Energy Markets I: First Models

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Plan of the Course

- **Commodity Markets**
  - Production, Transportation, Storage, Delivery
  - Spot / Forward Markets
- **Spread Option Valuation**
  - Why Spread Options
  - First Asset Valuation
- **Gas and Power Markets**
  - Physical / Financial Contracts
  - Price Formation
  - Load and Temperature
- **Weather Markets**
  - Weather Exposure
  - Temperature Options
- **More Asset Valuation**
  - Plant Optionality Valuation
  - Financial Valuation
  - Valuing Storage Facilities
- **Emission Markets**

No More Utilities *monopolies*

Vertical Integration of *production, transportation, distribution* of electricity

Unbundling

Open competitive markets for production and retail
(Typically, grid remains under control)

New Price Formation

Constant *supply - demand* balance (Market forces)
Commodities form a **separate asset class**!

LOCAL STACK – **MERIT ORDER** (plant on the margin)

Role of Financial Mathematics & Financial Engineering

Support portfolio management
(producer, retailer, utility, **investment banks**, . . .)

- Different **data analysis**
  (spot, day-ahead, on-peak, off-peak, firm, non-firm, forward,$\cdots$, negative prices)
- New instrument **valuation**
  (swing / recall / take-or-pay options, weather and credit derivatives, gas storage, cross commodity derivatives, ...)
- New forms of **hedging** using physical assets
  Perfected by **GS & MS** (power plants, pipelines, tankers, ...)
- Marking to market and new forms of **risk** measures

FE for the post-ENRON Power Markets

Degradation of credit exacerbated liquidity problems

- **Credit risk**
  - Understanding the statistics of credit migration
  - Including counter-party risk in valuation
  - Credit derivatives and credit **enhancement**
- **Reporting** and indexes
- Could **clearing** be a solution?
  - Exchange traded instruments pretty much standardized, but OTC!
  - Design of a minimal set of instruments for **standardization**
- **Collateral** requirements / **margin** calls
  - **Objective valuation** algorithms widely accepted for frequent Mark-to-Market
  - **Netting**
    - Challenge of the dependencies (correlations, copulas, ...)
    - Integrated approach to risk control

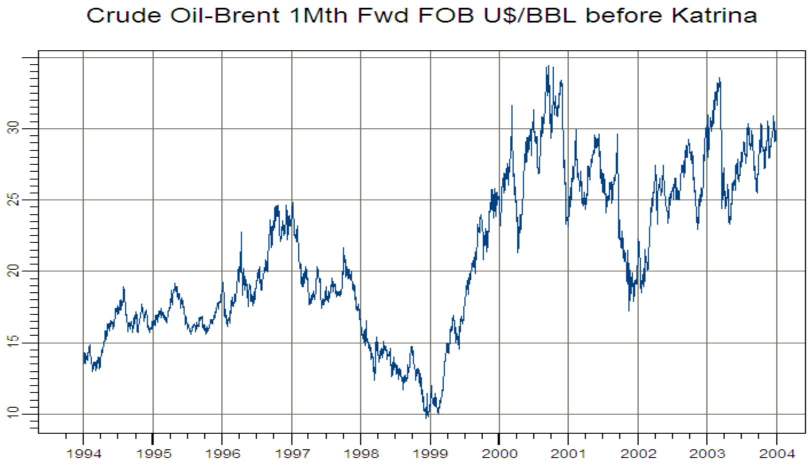
Commodities

- **Physical Markets**
  - Spot (immediate delivery) Markets
  - Forward Markets
- **Volume Explosion with Financially Settled Contracts**
  - Physical / Financial Contracts
  - Exchanges serve as **Clearing Houses**
  - Speculators *provide* **Liquidity**
- In IB, part of **Fixed Income Desk**
- **Seasonality / Storage / Convenience Yield**

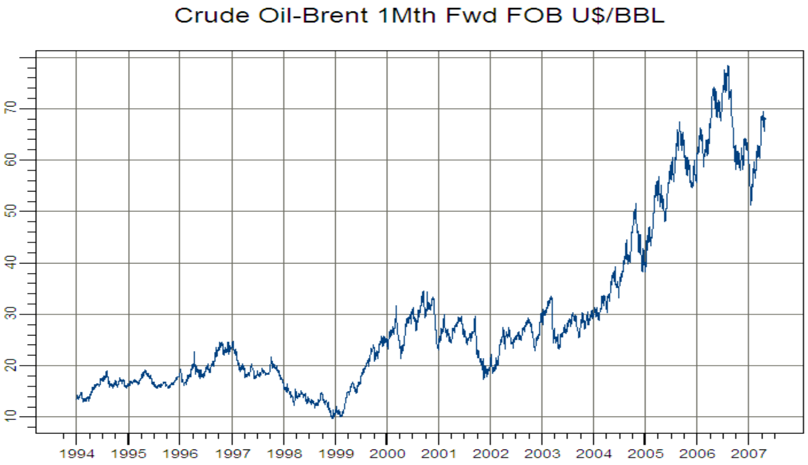
First Challenge: Constructing Forward Curves

- **How can it be a challenge?**
  - **Just do a PCA !**
    - *"OK"* for Crude Oil (backwardation/contango $\longrightarrow$ 3 factors)
    - Not settled for Gas
    - Does not work for Electricity
  - Extreme **complexity** & **size** of the data (location, grade, peak/off peak, firm/non firm, interruptible, swings, etc)
  - Incomplete and inconsistent sources of information
  - **Liquidity** and wide **Bid-Ask** spreads (**smoothing**)
  - **Length** of the curve (**extrapolation**)
- **Dynamic models *à la HJM***:
  Seasonality? Mean reversion? Jumps? Spot models? Factor Models? Cost of carry / convenience yield? Consistency? Historical? Risk neutral models? .....

