}$$

- Momentum tendency due to:
  - advection (transport by flow; secondary importance on daily timescales)
  - pressure gradient force
  - Coriolis force
  - turbulent momentum flux (in vertical)
- Integrated momentum budget for slab of thickness h:

UVic

Boundary Layer Dynamics

- Horizontal momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho} \frac{\partial(\rho \overline{\mathbf{u}' u_3'})}{\partial z}$$

- Momentum tendency due to:
  - advection (transport by flow; secondary importance on daily timescales)
  - pressure gradient force
  - Coriolis force
  - turbulent momentum flux (in vertical)
- Integrated momentum budget for slab of thickness h:

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho} \nabla p - f\hat{\mathbf{k}} \times \mathbf{u} + \frac{1}{h} \left(\overline{\mathbf{u}' u_3'}(0) - \overline{\mathbf{u}' u_3'}(h) \right)$$

Surface Wind Stress

- Surface wind stress is turbulent momentum flux across air/sea interface:

$$\tau_s = \rho_a \overline{\mathbf{u}' u_3'}(0)$$

Surface Wind Stress

- Surface wind stress is turbulent momentum flux across air/sea interface:

$$\tau_s = \rho_a \overline{\mathbf{u}' u_3'}(0)$$

where
$$\begin{aligned} u &= \text{along-mean wind component} \\ v &= \text{cross-mean wind component} \\ \mathbf{u} &= (u, v) \\ u_3 &= \text{vertical wind component} \end{aligned}$$

UVic

Surface Wind Stress

- Surface wind stress is turbulent momentum flux across air/sea interface:

$$\tau_s = \rho_a \overline{\mathbf{u}' u_3'}(0)$$

where
$$
\begin{aligned}
u &= \text{along-mean wind component} \\
v &= \text{cross-mean wind component} \\
\mathbf{u} &= (u, v) \\
u_3 &= \text{vertical wind component}
\end{aligned}
$$

- Flux parameterised in terms of **u** by bulk drag formula:

$$\tau_s = \rho_a c_d w \mathbf{u}$$

where $w = \| \mathbf{u} \|$ is the wind speed.

UVic

Drag Coefficient

- Drag coefficient influenced by

Drag Coefficient

- Drag coefficient influenced by

  surface roughness z_0

Drag Coefficient

- Drag coefficient influenced by

  surface roughness z_0

  Obukhov length L (buoyancy flux, stratification)

UVic

Drag Coefficient

- Drag coefficient influenced by

  surface roughness z_0

  Obukhov length L (buoyancy flux, stratification)

$$c_d = k^2 \left[\ln\left(\frac{z_a}{z_0}\right) - \psi_m\left(\frac{z_a}{L}\right) \right]^{-2}$$

UVic

Drag Coefficient

- Drag coefficient influenced by

  surface roughness z_0

  Obukhov length L (buoyancy flux, stratification)

$$c_d = k^2 \left[\ln\left(\frac{z_a}{z_0}\right) - \psi_m\left(\frac{z_a}{L}\right) \right]^{-2}$$

where

UVic

Drag Coefficient

- Drag coefficient influenced by

  surface roughness z_0

  Obukhov length L (buoyancy flux, stratification)

$$c_d = k^2 \left[\ln\left(\frac{z_a}{z_0}\right) - \psi_m\left(\frac{z_a}{L}\right) \right]^{-2}$$

where

  ψ_m is an empirical function

Drag Coefficient

- Drag coefficient influenced by

  surface roughness z_0

  Obukhov length L (buoyancy flux, stratification)

$$c_d = k^2 \left[\ln\left(\frac{z_a}{z_0}\right) - \psi_m\left(\frac{z_a}{L}\right) \right]^{-2}$$

where

ψ_m is an empirical function

z_a is the anemometer height, typically 10 m

UVic

Drag Coefficient

- Drag coefficient influenced by

  surface roughness z_0

  Obukhov length L (buoyancy flux, stratification)

$$c_d = k^2 \left[\ln\left(\frac{z_a}{z_0}\right) - \psi_m\left(\frac{z_a}{L}\right)\right]^{-2}$$

where

  ψ_m is an empirical function

  z_a is the anemometer height, typically 10 m

- Surface winds modify surface wave field

UVic

Drag Coefficient

- Drag coefficient influenced by

  surface roughness z_0

  Obukhov length L (buoyancy flux, stratification)

$$c_d = k^2 \left[\ln\left(\frac{z_a}{z_0}\right) - \psi_m\left(\frac{z_a}{L}\right) \right]^{-2}$$

  where

  ψ_m is an empirical function

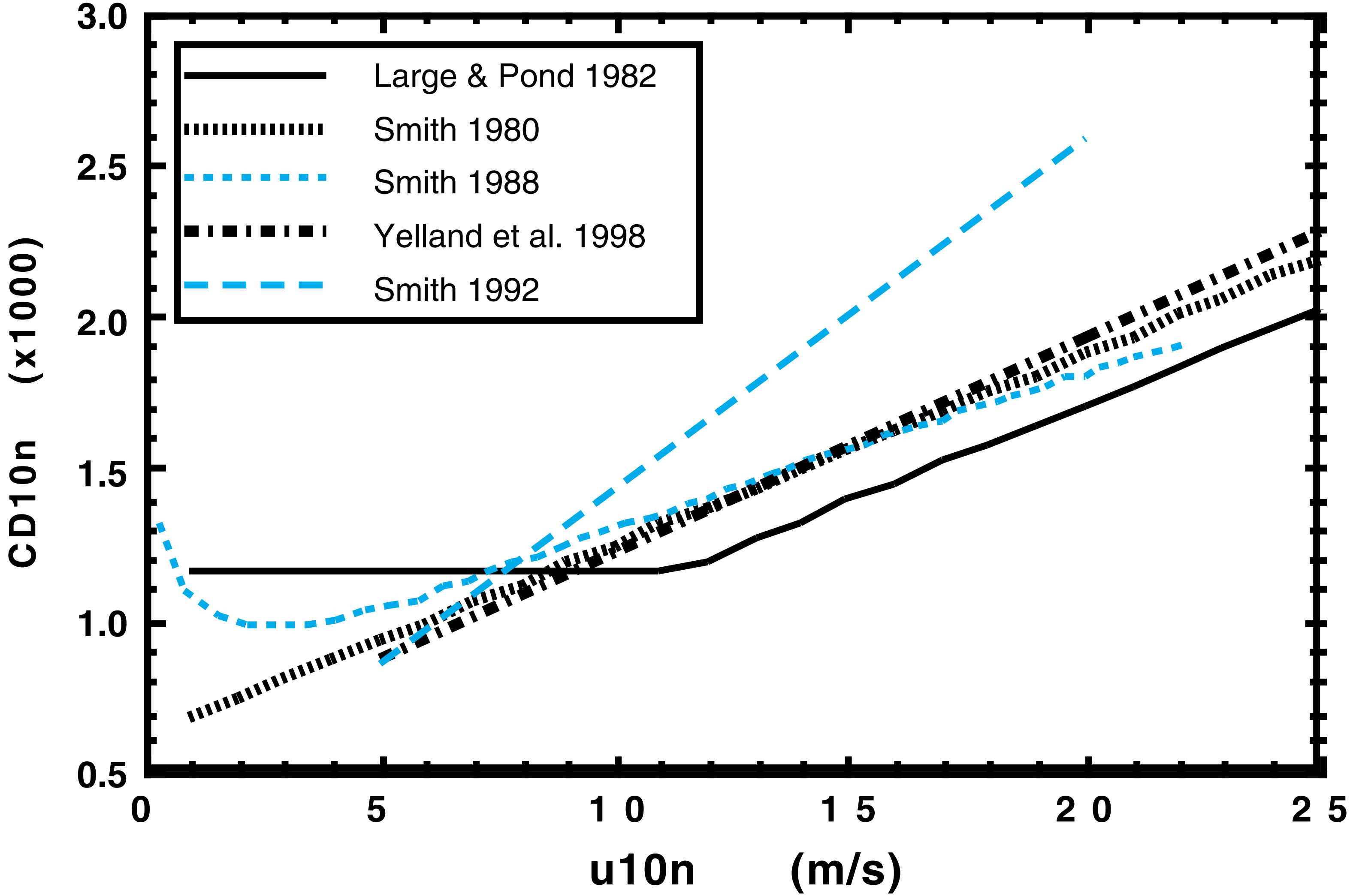
  z_a is the anemometer height, typically 10 m

- Surface winds modify surface wave field

  $\Rightarrow z_0$ depends on w

UVic

Neutral Drag Coefficient: Observations



UVic

Taylor et al. (2000) WMO/TD No. 1036

Surface Momentum Budget

- To close momentum budget, need parameterisation of turbulent momentum flux at $z = h$

UVic

Surface Momentum Budget

- To close momentum budget, need parameterisation of turbulent momentum flux at $z = h$
- Use "finite-differenced" eddy viscosity:

Surface Momentum Budget

- To close momentum budget, need parameterisation of turbulent momentum flux at $z = h$
- Use "finite-differenced" eddy viscosity:

$$\overline{\mathbf{u}' u_3'}(h) = \frac{K}{h}(\mathbf{U} - \mathbf{u})$$

Surface Momentum Budget

- To close momentum budget, need parameterisation of turbulent momentum flux at $z = h$
- Use "finite-differenced" eddy viscosity:

$$\overline{\mathbf{u}' u_3'}(h) = \frac{K}{h}(\mathbf{U} - \mathbf{u})$$

$\Rightarrow$ Surface layer momentum budget

Surface Momentum Budget

- To close momentum budget, need parameterisation of turbulent momentum flux at $z = h$
- Use "finite-differenced" eddy viscosity:

$$\overline{\mathbf{u}' u_3'}(h) = \frac{K}{h}(\mathbf{U} - \mathbf{u})$$

$\Rightarrow$ Surface layer momentum budget

$$\begin{aligned}\frac{d\mathbf{u}}{dt} &= -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{c_d}{h} w\mathbf{u} + \frac{K}{h^2}(\mathbf{U} - \mathbf{u}) \\ &= \mathbf{\Pi} - \frac{c_d}{h} w\mathbf{u} - \frac{K}{h^2}\mathbf{u}\end{aligned}$$

Surface Momentum Budget

- To close momentum budget, need parameterisation of turbulent momentum flux at $z = h$
- Use "finite-differenced" eddy viscosity:

$$\overline{\mathbf{u}' u_3'}(h) = \frac{K}{h}(\mathbf{U} - \mathbf{u})$$

$\Rightarrow$ Surface layer momentum budget

$$\begin{aligned} \frac{d\mathbf{u}}{dt} &= -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{c_d}{h} w \mathbf{u} + \frac{K}{h^2}(\mathbf{U} - \mathbf{u}) \\ &= \mathbf{\Pi} - \frac{c_d}{h} w \mathbf{u} - \frac{K}{h^2}\mathbf{u} \end{aligned}$$

where

$$\mathbf{\Pi} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} + \frac{K}{h^2}\mathbf{U}$$

UVic

Mechanistic Model: SDE

- Decomposing $\mathbf{\Pi}$ into mean and fluctuations:

$$\begin{aligned} \Pi_u(t) &= \langle \Pi_u \rangle + \sigma \dot{W}_1(t) \\ \Pi_v(t) &= \sigma \dot{W}_2(t) \end{aligned}$$

UVic

Mechanistic Model: SDE

- Decomposing $\mathbf{\Pi}$ into mean and fluctuations:

$$\begin{aligned}\Pi_u(t) &= \langle \Pi_u \rangle + \sigma \dot{W}_1(t) \\ \Pi_v(t) &= \sigma \dot{W}_2(t)\end{aligned}$$

where $\dot{W}_i$ is Gaussian white noise

$$\left\langle \dot{W}_i(t_1) \dot{W}_j(t_2) \right\rangle = \delta_{ij} \delta(t_1 - t_2)$$

UVic

Mechanistic Model: SDE

- Decomposing $\mathbf{\Pi}$ into mean and fluctuations:

$$\Pi_u(t) = \langle \Pi_u \rangle + \sigma \dot{W}_1(t)$$
$$\Pi_v(t) = \sigma \dot{W}_2(t)$$

where $\dot{W}_i$ is Gaussian white noise

$$\left\langle \dot{W}_i(t_1)\dot{W}_j(t_2) \right\rangle = \delta_{ij}\delta(t_1 - t_2)$$

we obtain stochastic differential equation

$$\frac{du}{dt} = \langle \Pi_u \rangle - \frac{c_d}{h} wu - \frac{K}{h^2} u + \sigma \dot{W}_1$$
$$\frac{dv}{dt} = -\frac{c_d}{h} wv - \frac{K}{h^2} v + \sigma \dot{W}_2$$

UVic

Mechanistic Model: pdf

- Solution of associated Fokker-Planck equation for stationary pdf:

$$p_{uv}(u,v) = \mathcal{N}_1 \exp\left(\frac{2}{\sigma^2}\left\{\langle\Pi_u\rangle\, u - \frac{K}{2h^2}(u^2+v^2) - \frac{1}{h}\int_0^{\sqrt{u^2+v^2}} c_d(w')w'^2\, dw'\right\}\right)$$

Mechanistic Model: pdf

- Solution of associated Fokker-Planck equation for stationary pdf:

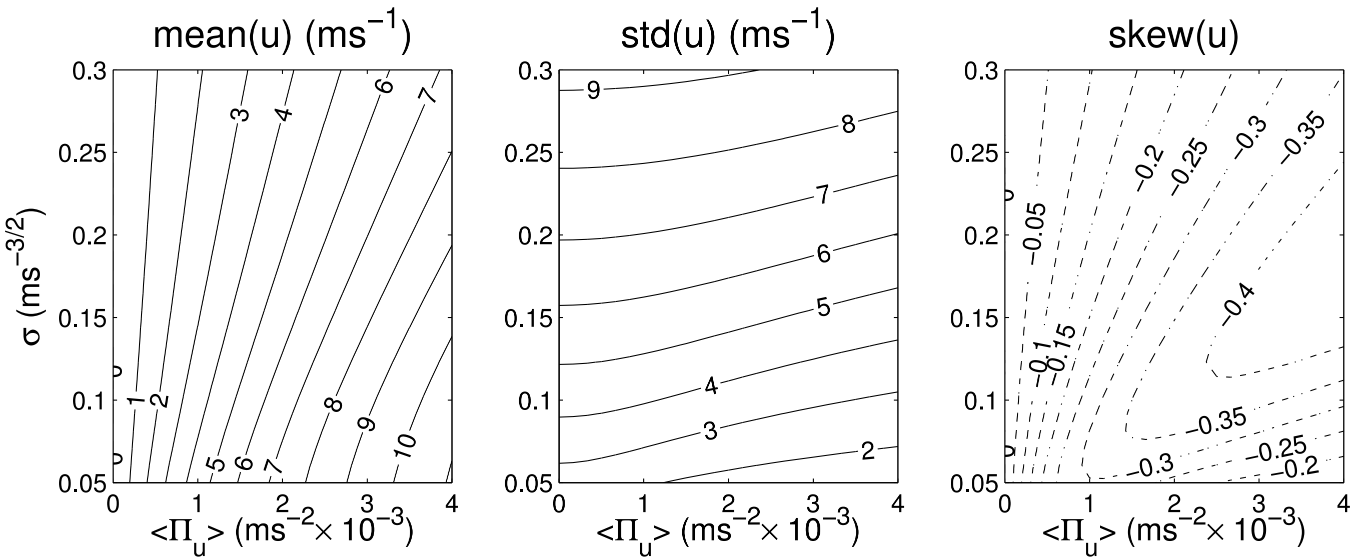
$$p_{uv}(u,v) = \mathcal{N}_1 \exp\left(\frac{2}{\sigma^2}\left\{\langle\Pi_u\rangle\, u - \frac{K}{2h^2}(u^2+v^2) - \frac{1}{h}\int_0^{\sqrt{u^2+v^2}} c_d(w')w'^2\, dw'\right\}\right)$$

- Changing to polar coordinates and integrating over angle gives wind speed pdf:

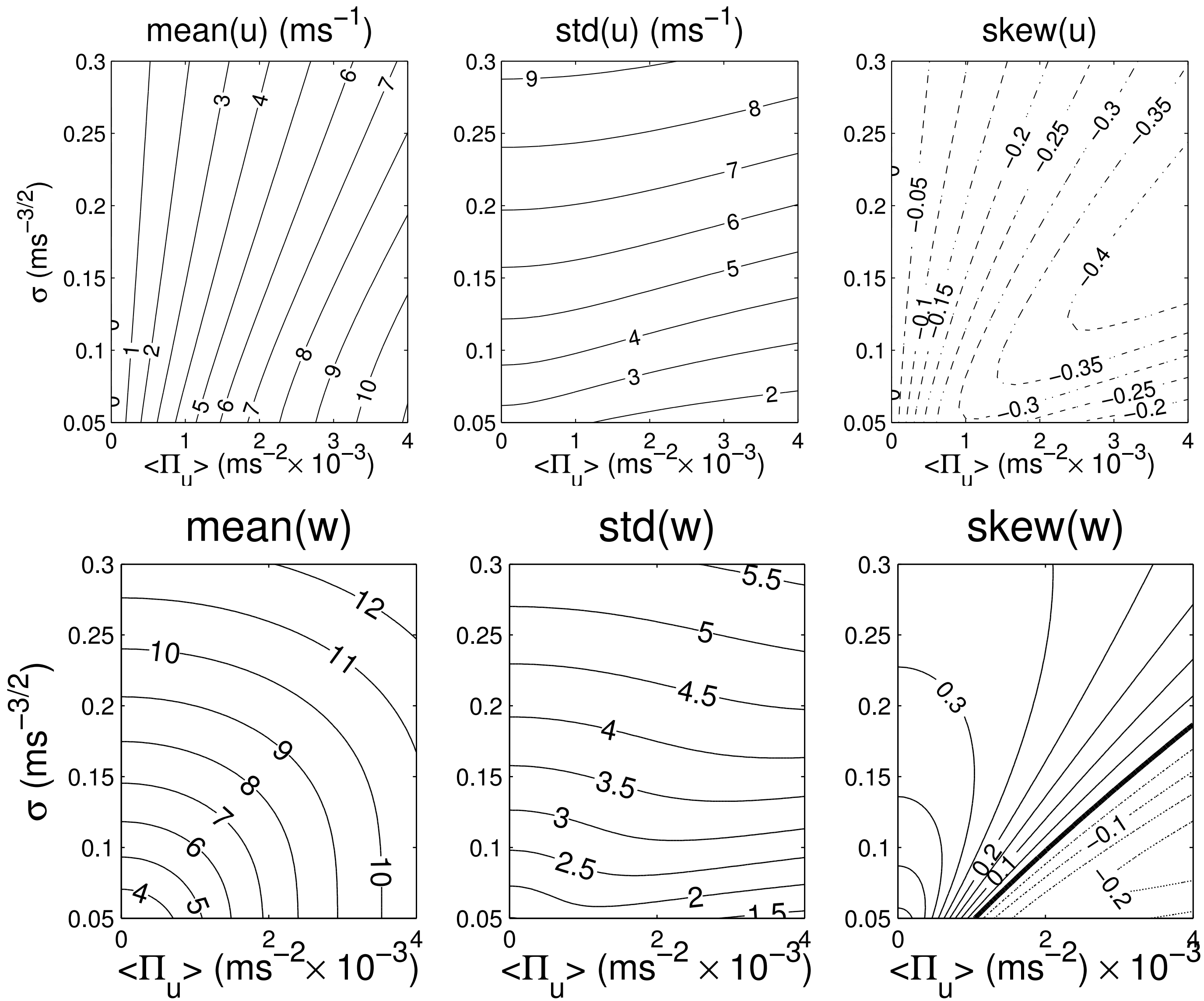
$$p_w(w) = \mathcal{N} w I_0\left(\frac{2\,\langle\Pi_u\rangle\, w}{\sigma^2}\right)\exp\left(-\frac{2}{\sigma^2}\left\{\frac{K}{2h^2}w^2 + \frac{1}{h}\int_0^{w} c_d(w')w'^2\, dw'\right\}\right)$$

UVic

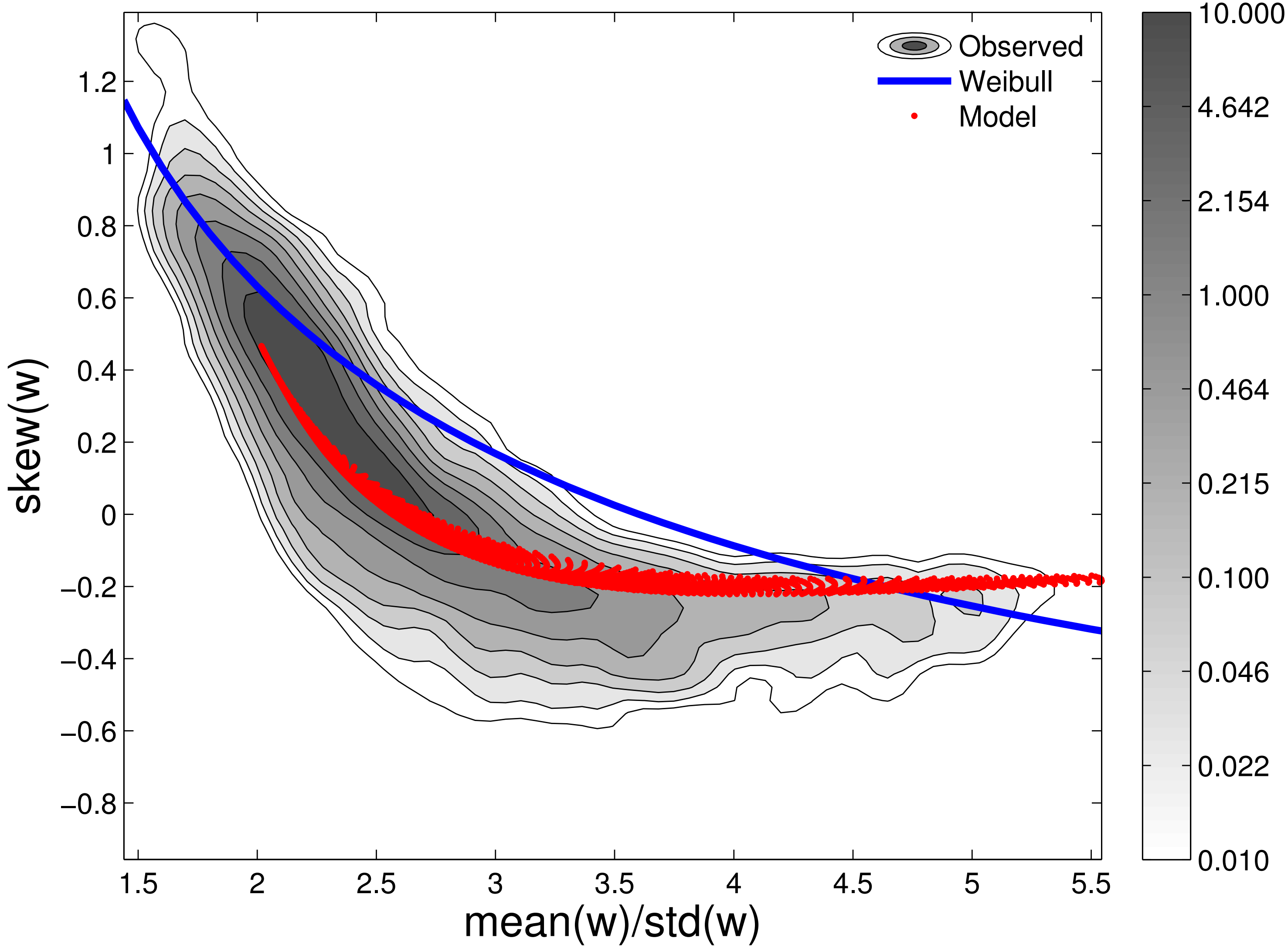
Mechanistic Model: Predictions



Mechanistic Model: Predictions



Mechanistic Model: Comparison with Observations



UVic

Case Study I: Conclusions

- Sea surface wind pdfs characterised by relationships between moments

Case Study I: Conclusions

- Sea surface wind pdfs characterised by relationships between moments
- These moment relationships reflect physical processes producing distributions

UVic

Case Study I: Conclusions

- Sea surface wind pdfs characterised by relationships between moments
- These moment relationships reflect physical processes producing distributions
- Idealised stochastic models can be constructed from basic physical principles to (qualitatively) explain physical origin of pdf structure

UVic

Case Study I: Conclusions

- Sea surface wind pdfs characterised by relationships between moments
- These moment relationships reflect physical processes producing distributions
- Idealised stochastic models can be constructed from basic physical principles to (qualitatively) explain physical origin of pdf structure
- More accurate quantitative simulation requires a more sophisticated model; qualitative utility of relatively simple model suggests it captures essential physics

UVic

El Niño - Southern Oscillation (ENSO)

- ENSO is the dominant mode of climate variability on interannual timescales, involving coupled interactions between the ocean and the atmosphere

UVic

El Niño - Southern Oscillation (ENSO)

- ENSO is the dominant mode of climate variability on interannual timescales, involving coupled interactions between the ocean and the atmosphere
- Dynamics primarily contained in equatorial Pacific; impacts felt globally

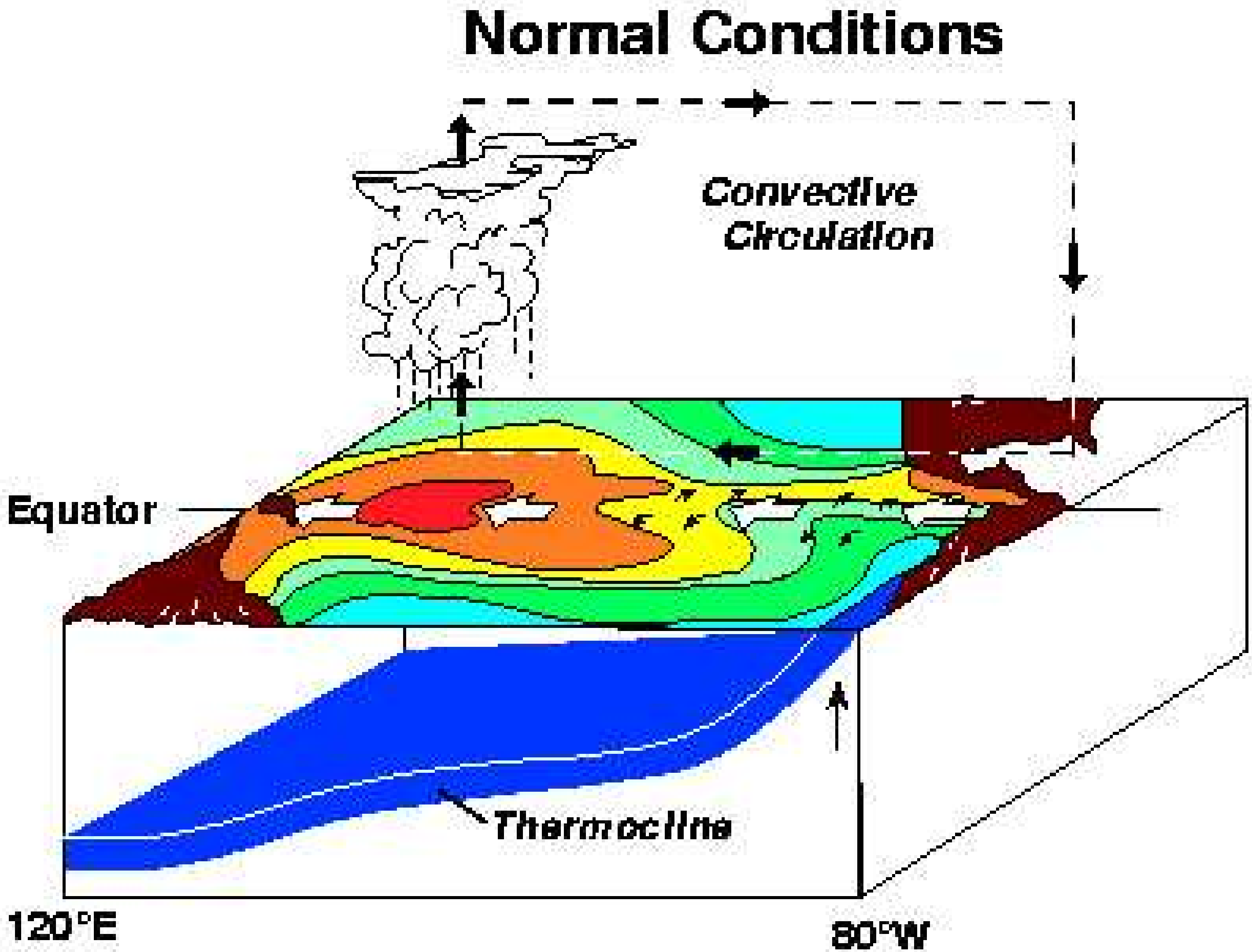
UVic

El Niño - Southern Oscillation (ENSO)

- ENSO is the dominant mode of climate variability on interannual timescales, involving coupled interactions between the ocean and the atmosphere
- Dynamics primarily contained in equatorial Pacific; impacts felt globally
- Skillful ENSO forecasts are believed to be primary potential source of skill for seasonal climate forecasting

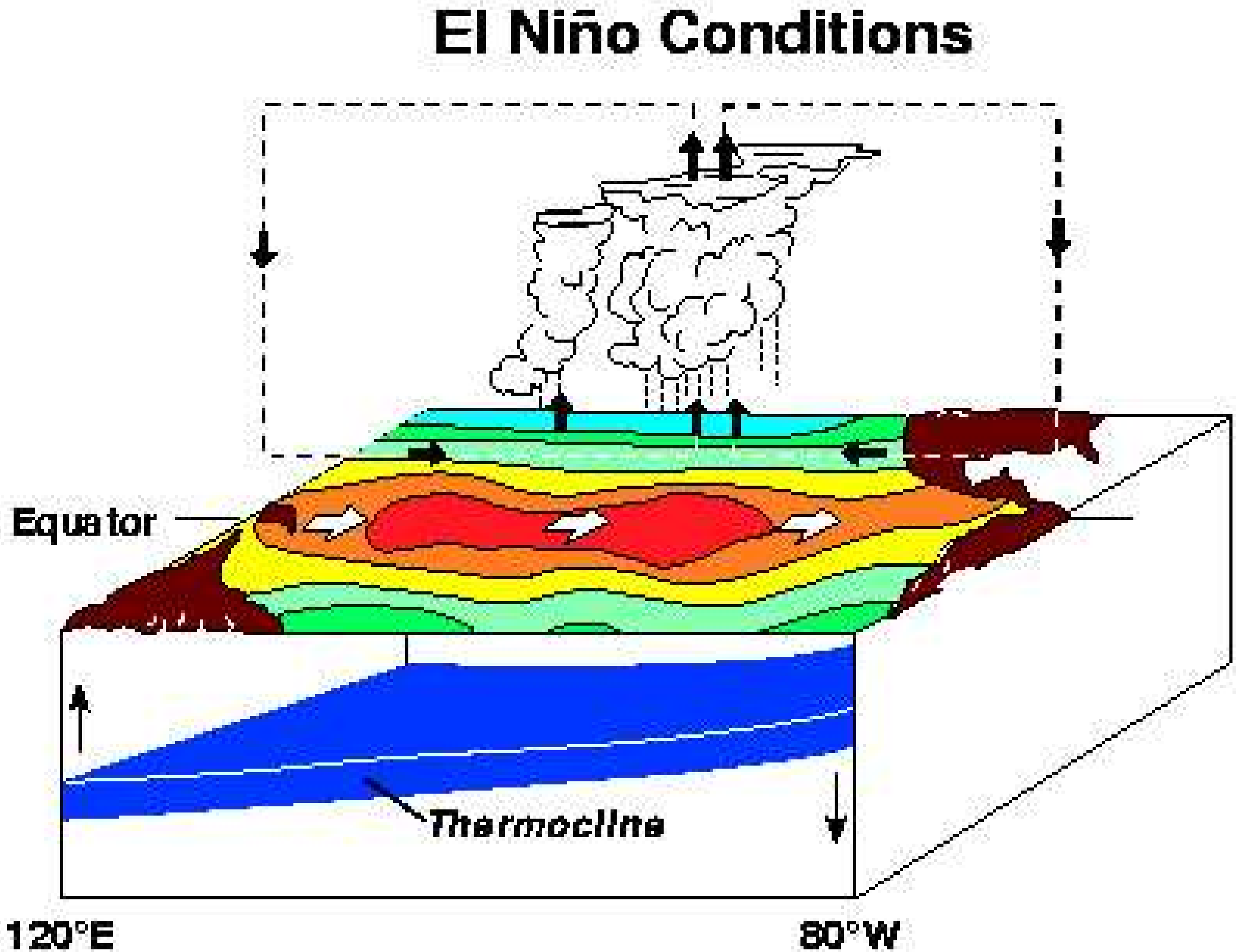
UVic

Tropical Pacific: Mean State



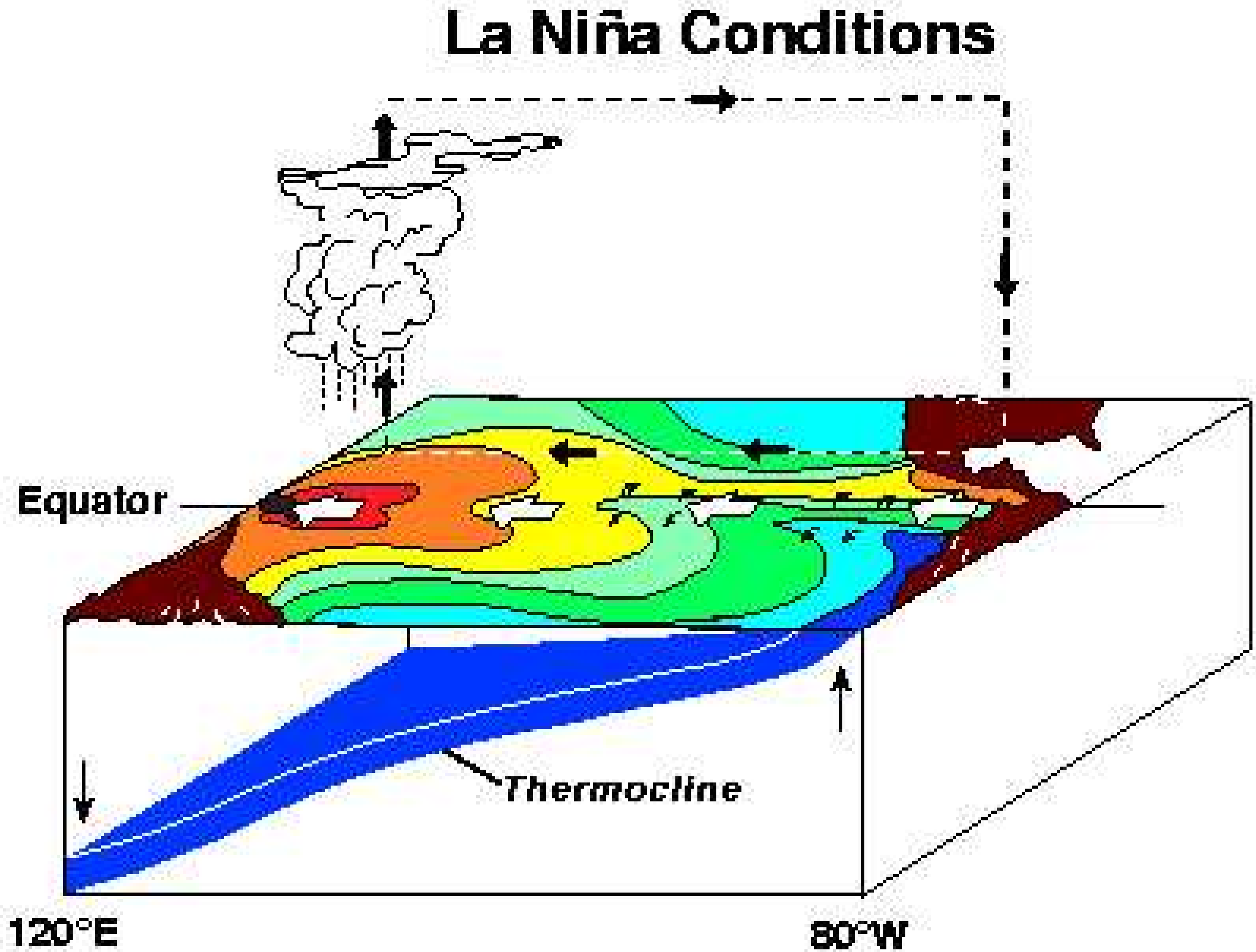
From www.pmel.noaa.gov/tao/elnino/nino-home.html

Tropical Pacific: El Niño State



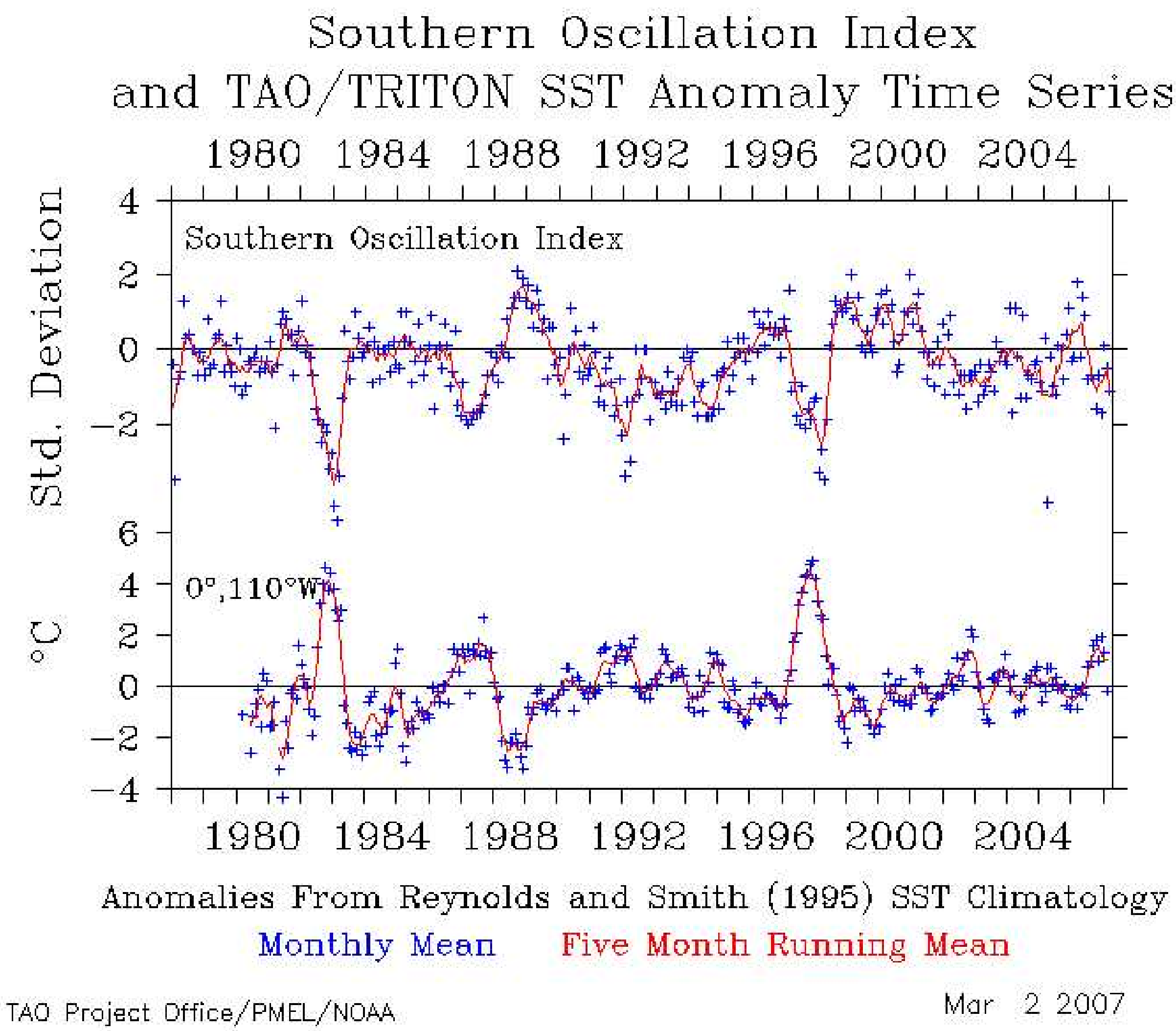
UVic From www.pmel.noaa.gov/tao/elnino/nino-home.html