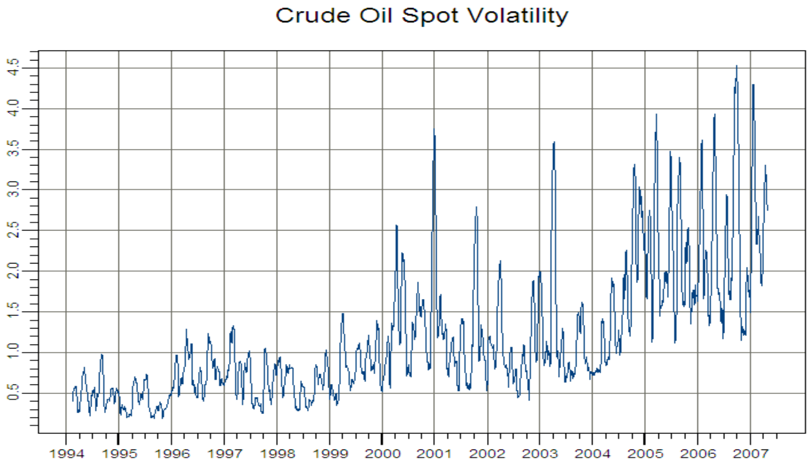
Crude Oil


Crude Oil-Brent 1Mth Fwd FOB U$/BBL before Katrina
10
15
20
25
30
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004

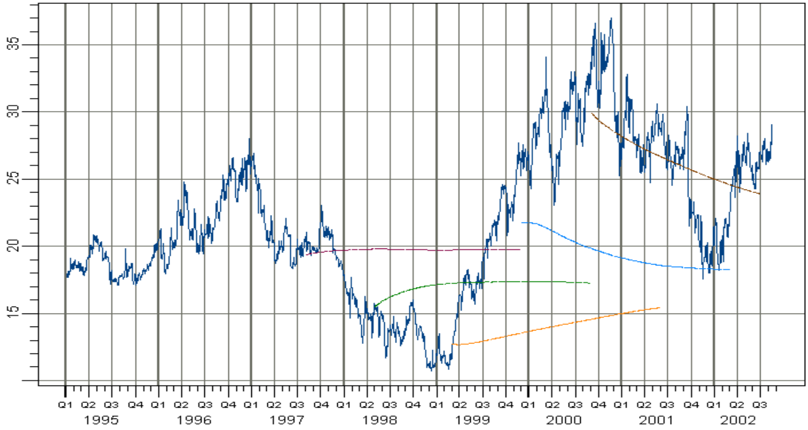
More Crude Oil Data

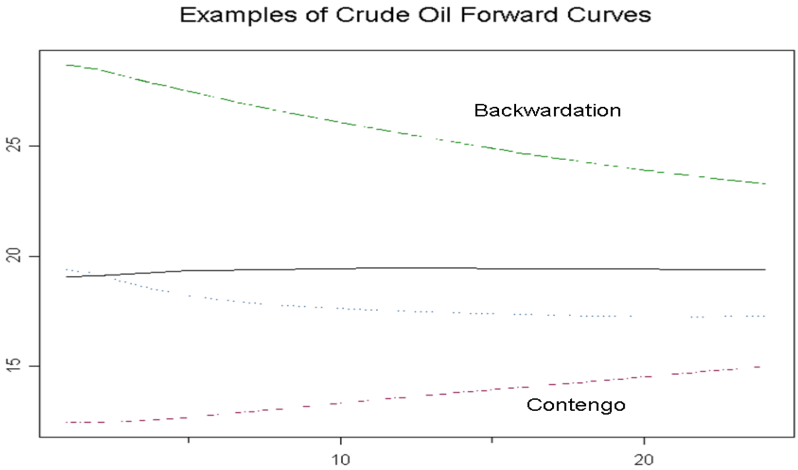


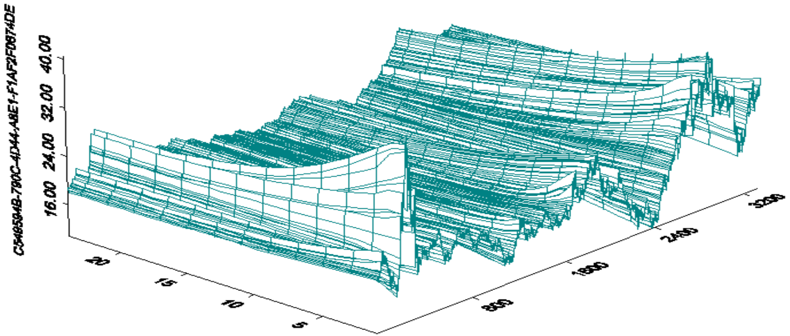
Spot Volatility



Is the Forward the Expected Value of Future Spots?




Examples of Crude Oil Forward Curves
Backwardation
Contengo
25
20
15
10
20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3200

Spot Forward Relationship

In financial models where one can hold positions at no cost

$$F(t, T) = S(t)e^{r(T-t)}$$

by a simple **cash & carry arbitrage** argument. In particular

$$F(t, T) = \mathbb{E}\{S(T) \,|\, \mathcal{F}_t\}$$

for risk neutral expectations.

Perfect Price Discovery

In general (theory of normal **backwardation**)

- $F(t, T)$ is a **downward biased** estimate of $S(T)$
- Spot price exceeds the forward prices

Notion of Convenience Yield

Forward Price = (risk neutral)
conditional expectation of future values of **Spot Price**

- No **cash & carry** arbitrage argument
  - Is the spot really tradable?
  - What are its dynamics?
  - How do we *risk-adjust* them?
- **Convenience Yield** for storable commodities
  - Natural Gas, Crude Oil, . . .
  - Correct interest rate to compute present values
  - Does not apply to Electricity

Spot-Forward Relationship in Commodity Markets

For **storable** commodities (still same **cash & carry arbitrage** argument)

$$F(t,T) = S(t)e^{(r-\delta)(T-t)}$$

for $\delta \geq 0$ called **convenience yield**. (**NOT FOR ELECTRICITY !**)

Decompose $\delta = \delta_1 - c$ with

- δ_1 benefit from owning the physical commodity
- c cost of storage

Then

$$f(t,T) = e^{r(T-t)}e^{-\delta_1(T-t)}e^{-c(T-t)}$$

- $e^{r(T-t)}$ cost of **financing** the purchase
- $e^{c(T-t)}$ cost of **storage**
- $e^{-\delta_1(T-t)}$ sheer **benefit from owning** the physical commodity

Backwardation / Contango Duality

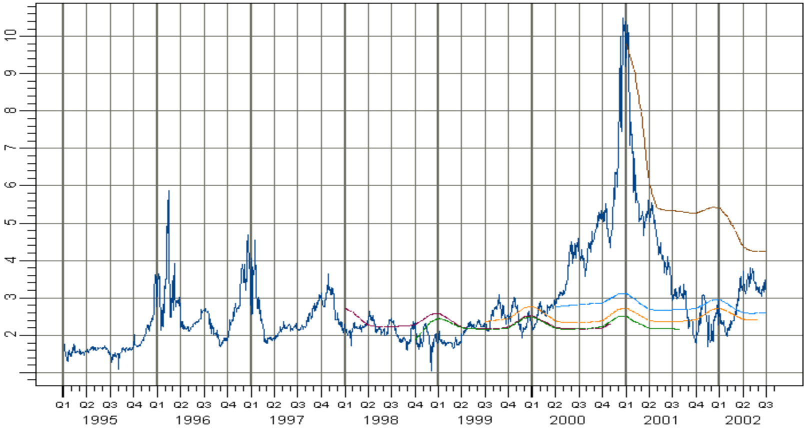
Backwardation

- $T \hookrightarrow F(t,T) = S(t)e^{(r+c-\delta_1)(T-t)}$ decreasing if $r + c < \delta_1$
  - Low cost of storage
  - Low interest rate
  - High benefit in holding the commodity

Contango

- $T \hookrightarrow F(t,T) = S(t)e^{(r+c-\delta_1)(T-t)}$ increasing if $r + c \geq \delta_1$

Natural Gas


2
3
4
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7
8
9
10
Q1 Q2 Q3 Q4 Q1 Q2 Q3 Q4 Q1 Q2 Q3 Q4 Q1 Q2 Q3 Q4 Q1 Q2 Q3 Q4 Q1 Q2 Q3 Q4 Q1 Q2 Q3 Q4 Q1 Q2 Q3
1995 1996 1997 1998 1999 2000 2001 2002

Commodity Convenience Yield Models

Gibson-Schwartz Two-factor model

- S_t commodity spot price
- δ_t convenience yield

Risk Neutral Dynamics

$$dS_t = (r_t - \delta_t)S_t\,dt + \sigma S_t\,dW_t^1,$$
$$d\delta_t = \kappa(\theta - \delta_t)dt + \sigma_\delta\,dW_t^2$$

Major Problems

- Explicit formulae (exponential affine model)
- Convenience yield implied from forward contract prices
- Unstable & Inconsistent (**R.C.-M. Ludkovski**)