Tropical Pacific: La Niña State



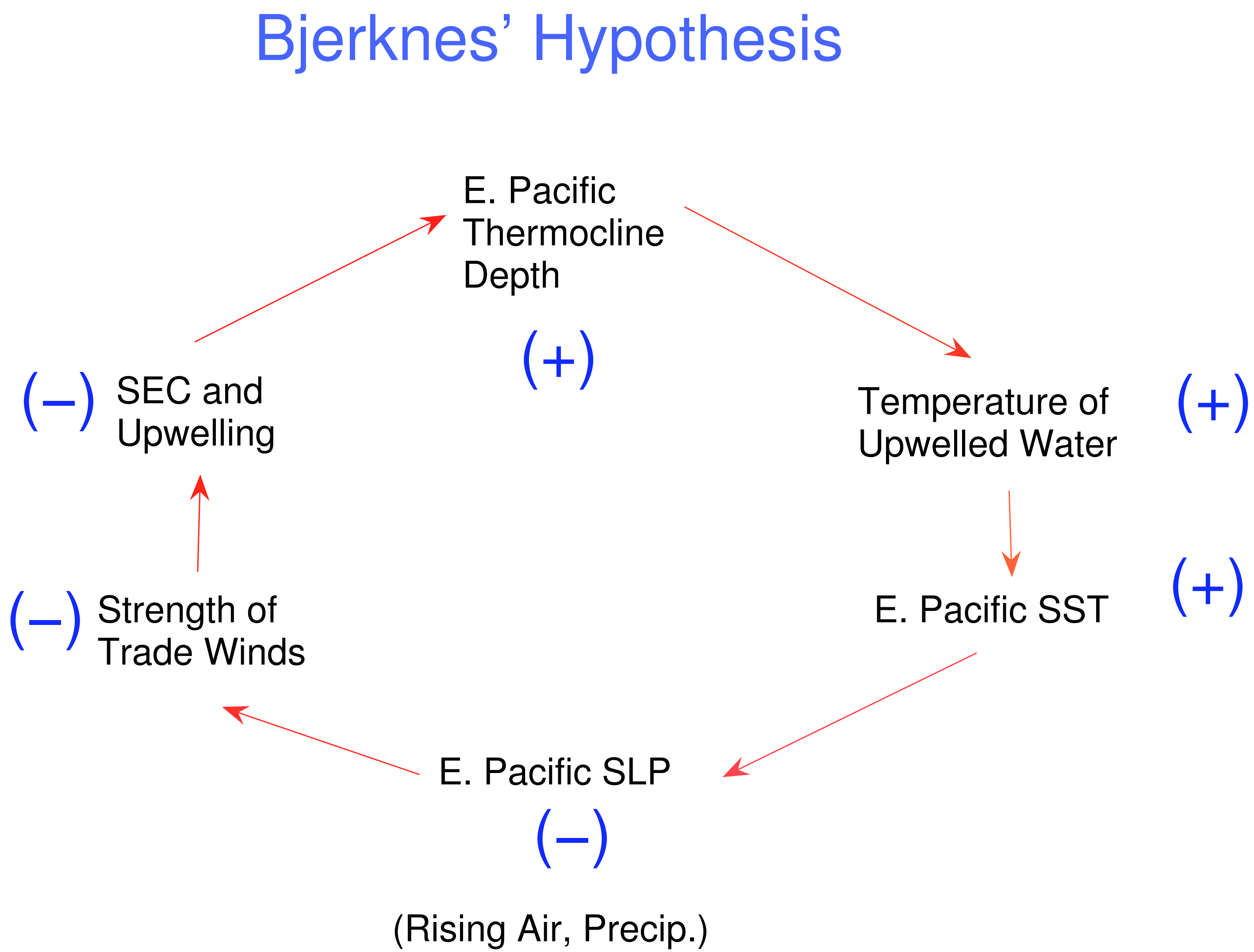
From www.pmel.noaa.gov/tao/elnino/nino-home.html

ENSO Indices: SOI and East Pacific SST

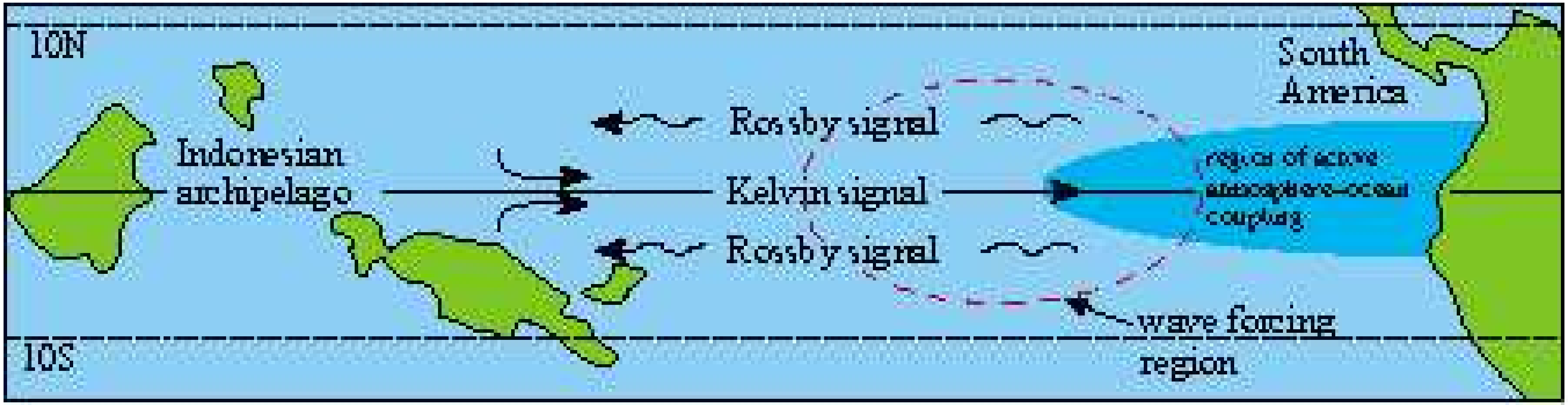


UVic From www.pmel.noaa.gov/tao/elnino/nino-home.html

El Niño Growth: Bjerknes' Hypothesis

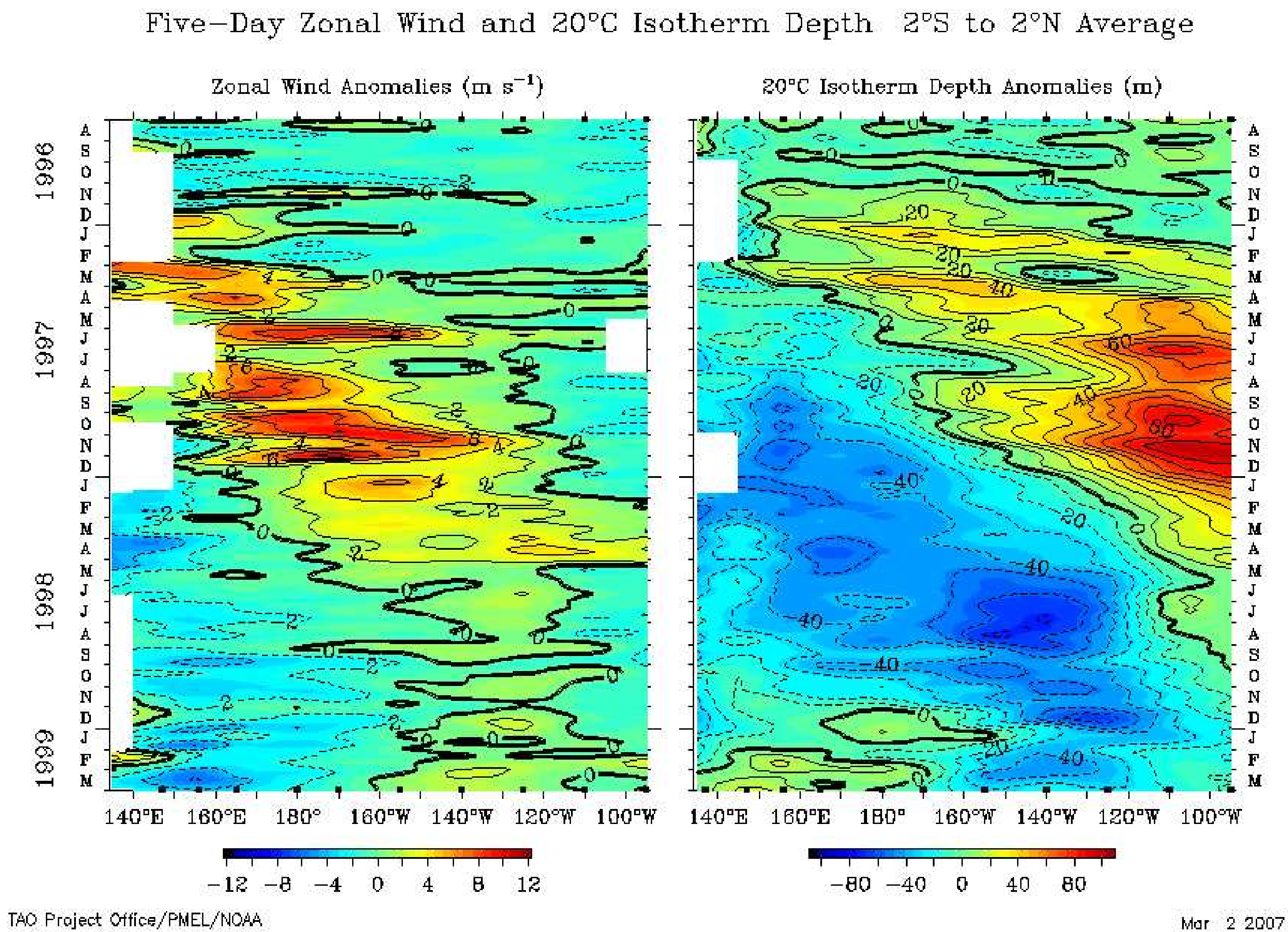


ENSO Cycles: Delayed Oscillator Mechanism



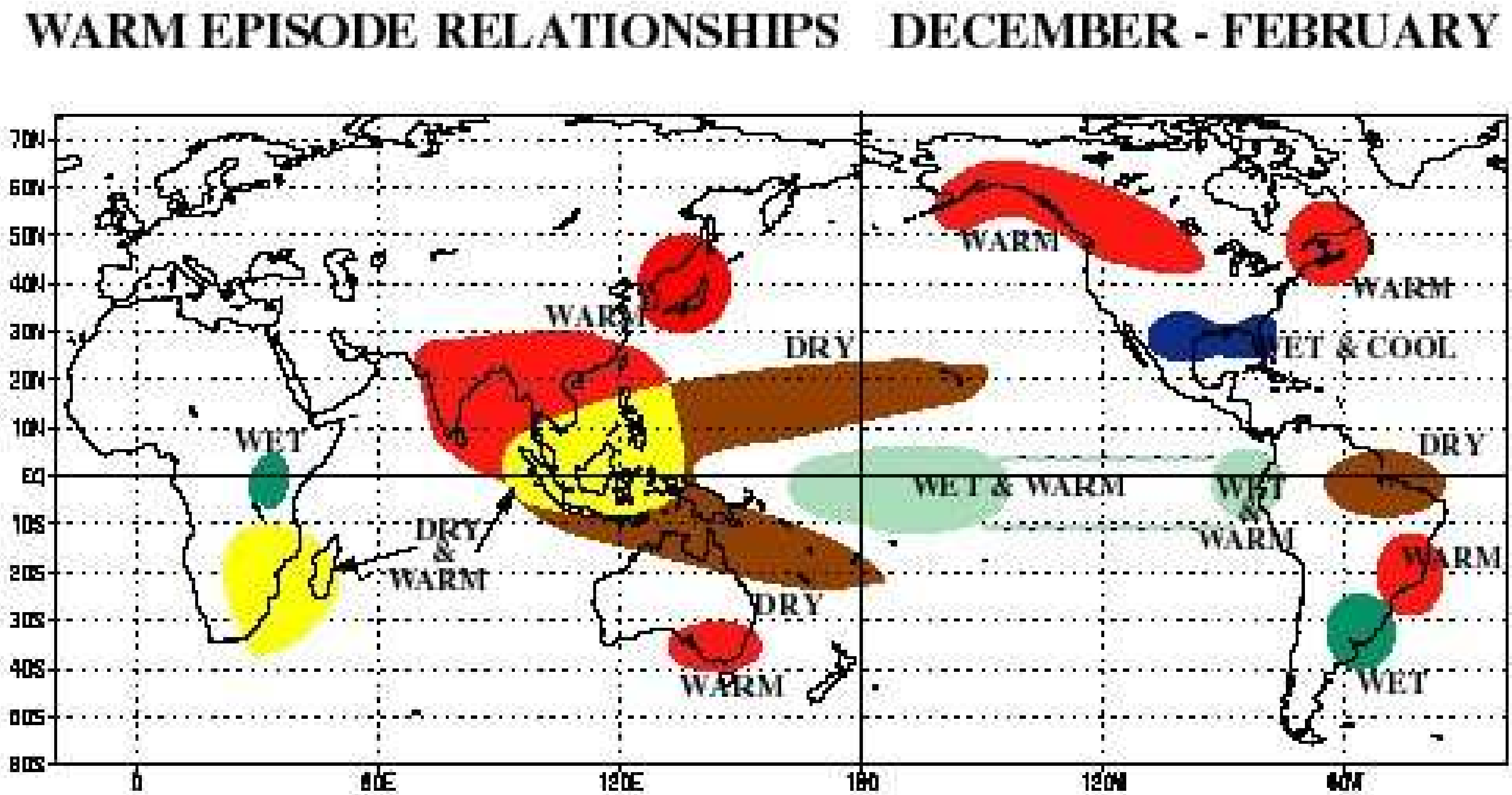
From Chang and Battisti, *Physics World*, 1998.