Lack of Consistency

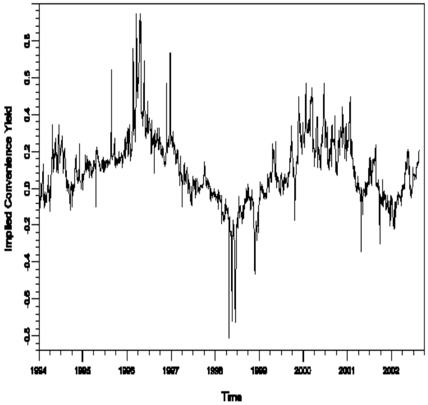
Exponential Affine Model

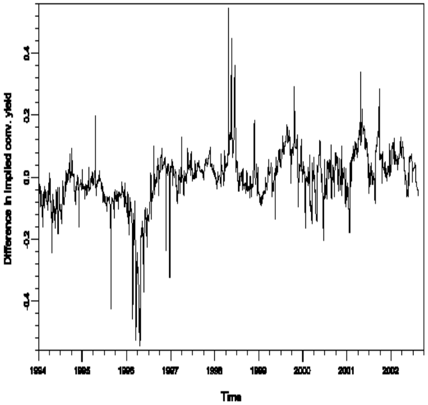
$$F(t,T) = S_t e^{\int_t^T r_s ds} e^{B(t,T)\delta_t + A(t,T)}$$

where

$$\begin{aligned} B(t,T) &= \frac{e^{-\kappa(T-t)} - 1}{\kappa}, \\ A(t,T) &= \frac{\kappa\theta + \rho\sigma_s\gamma}{\kappa^2}(1 - e^{-\kappa(T-t)} - \kappa(T-t)) + \\ &\quad + \frac{\gamma^2}{\kappa^3}(2\kappa(T-t) - 3 + 4e^{-\kappa(T-t)} - e^{-2\kappa(T-t)}). \end{aligned}$$

- For each T, one can imply δ_t from $F(t,T)$
- Inconsistency in the implied δ_t
- Ignores **Maturity Specific** effects





Crude Oil convenience yield implied by a 3 month futures contract (left)
Difference in implied convenience yields between 3 and 12 month contracts.

Convenience Yield Models Revisited

Use **forward** $F_t = F(t, T_0)$ instead of **spot** S_t (T_0 fixed maturity)

Historical Dynamics

$$\begin{aligned} dF_t &= (\mu_t - \delta_t) F_t \, dt + \sigma F_t \, dW_t^1, \\ d\delta_t &= \kappa(\theta - \delta_t) dt + \sigma_\delta \, dW_t^2 \end{aligned}$$

or more generally

$$d\delta_t = b(\delta_t, F_t) dt + \sigma_\delta(\delta_t, F_t) dW_t^2$$

We assume

- F_t is **tradable** (hence **observable**)
- (Forward) convenience yield δ_t **not observable** (**filtering**)

Different from **Bjork-Landen**'s **Risk Neutral Term Structure of Convenience Yield**

The Case of Power

Several obstructions

- Cannot store the physical commodity
- Which spot price? Real time? Day-ahead? Balance-of-the-week? month? on-peak? off-peak? etc
- Does the forward price converge as the time to maturity goes to 0?

Mathematical spot?

$$S(t) = \lim_{T \downarrow t} F(t, T)$$

Sparse Forward Data

- Lack of **transparency** (manipulated indexes)
- Poor (or lack of) **reporting** by fear of law suits
- **CCRO** white paper(s)

Dynamic Model for Forward Curves

n-factor forward curve model

$$\frac{dF(t,T)}{F(t,T)} = \mu(t,T)dt + \sum_{k=1}^{n} \sigma_k(t,T)dW_k(t) \qquad t \le T$$

- $\boldsymbol{W} = (W_1, \ldots, W_n)$ is a n-dimensional standard Brownian motion,
- drift μ and volatilities σ_k are deterministic functions of t and time-of-maturity T
- $\mu(t,T) \equiv 0$ for pricing
- $\mu(t,T)$ calibrated to historical data for risk management

Explicit Solution

$$F(t,T) = F(0,T)\exp\left[\int_0^t \left[\mu(s,T) - \frac{1}{2}\sum_{k=1}^n \sigma_k(s,T)^2\right] ds + \sum_{k=1}^n \int_0^t \sigma_k(s,T)dW_k(s)\right]$$

Forward prices are **log-normal** (deterministic coefficients)

$$F(t,T) = \alpha e^{\beta X - \beta^2/2}$$

with $X \sim N(0,1)$ and

$$\alpha = F(0,T)\exp\left[\int_0^t \mu(s,T)ds\right], \qquad \text{and} \qquad \beta = \sqrt{\sum_{k=1}^n \int_0^t \sigma_k(s,T)^2 ds}$$

Dynamics of the Spot Price

Spot price left hand of forward curve

$$S(t) = F(t, t)$$

We get

$$S(t) = F(0, t) \exp \left[\int_0^t [\mu(s, t) - \frac{1}{2} \sum_{k=1}^n \sigma_k(s, t)^2] ds + \sum_{k=1}^n \int_0^t \sigma_k(s, t) dW_k(s) \right]$$

and differentiating both sides we get:

$$dS(t) = S(t) \Bigg[\Bigg(\frac{1}{F(0, t)} \frac{\partial F(0, t)}{\partial t} + \mu(t, t) + \int_0^t \frac{\partial \mu(s, t)}{\partial t} ds - \frac{1}{2} \sigma_S(t)^2$$
$$- \sum_{k=1}^n \int_0^t \sigma_k(s, t) \frac{\partial \sigma_k(s, t)}{\partial t} ds + \sum_{k=1}^n \int_0^t \frac{\partial \sigma_k(s, t)}{\partial t} dW_k(s) \Bigg) dt + \sum_{k=1}^n \sigma_k(t, t) dW_k(t) \Bigg]$$

Spot volatility

$$\sigma_S(t)^2 = \sum_{k=1}^n \sigma_k(t, t)^2. \qquad (1)$$

Spot Dynamics cont.