ENSO Irregularity: Stochastic Oscillator Mechanism



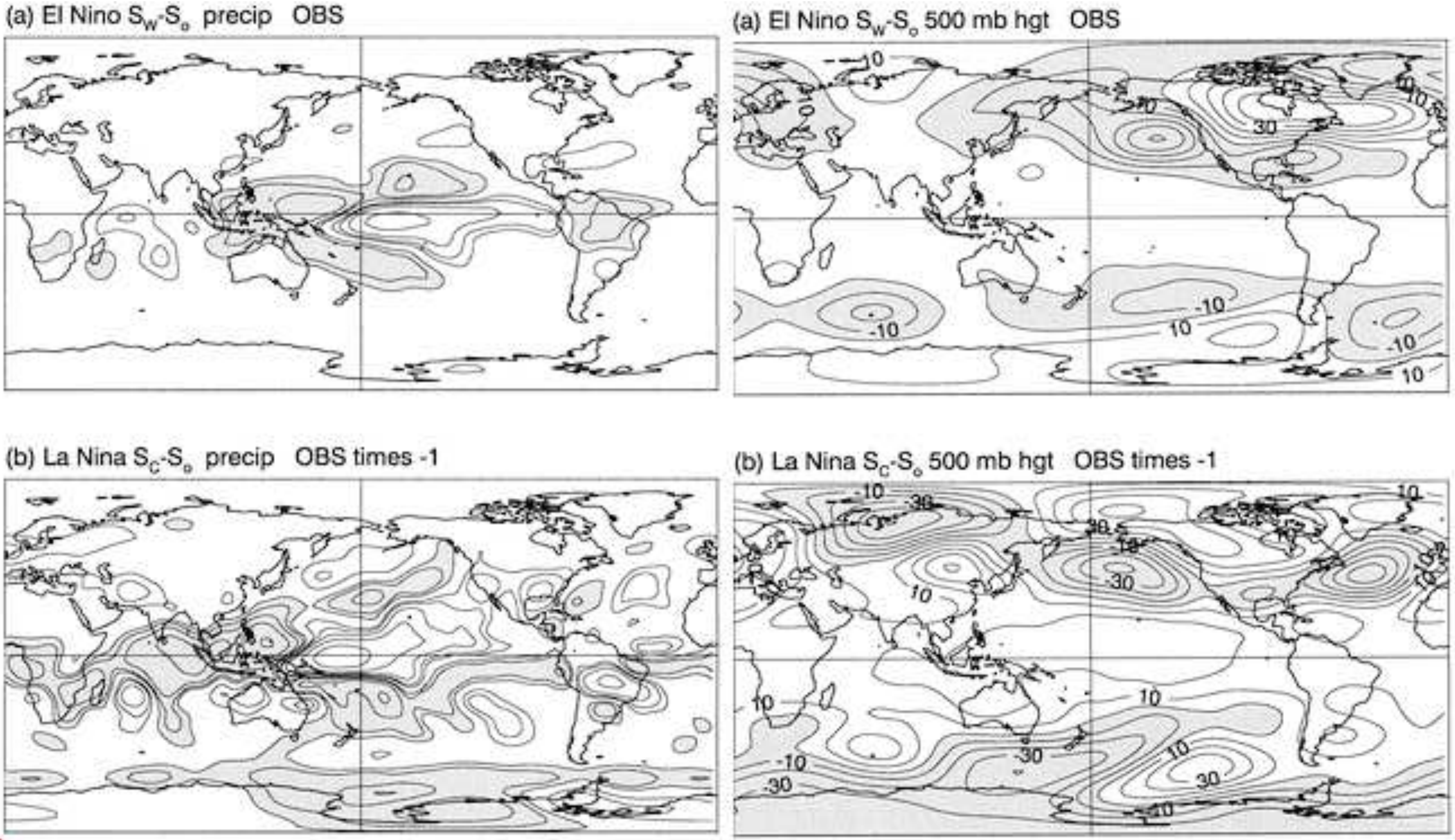
From www.pmel.noaa.gov/tao/elnino/nino-home.html

El Niño Impacts: Northern Hemisphere Winter



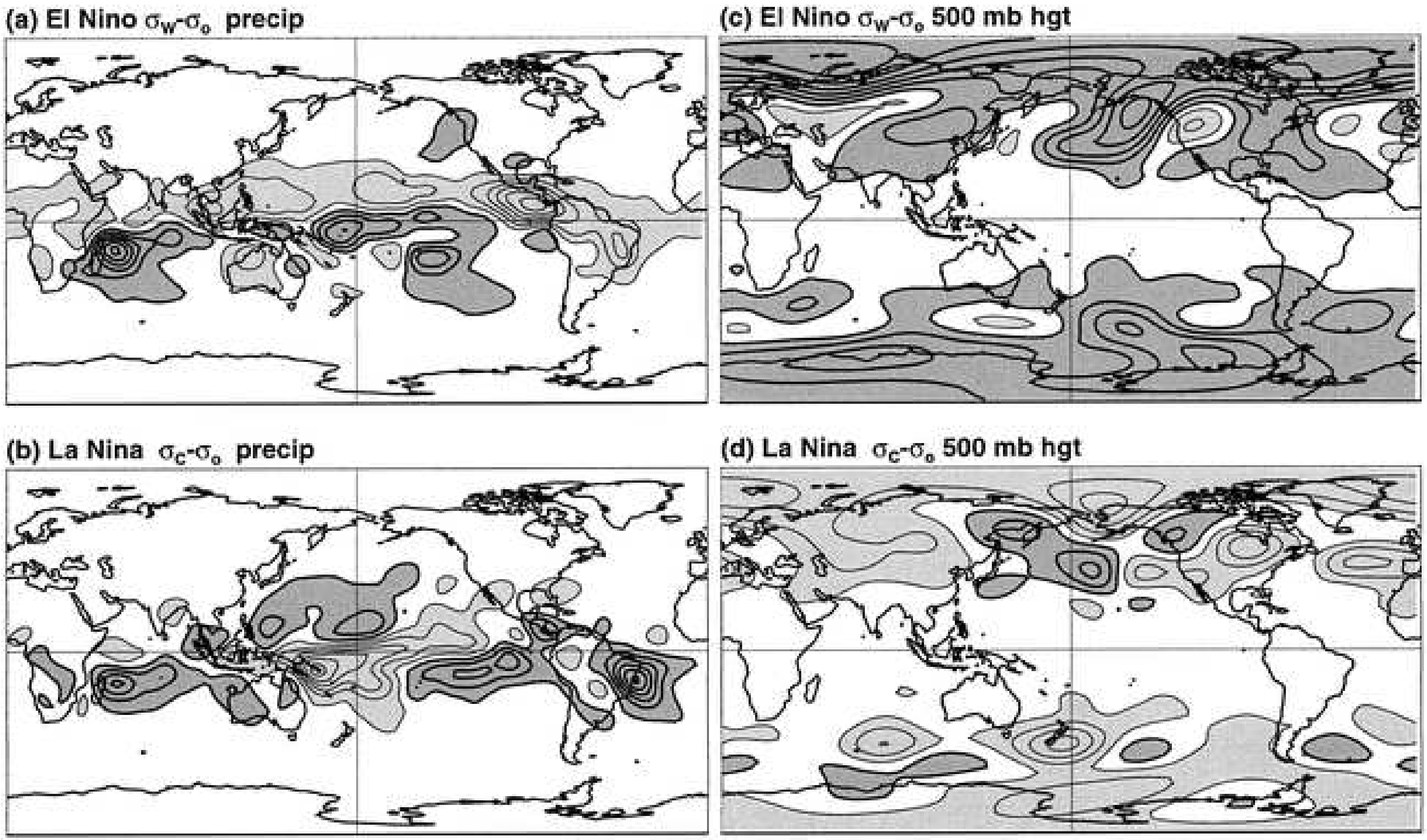
From iri.columbia.edu/climate/ENSO/

ENSO Impacts: Changes in Mean Climate



Adapted from Sardeshmukh et al., J. Clim (2000)

ENSO Impacts: Changes in Climate Variability



From Sardeshmukh et al., J. Clim (2000)

ENSO Impacts: Extreme Events

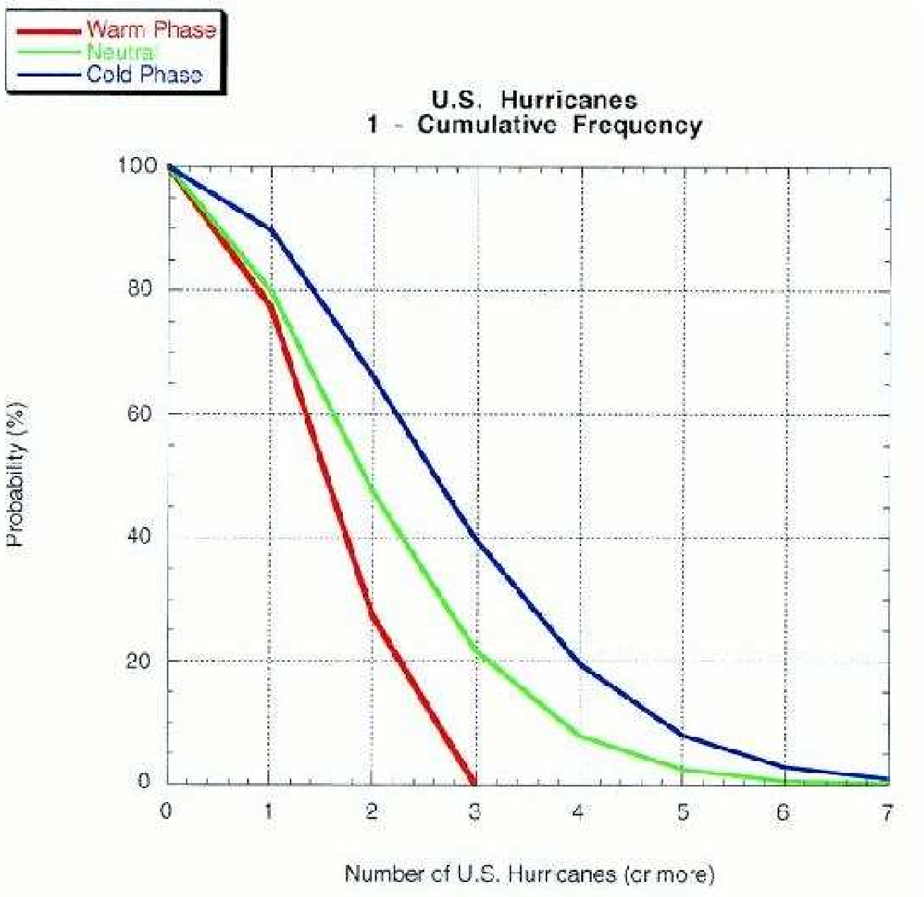


FIG. 1. Inverse cumulative frequency distributions of U.S. landfalling hurricanes, 1900–97. Red line indicates warm phase of ENSO, blue line indicates cold phase of ENSO, green line indicates neutral ENSO conditions.

UVic

From Bove et al., J. Clim (1998)

ENSO Dynamics

- As a first approximation, ENSO dynamics modelled as stable linear dynamical system driven by noise:

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + B(\mathbf{x}) \circ \dot{\mathbf{W}}$$

ENSO Dynamics

- As a first approximation, ENSO dynamics modelled as stable linear dynamical system driven by noise:

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + B(\mathbf{x}) \circ \dot{\mathbf{W}}$$

where

$\mathbf{x}$	is the state vector (e.g. sea surface temperature field)
A	is the linearised dynamical operator
$B(\mathbf{x})$	is the noise amplitude matrix
$\dot{\mathbf{W}}$	is a vector of independent white noise processes

UVic

ENSO Dynamics

- As a first approximation, ENSO dynamics modelled as stable linear dynamical system driven by noise:

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + B(\mathbf{x}) \circ \dot{\mathbf{W}}$$

  where

$\mathbf{x}$	is the state vector (e.g. sea surface temperature field)
A	is the linearised dynamical operator
$B(\mathbf{x})$	is the noise amplitude matrix
$\dot{\mathbf{W}}$	is a vector of independent white noise processes

- For simplicity, will assume that B is state-independent (additive noise)

UVic

ENSO Dynamics

- Analytic solution to SDE:

ENSO Dynamics

- Analytic solution to SDE:

$$\mathbf{x}(t) = P(t)\mathbf{x}(0) + \int_0^t P(t - t')B\dot{\mathbf{W}}(t')dt'$$

ENSO Dynamics

- Analytic solution to SDE:

$$\mathbf{x}(t) = P(t)\mathbf{x}(0) + \int_0^t P(t-t')B\dot{\mathbf{W}}(t')dt'$$

where

ENSO Dynamics

- Analytic solution to SDE:

$$\mathbf{x}(t) = P(t)\mathbf{x}(0) + \int_0^t P(t-t')B\dot{\mathbf{W}}(t')dt'$$

where

$$P(t) = \exp(At)$$

UVic

ENSO Dynamics

- Analytic solution to SDE:

$$\mathbf{x}(t) = P(t)\mathbf{x}(0) + \int_0^t P(t-t')B\dot{\mathbf{W}}(t')dt'$$

where

$$P(t) = \exp(At)$$

- Evolution of moments:

$$\langle \mathbf{x}(t) \rangle = P(t)\mathbf{x}(0) \;\; ; \;\; \left\langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \right\rangle = P(\tau) < \mathbf{x}(t)\mathbf{x}(t)^T >$$

ENSO Dynamics

- Analytic solution to SDE:

$$\mathbf{x}(t) = P(t)\mathbf{x}(0) + \int_0^t P(t-t')B\dot{\mathbf{W}}(t')dt'$$

where

$$P(t) = \exp(At)$$

- Evolution of moments:

$$\langle \mathbf{x}(t) \rangle = P(t)\mathbf{x}(0) \;\; ; \;\; \left\langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \right\rangle = P(\tau) < \mathbf{x}(t)\mathbf{x}(t)^T >$$

- Assuming max(Re(eig(A)))$<$ 0, stationary covariance

UVic

ENSO Dynamics

- Analytic solution to SDE:

$$\mathbf{x}(t) = P(t)\mathbf{x}(0) + \int_0^t P(t-t')B\dot{\mathbf{W}}(t')dt'$$

where

$$P(t) = \exp(At)$$

- Evolution of moments:

$$\langle \mathbf{x}(t) \rangle = P(t)\mathbf{x}(0) \;\; ; \;\; \langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \rangle = P(\tau) < \mathbf{x}(t)\mathbf{x}(t)^T >$$

- Assuming max(Re(eig(A)))< 0, stationary covariance

$$C = \langle \mathbf{x}\mathbf{x}^T \rangle$$

UVic

ENSO Dynamics

- Analytic solution to SDE:

$$\mathbf{x}(t) = P(t)\mathbf{x}(0) + \int_0^t P(t-t')B\dot{\mathbf{W}}(t')dt'$$

where

$$P(t) = \exp(At)$$

- Evolution of moments:

$$\langle \mathbf{x}(t) \rangle = P(t)\mathbf{x}(0) \;\; ; \;\; \left\langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \right\rangle = P(\tau) < \mathbf{x}(t)\mathbf{x}(t)^T >$$

- Assuming max(Re(eig(A)))<0, stationary covariance

$$C = \left\langle \mathbf{x}\mathbf{x}^T \right\rangle$$

satisfies fluctuation-dissipation relationship

UVic

ENSO Dynamics

- Analytic solution to SDE:

$$\mathbf{x}(t) = P(t)\mathbf{x}(0) + \int_0^t P(t-t')B\dot{\mathbf{W}}(t')dt'$$

where

$$P(t) = \exp(At)$$

- Evolution of moments:

$$\langle \mathbf{x}(t) \rangle = P(t)\mathbf{x}(0) \;\; ; \;\; \left\langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \right\rangle = P(\tau) < \mathbf{x}(t)\mathbf{x}(t)^T >$$

- Assuming max(Re(eig(A)))< 0, stationary covariance

$$C = \left\langle \mathbf{x}\mathbf{x}^T \right\rangle$$

satisfies fluctuation-dissipation relationship

UVic

$$AC + CA^T = -BB^T$$

Optimal Perturbations

- Linearised operator A will generally be *non-normal*, i.e.

$$AA^T \neq A^T A$$

UVic

Optimal Perturbations

- Linearised operator A will generally be *non-normal*, i.e.

$$AA^T \neq A^T A$$

so eigenvectors of A are not orthogonal, with consequences:

Optimal Perturbations

- Linearised operator A will generally be *non-normal*, i.e.

$$AA^T \neq A^T A$$

  so eigenvectors of A are not orthogonal, with consequences:
  - eigenvectors of covariance C (EOFs) do not coincide with eigenvectors of A (dynamical modes)

UVic

Optimal Perturbations

- Linearised operator A will generally be *non-normal*, i.e.

$$AA^T \neq A^T A$$

so eigenvectors of A are not orthogonal, with consequences:

  - eigenvectors of covariance C (EOFs) do not coincide with eigenvectors of A (dynamical modes)
  - perturbation norm (with metric M)

UVic

Optimal Perturbations

- Linearised operator A will generally be *non-normal*, i.e.

$$AA^T \neq A^T A$$

so eigenvectors of A are not orthogonal, with consequences:

  - eigenvectors of covariance C (EOFs) do not coincide with eigenvectors of A (dynamical modes)
  - perturbation norm (with metric M)

$$N(t) = \mathbf{x}(t)^T M \mathbf{x}(t) = \mathbf{x}(0)^T P(t)^T M P(t) \mathbf{x}(0)$$

UVic

Optimal Perturbations

- Linearised operator A will generally be *non-normal*, i.e.

$$AA^T \neq A^T A$$

so eigenvectors of A are not orthogonal, with consequences:

  - eigenvectors of covariance C (EOFs) do not coincide with eigenvectors of A (dynamical modes)
  - perturbation norm (with metric M)

$$N(t) = \mathbf{x}(t)^T M \mathbf{x}(t) = \mathbf{x}(0)^T P(t)^T M P(t) \mathbf{x}(0)$$

  may grow (by potentially large amount) over finite times even though asymptotically stable:

$$\lim_{t \to \infty} N(t) = 0$$

UVic

Optimal Perturbations

- Defining amplification factor

Optimal Perturbations

- Defining amplification factor

$$n(t) = \frac{\mathbf{x}(0)^T P(t)^T M P(t) \mathbf{x}(0)}{\mathbf{x}(0)^T M \mathbf{x}(0)}$$

Optimal Perturbations

- Defining amplification factor

$$n(t) = \frac{\mathbf{x}(0)^T P(t)^T M P(t) \mathbf{x}(0)}{\mathbf{x}(0)^T M \mathbf{x}(0)}$$

optimal perturbation $\mathbf{e}$ maximises $n(t)$ subject to constraint $\mathbf{x}(0)^T M \mathbf{x}(0) = 1 \Rightarrow$ generalised eigenvalue problem:

UVic

Optimal Perturbations

- Defining amplification factor

$$n(t) = \frac{\mathbf{x}(0)^T P(t)^T M P(t) \mathbf{x}(0)}{\mathbf{x}(0)^T M \mathbf{x}(0)}$$

optimal perturbation $\mathbf{e}$ maximises $n(t)$ subject to constraint $\mathbf{x}(0)^T M \mathbf{x}(0) = 1 \Rightarrow$ generalised eigenvalue problem:

$$P(t)^T M P(t) \mathbf{e} = \lambda M \mathbf{e}$$

UVic

Optimal Perturbations

- Defining amplification factor

$$n(t) = \frac{\mathbf{x}(0)^T P(t)^T M P(t) \mathbf{x}(0)}{\mathbf{x}(0)^T M \mathbf{x}(0)}$$

optimal perturbation $\mathbf{e}$ maximises $n(t)$ subject to constraint $\mathbf{x}(0)^T M \mathbf{x}(0) = 1 \Rightarrow$ generalised eigenvalue problem:

$$P(t)^T M P(t)\mathbf{e} = \lambda M \mathbf{e}$$

- Response to fluctuating forcing:

$$\text{var}(\mathbf{X}(t)) = B^T \left(\int_0^t P(s)^T M P(s) ds \right) B$$

UVic

Optimal Perturbations

- Defining amplification factor

$$n(t) = \frac{\mathbf{x}(0)^T P(t)^T M P(t) \mathbf{x}(0)}{\mathbf{x}(0)^T M \mathbf{x}(0)}$$

optimal perturbation $\mathbf{e}$ maximises $n(t)$ subject to constraint $\mathbf{x}(0)^T M \mathbf{x}(0) = 1 \Rightarrow$ generalised eigenvalue problem:

$$P(t)^T M P(t) \mathbf{e} = \lambda M \mathbf{e}$$

- Response to fluctuating forcing:

$$\text{var}(\mathbf{X}(t)) = B^T \left(\int_0^t P(s)^T M P(s) ds \right) B$$

$\Rightarrow$ importance of projection of noise structure on “average” optimals for maintaining variance;

“stochastic optimals”

UVic

Stochastic ENSO Models

- How are A and B determined?

Stochastic ENSO Models

- How are A and B determined?

1. **Empirical: Linear Inverse Modelling**

UVic

Stochastic ENSO Models

- How are A and B determined?

1. **Empirical: Linear Inverse Modelling**
    - estimate covariances from observations and compute

$$A = \frac{1}{\tau} \ln \left(\left\langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \right\rangle \left\langle \mathbf{x}(t)\mathbf{x}(t)^T \right\rangle^{-1} \right)$$

Stochastic ENSO Models

- How are A and B determined?

1. **Empirical: Linear Inverse Modelling**
   - estimate covariances from observations and compute

$$A = \frac{1}{\tau} \ln \left(\left\langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \right\rangle \left\langle \mathbf{x}(t)\mathbf{x}(t)^T \right\rangle^{-1} \right)$$

   - Compute B from Lyapunov equation

$$A \left\langle \mathbf{x}\mathbf{x}^T \right\rangle + \left\langle \mathbf{x}\mathbf{x}^T \right\rangle A^T = -BB^T$$

UVic

Stochastic ENSO Models

- How are A and B determined?

1. **Empirical: Linear Inverse Modelling**
    - estimate covariances from observations and compute

$$A = \frac{1}{\tau} \ln \left(\left\langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \right\rangle \left\langle \mathbf{x}(t)\mathbf{x}(t)^T \right\rangle^{-1} \right)$$

    - Compute B from Lyapunov equation

$$A \left\langle \mathbf{x}\mathbf{x}^T \right\rangle + \left\langle \mathbf{x}\mathbf{x}^T \right\rangle A^T = -BB^T$$

    - Issues:

UVic

Stochastic ENSO Models

- How are A and B determined?

1. **Empirical: Linear Inverse Modelling**

   - estimate covariances from observations and compute

   $$A = \frac{1}{\tau} \ln \left(\left\langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \right\rangle \left\langle \mathbf{x}(t)\mathbf{x}(t)^T \right\rangle^{-1} \right)$$

   - Compute B from Lyapunov equation

   $$A \left\langle \mathbf{x}\mathbf{x}^T \right\rangle + \left\langle \mathbf{x}\mathbf{x}^T \right\rangle A^T = -BB^T$$

   - Issues:

     i. if $\mathbf{x}(t)$ not truly Markov, estimates will depend on lag τ

UVic

Stochastic ENSO Models

- How are A and B determined?

1. **Empirical: Linear Inverse Modelling**
   - estimate covariances from observations and compute

$$A = \frac{1}{\tau} \ln \left(\left\langle \mathbf{x}(t+\tau)\mathbf{x}(t)^T \right\rangle \left\langle \mathbf{x}(t)\mathbf{x}(t)^T \right\rangle^{-1} \right)$$

   - Compute B from Lyapunov equation

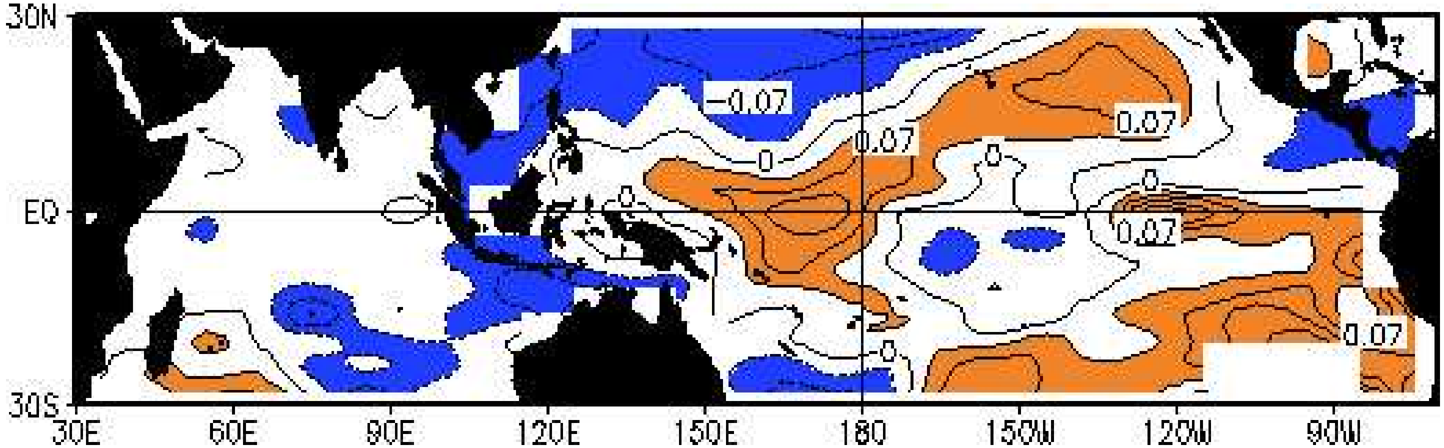
$$A \left\langle \mathbf{x}\mathbf{x}^T \right\rangle + \left\langle \mathbf{x}\mathbf{x}^T \right\rangle A^T = -BB^T$$

   - Issues:
     i. if $\mathbf{x}(t)$ not truly Markov, estimates will depend on lag τ
     ii. must enforce positive-definiteness of BB^T