Hence

$$\frac{dS(t)}{S(t)} = \left[\frac{\partial \log F(0,t)}{\partial t} + D(t)\right] dt + \sum_{k=1}^{n} \sigma_k(t,t) dW_k(t)$$

with drift

$$D(t) = \mu(t,t) - \frac{1}{2}\sigma_S(t)^2 + \int_0^t \frac{\partial \mu(s,t)}{\partial t} ds - \sum_{k=1}^{n} \int_0^t \sigma_k(s,t) \frac{\partial \sigma_k(s,t)}{\partial t} ds$$
$$+ \sum_{k=1}^{n} \int_0^t \frac{\partial \sigma_k(s,t)}{\partial t} dW_k(s)$$

Remarks

- Interpretation of drift (in a risk-neutral setting)
  - logarithmic derivative of the forward can be interpreted as a discount rate (*i.e.*, the running interest rate)
  - $D(t)$ can be interpreted as a convenience yield
- Drift generally **not Markovian**
- Particular case $n = 1$, $\mu(t, T) \equiv 0$, $\sigma_1(t, T) = \sigma e^{-\lambda(T-t)}$

$$D(t) = \lambda[\log F(0, t) - \log S(t)] + \frac{\sigma^2}{4}(1 - e^{-2\lambda t})$$

$$\frac{dS(t)}{S(t)} = [\mu(t) - \lambda \log S(t)]dt + \sigma dW(t)$$

exponential OU

Changing Variables

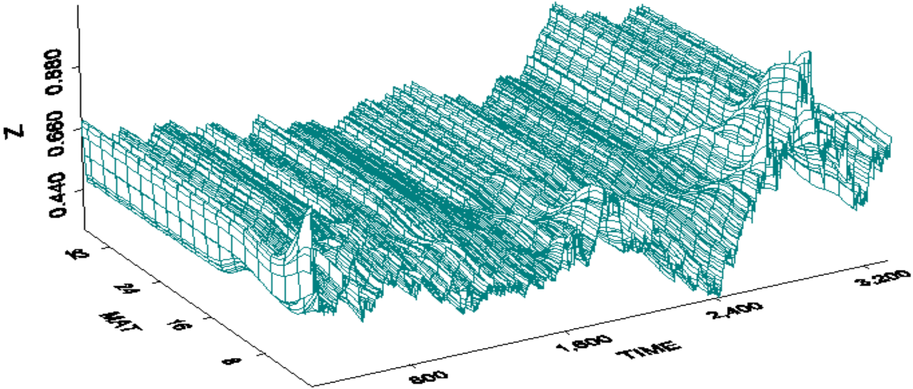
time-of-maturity $T \quad \rightrightarrows \quad$ **time-to-maturity** τ