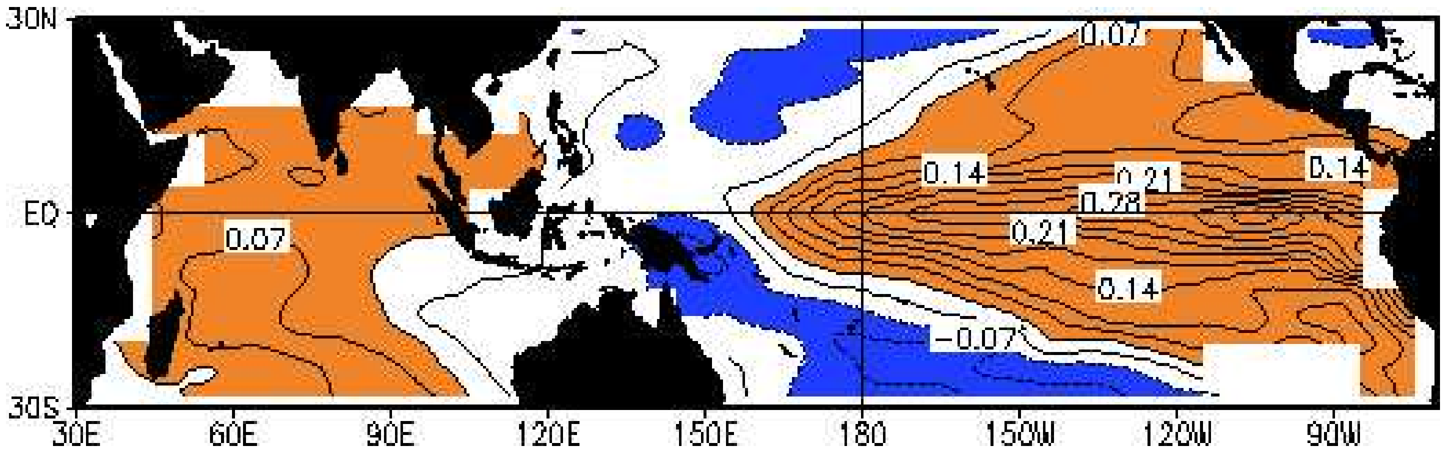
UVic

LIM: Optimal Perturbations from Observations

Optimal perturbation $\mathbf{e}$



Perturbation $P(t)e$ at $t = 7$ months



From Penland and Sardeshmukh (1995)

LIM: Response to Stochastic Forcing

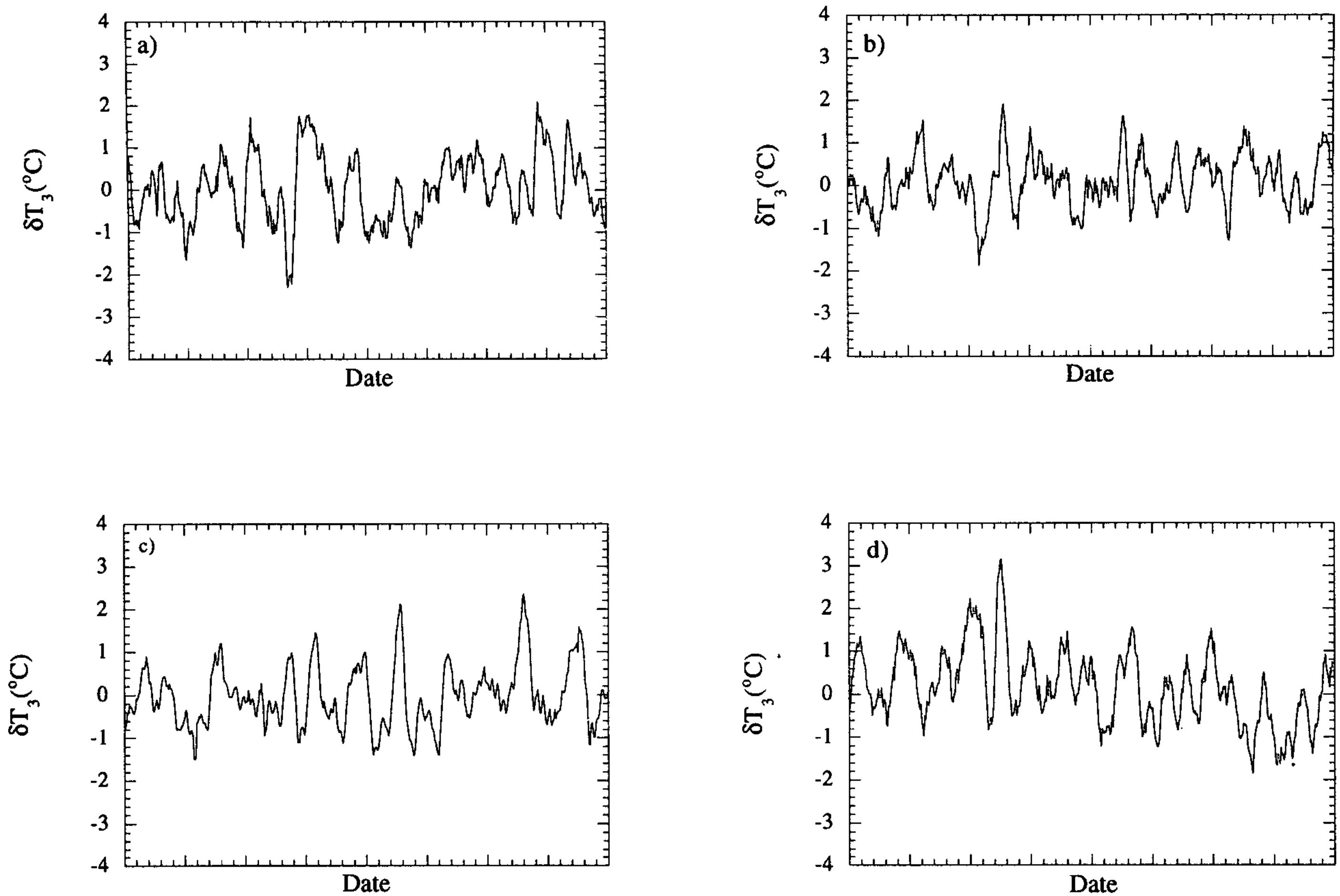


FIG. 15. (a)–(d) Three 40-yr segments of the Niño-3 SST anomaly calculated from output generated by the linear model. Also shown is the measured record. Which is which?

UVic

From Penland and Sardeshmukh (1995)

Stochastic ENSO Models

2. **Mechanistic: Stochastic Reduction of Tangent Linear Model**

Stochastic ENSO Models

2. **Mechanistic: Stochastic Reduction of Tangent Linear Model**

- Partition dynamics into "slow" and "fast" variables $\mathbf{x}$ and $\mathbf{y}$:

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{f}(\mathbf{x}, \mathbf{y}) \\ \frac{d\mathbf{y}}{dt} &= \frac{1}{\epsilon}\mathbf{g}(\mathbf{x}, \mathbf{y}) \end{aligned}$$

UVic

Stochastic ENSO Models

2. **Mechanistic: Stochastic Reduction of Tangent Linear Model**

- Partition dynamics into "slow" and "fast" variables $\mathbf{x}$ and $\mathbf{y}$:

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{f}(\mathbf{x}, \mathbf{y}) \\ \frac{d\mathbf{y}}{dt} &= \frac{1}{\epsilon}\mathbf{g}(\mathbf{x}, \mathbf{y}) \end{aligned}$$

- Reduce coupled system to effective stochastic dynamics for $\mathbf{x}$:

$$\frac{d\mathbf{x}}{dt} = L\mathbf{x} + N(\mathbf{x}, \mathbf{x}) + S(\mathbf{x}) \circ \dot{\mathbf{W}}$$

UVic

Stochastic ENSO Models

2. **Mechanistic: Stochastic Reduction of Tangent Linear Model**

- Partition dynamics into "slow" and "fast" variables $\mathbf{x}$ and $\mathbf{y}$:

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{f}(\mathbf{x},\mathbf{y}) \\ \frac{d\mathbf{y}}{dt} &= \frac{1}{\epsilon}\mathbf{g}(\mathbf{x},\mathbf{y}) \end{aligned}$$

- Reduce coupled system to effective stochastic dynamics for $\mathbf{x}$:

$$\frac{d\mathbf{x}}{dt} = L\mathbf{x} + N(\mathbf{x},\mathbf{x}) + S(\mathbf{x}) \circ \dot{\mathbf{W}}$$

(many ways of doing this; some formal, some ad hoc)

UVic

Stochastic ENSO Models

2. **Mechanistic: Stochastic Reduction of Tangent Linear Model**

- Partition dynamics into "slow" and "fast" variables $\mathbf{x}$ and $\mathbf{y}$:

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{f}(\mathbf{x}, \mathbf{y}) \\ \frac{d\mathbf{y}}{dt} &= \frac{1}{\epsilon}\mathbf{g}(\mathbf{x}, \mathbf{y}) \end{aligned}$$

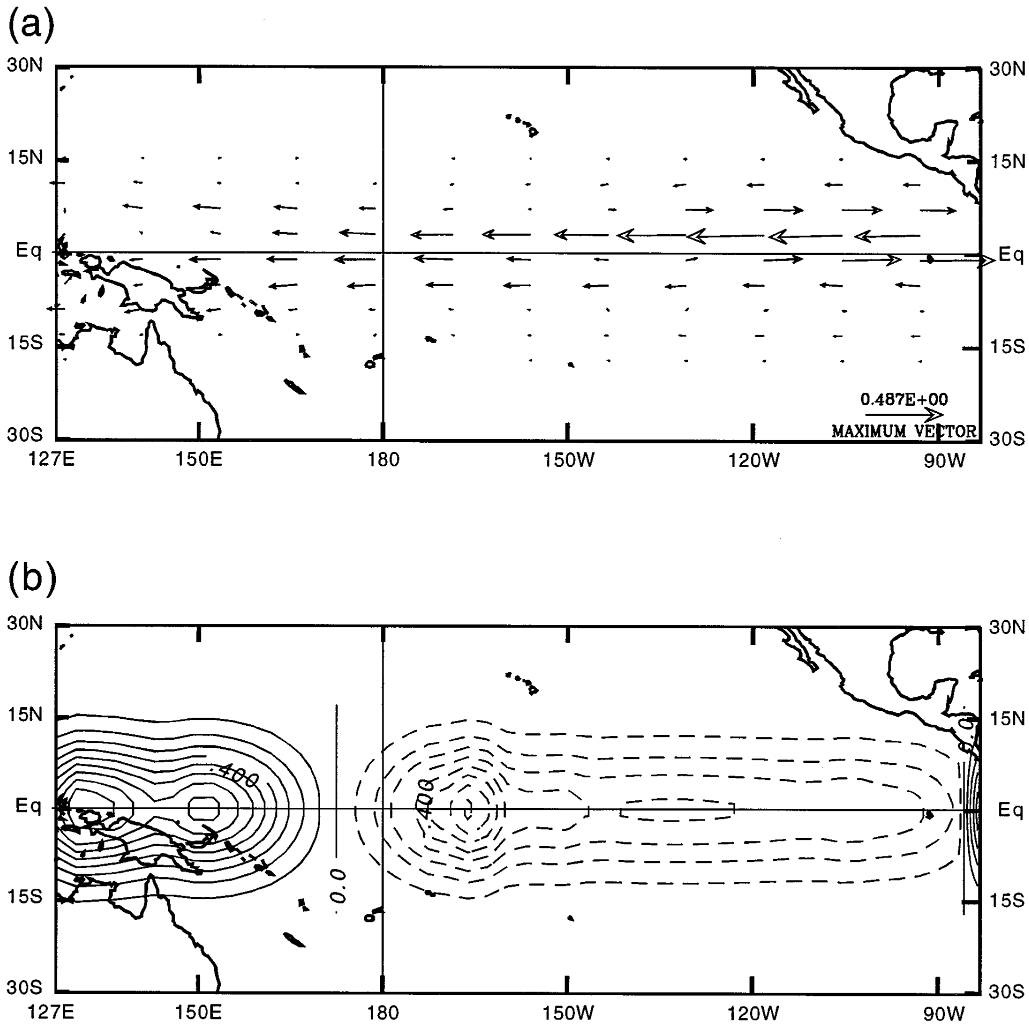
- Reduce coupled system to effective stochastic dynamics for $\mathbf{x}$:

$$\frac{d\mathbf{x}}{dt} = L\mathbf{x} + N(\mathbf{x}, \mathbf{x}) + S(\mathbf{x}) \circ \dot{\mathbf{W}}$$

  (many ways of doing this; some formal, some ad hoc)

- Linearise model around appropriate state (e.g. climatological mean)

Mechanistic Model: 6-Month Stochastic Optimals



From Kleeman and Moore, *J. Atmos. Sci.*, 1997.

Prediction Utility

- Can assess "utility" of ensemble prediction $p(\mathbf{x})$ by comparing forecast pdf with "climatological" pdf $q(\mathbf{x})$, using relative entropy

Prediction Utility

- Can assess "utility" of ensemble prediction $p(\mathbf{x})$ by comparing forecast pdf with "climatological" pdf $q(\mathbf{x})$, using relative entropy

$$R = \int p(\mathbf{x}) \ln \left(\frac{p(\mathbf{x})}{q(\mathbf{x})} \right) d\mathbf{x}$$

UVic

Prediction Utility

- Can assess "utility" of ensemble prediction $p(\mathbf{x})$ by comparing forecast pdf with "climatological" pdf $q(\mathbf{x})$, using relative entropy

$$R = \int p(\mathbf{x}) \ln\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) d\mathbf{x}$$

- Assuming p and q are both Gaussian, can decompose R:

Prediction Utility

- Can assess "utility" of ensemble prediction $p(\mathbf{x})$ by comparing forecast pdf with "climatological" pdf $q(\mathbf{x})$, using relative entropy

$$R = \int p(\mathbf{x}) \ln \left(\frac{p(\mathbf{x})}{q(\mathbf{x})} \right) d\mathbf{x}$$

- Assuming p and q are both Gaussian, can decompose R:

$$R = \text{Signal} + \text{Dispersion}$$

UVic

Prediction Utility

- Can assess “utility” of ensemble prediction $p(\mathbf{x})$ by comparing forecast pdf with “climatological” pdf $q(\mathbf{x})$, using relative entropy

$$R = \int p(\mathbf{x}) \ln \left(\frac{p(\mathbf{x})}{q(\mathbf{x})} \right) d\mathbf{x}$$

- Assuming p and q are both Gaussian, can decompose R:

$$R = \text{Signal} + \text{Dispersion}$$

where

UVic

Prediction Utility

- Can assess "utility" of ensemble prediction $p(\mathbf{x})$ by comparing forecast pdf with "climatological" pdf $q(\mathbf{x})$, using relative entropy

$$R = \int p(\mathbf{x}) \ln \left(\frac{p(\mathbf{x})}{q(\mathbf{x})} \right) d\mathbf{x}$$

- Assuming p and q are both Gaussian, can decompose R:

$$R = \text{Signal} + \text{Dispersion}$$

where

$$\begin{aligned} \text{Signal} &= \frac{1}{2}(\mu_p - \mu_q)^T \Sigma_q^{-2} (\mu_p - \mu_q) - \frac{n}{2} \\ \text{Disp} &= \frac{1}{2} \ln \left(\frac{\det(\Sigma_q^2)}{\det(\Sigma_p^2)} \right) + \text{tr}(\Sigma_p^2 \Sigma_q^{-2}) \end{aligned}$$

UVic

“Observed” Prediction Utility

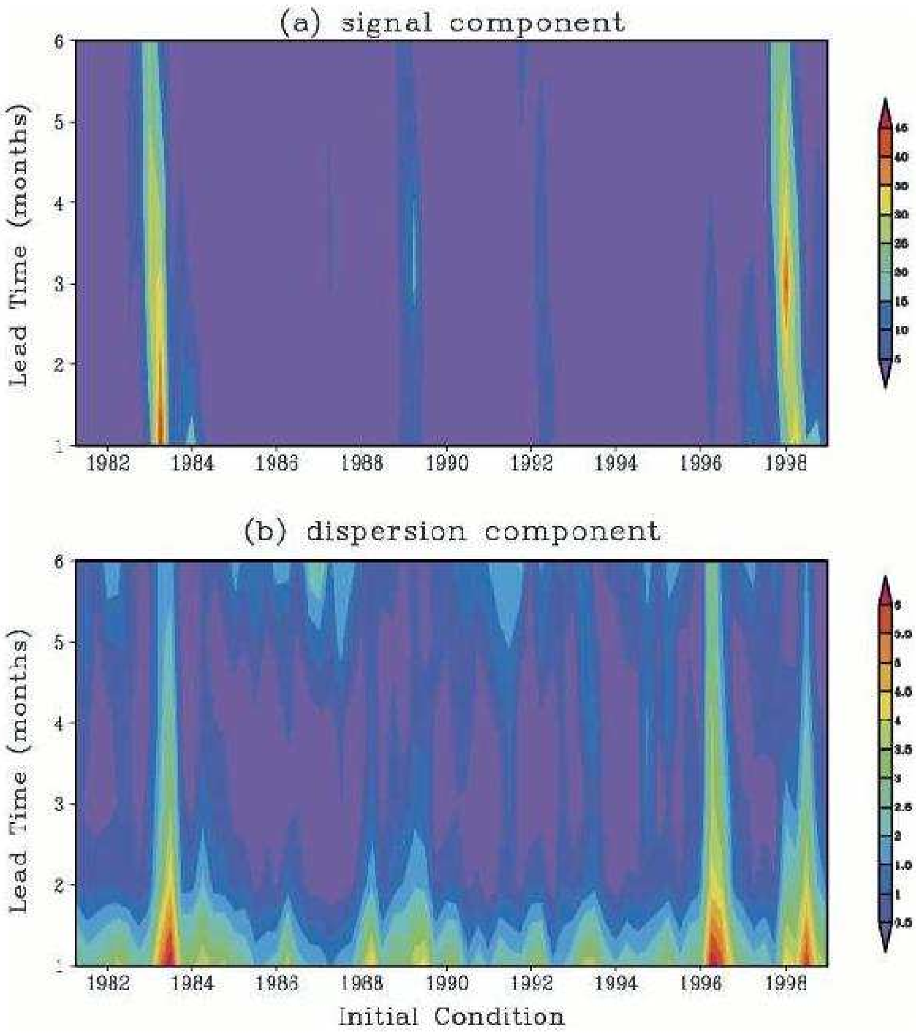
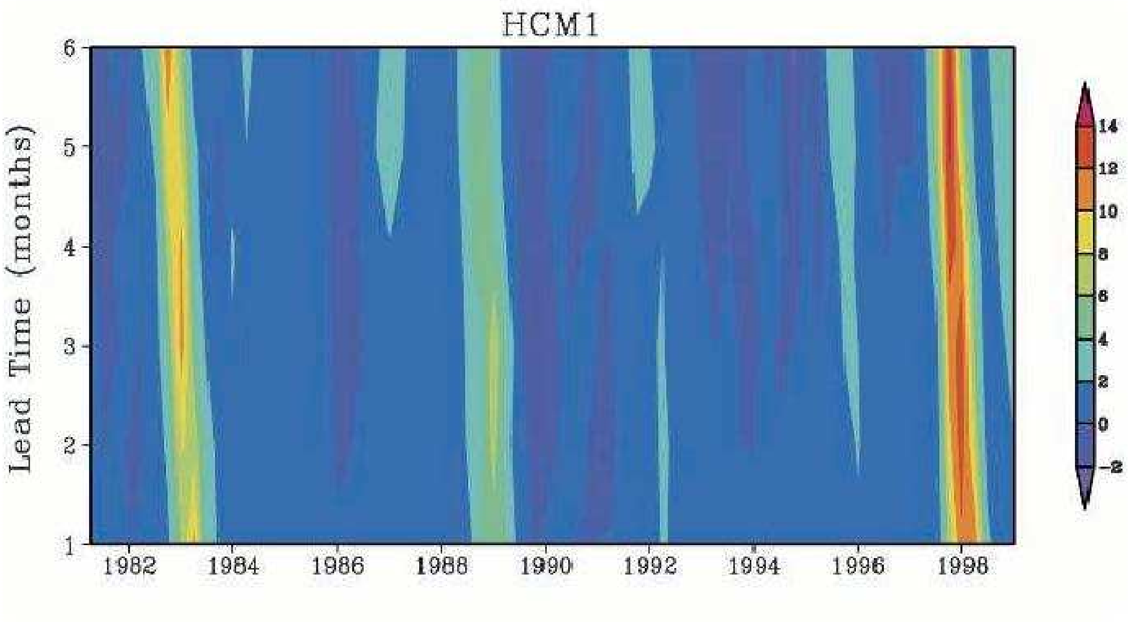


FIG. 11. Signal component (SC) and dispersion component (DC) of R for HCM1.

From Tang, Kleeman, & Moore, *J. Atmos. Sci.*, 2005.

Individual Forecast Contributions to Correlation

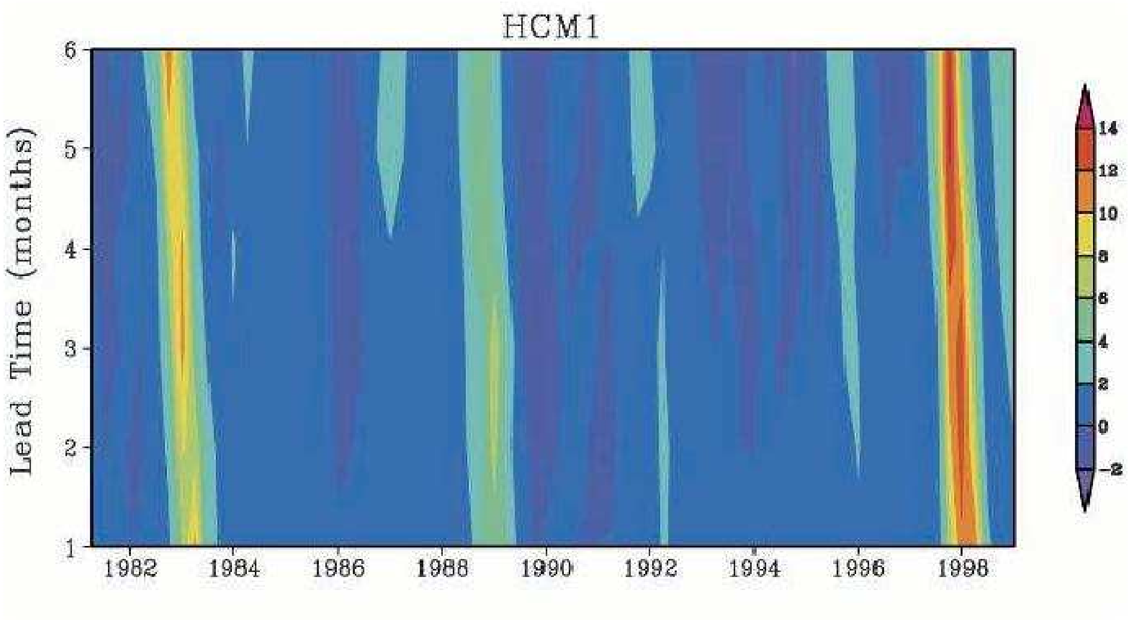


From Tang, Kleeman, & Moore, *J. Atmos. Sci.*, 2005.

- Predictive utility relates well to “skillful” forecasts

UVic

Individual Forecast Contributions to Correlation



From Tang, Kleeman, & Moore, *J. Atmos. Sci.*, 2005.

- Predictive utility relates well to “skillful” forecasts
- ENSO forecast skill derives mostly from initial conditions

UVic

Case Study II: Conclusions

- State of Tropical Pacific has influence on “weather” and “climate” pdfs globally

UVic

Case Study II: Conclusions

- State of Tropical Pacific has influence on “weather” and “climate” pdfs globally
- Linearised stochastic dynamics can be constructed giving insight to:

UVic

Case Study II: Conclusions

- State of Tropical Pacific has influence on “weather” and “climate” pdfs globally
- Linearised stochastic dynamics can be constructed giving insight to:
  - maintenance of ENSO variability

UVic

Case Study II: Conclusions

- State of Tropical Pacific has influence on “weather” and “climate” pdfs globally
- Linearised stochastic dynamics can be constructed giving insight to:
  - maintenance of ENSO variability
  - sources of predictability

UVic

Case Study II: Conclusions

- State of Tropical Pacific has influence on “weather” and “climate” pdfs globally
- Linearised stochastic dynamics can be constructed giving insight to:
  - maintenance of ENSO variability
  - sources of predictability
  - importance of “non-normal” dynamics

UVic

Case Study II: Conclusions

- State of Tropical Pacific has influence on “weather” and “climate” pdfs globally
- Linearised stochastic dynamics can be constructed giving insight to:
  - maintenance of ENSO variability
  - sources of predictability
  - importance of “non-normal” dynamics
- Other studies have relaxed some of the above assumptions, allowing for e.g. multiplicative noise and nonlinearity

Overall Conclusions

- Climate science is fundamentally probabilistic

Overall Conclusions

- Climate science is fundamentally probabilistic
- Need for probabilistic approach arises from essential complexity of system; some of these complexities can be addressed by better models or observations, but some are irreducible

Overall Conclusions

- Climate science is fundamentally probabilistic
- Need for probabilistic approach arises from essential complexity of system; some of these complexities can be addressed by better models or observations, but some are irreducible
- Fundamental challenges:

Overall Conclusions

- Climate science is fundamentally probabilistic
- Need for probabilistic approach arises from essential complexity of system; some of these complexities can be addressed by better models or observations, but some are irreducible
- Fundamental challenges:
  - characterise climate state pdfs

Overall Conclusions

- Climate science is fundamentally probabilistic
- Need for probabilistic approach arises from essential complexity of system; some of these complexities can be addressed by better models or observations, but some are irreducible
- Fundamental challenges:
  - characterise climate state pdfs
  - understand physical origin of pdfs

UVic

Overall Conclusions

- Climate science is fundamentally probabilistic
- Need for probabilistic approach arises from essential complexity of system; some of these complexities can be addressed by better models or observations, but some are irreducible
- Fundamental challenges:
  - characterise climate state pdfs
  - understand physical origin of pdfs
  - use pdfs to improve forecasts/predictions

UVic

Overall Conclusions

- Climate science is fundamentally probabilistic
- Need for probabilistic approach arises from essential complexity of system; some of these complexities can be addressed by better models or observations, but some are irreducible
- Fundamental challenges:
  - characterise climate state pdfs
  - understand physical origin of pdfs
  - use pdfs to improve forecasts/predictions
- Individual processes/phenomena have been investigated with considerable success, but much remains to be done

UVic

References

1. Imkeller, P. and J.-S. von Storch (eds), 2001: *Stochastic Climate Models*. Birkhäuser, 432 pp.
2. Imkeller, P. and A. Monahan, 2002: Conceptual stochastic climate models. *Stoch. and Dynamics*, **2**, 311-326.
3. Palmer, T. et al., 2005: Representing model uncertainty in weather and climate prediction. *Ann. Rev. Earth Planet. Sci.*, **33**, 163-93.
4. Palmer, T. and P. Williams: Stochastic Physics and Climate Modelling. *Phil. Trans. Roy. Soc.*, to appear in 2008.
5. Penland, C. 2003: Noise out of chaos and why it won't go away. *Bull. Amer. Met. Soc.* **84**, 921-925.

UVic

Tropical Multiscale Convective Systems

UVic