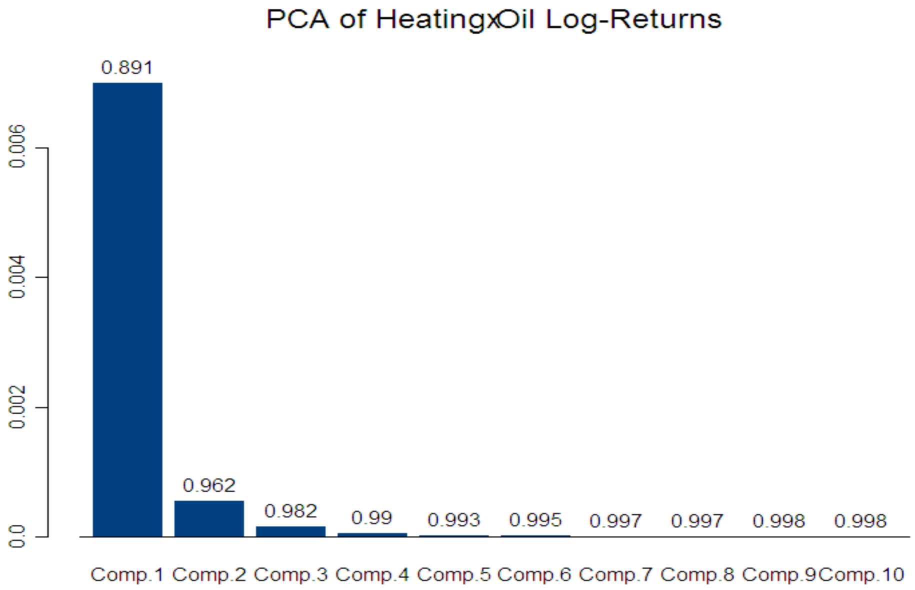
changes dependence upon t

$$t \hookrightarrow F(t, T) = F(t, t + \tau) = \tilde{F}(t, \tau)$$

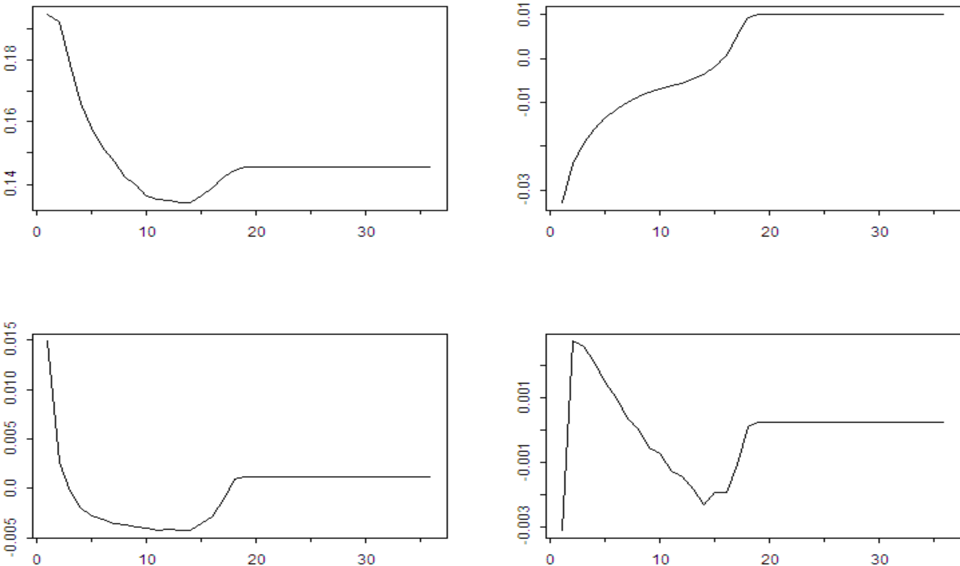
Fixed Domain $[0, \infty)$ for $\tau \hookrightarrow \tilde{F}(t(\tau)$

Heating Oil Forward Surface

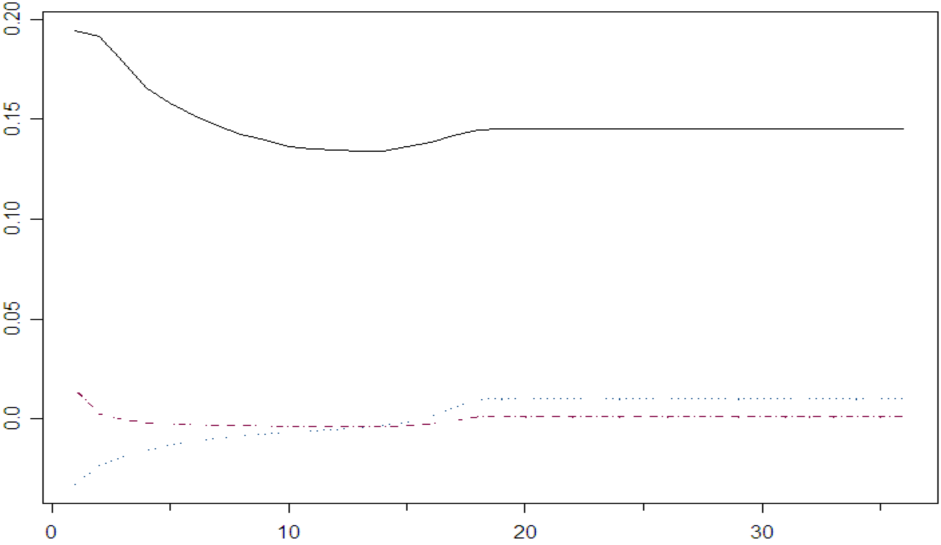



PCA of HeatingxOil Log-Returns
0.891
0.962
0.982
0.99
0.993
0.995
0.997
0.997
0.998
0.998
0.0
0.002
0.004
0.006
Comp.1
Comp.2
Comp.3
Comp.4
Comp.5
Comp.6
Comp.7
Comp.8
Comp.9
Comp.10

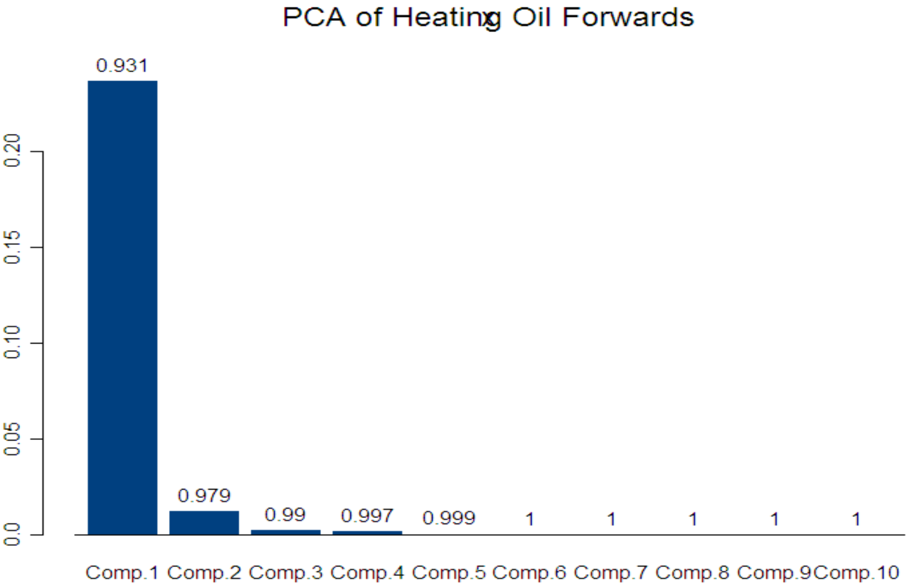
HO PCA Loadings



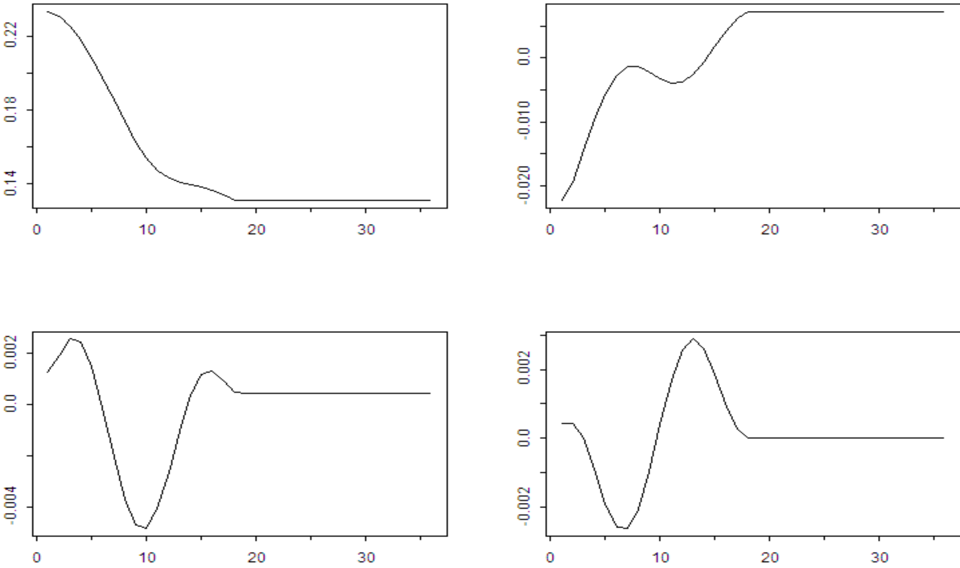
HO Loadings on their Importance Scale



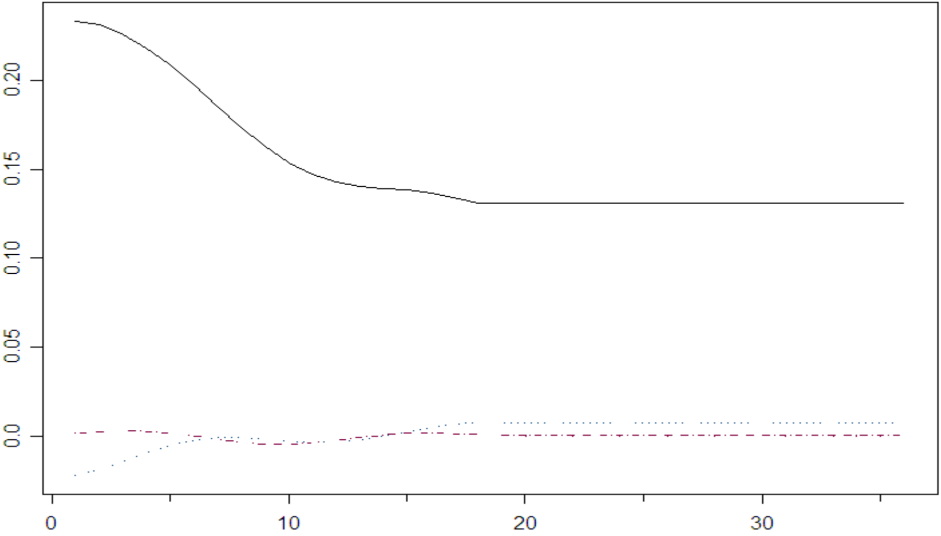
Plain Forward HO PCA

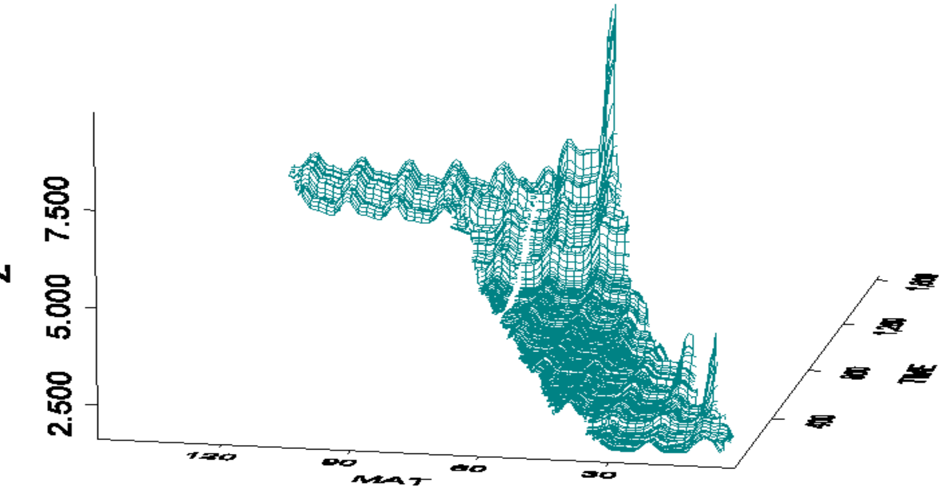


HO PCA Loadings



HO Loadings on their Importance Scale





Z
7.500
5.000
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120
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60
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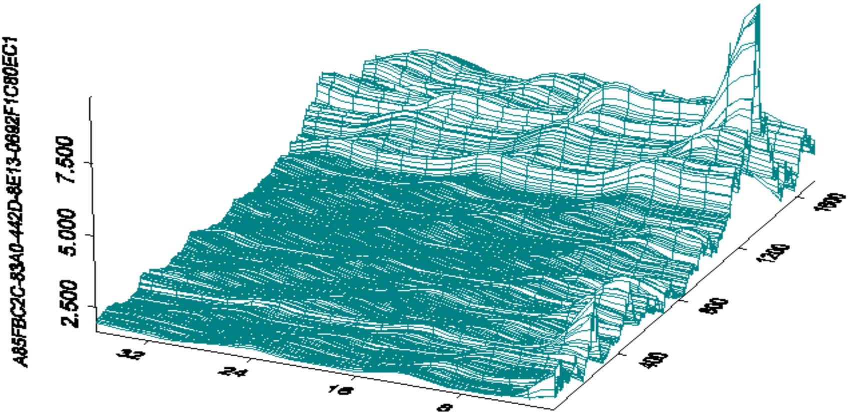
MAT


A85FBC2C-83A0-442D-8E13-0892F1C80EC1
2.500
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7.500
32
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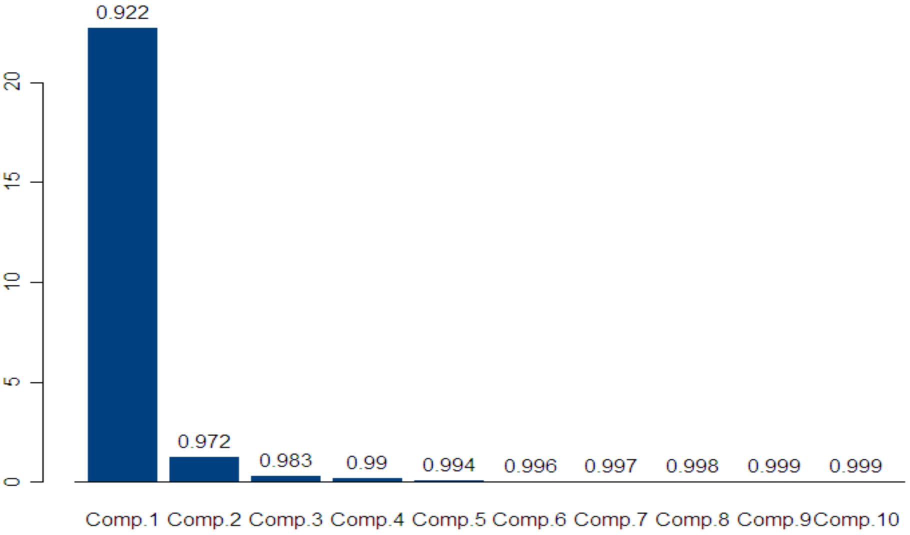
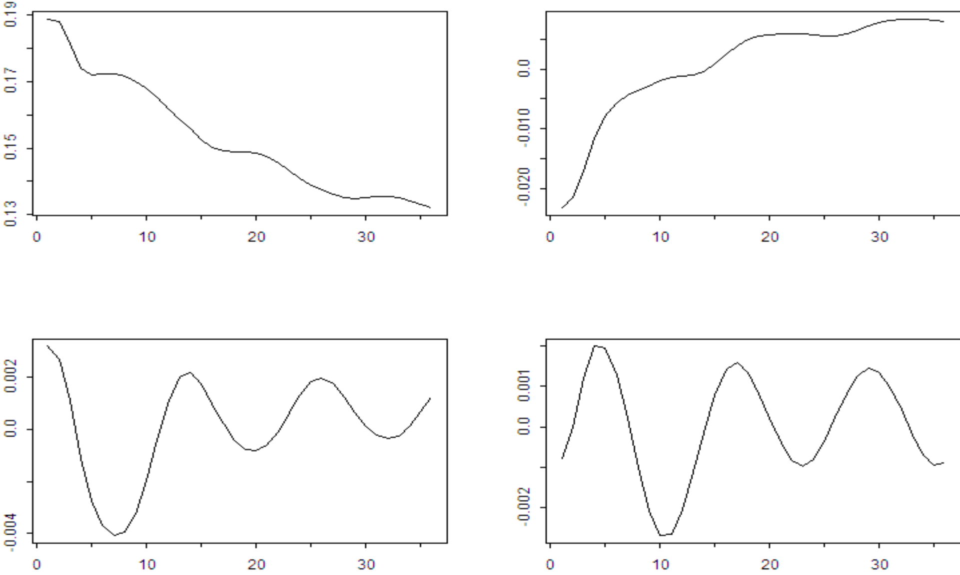
1600

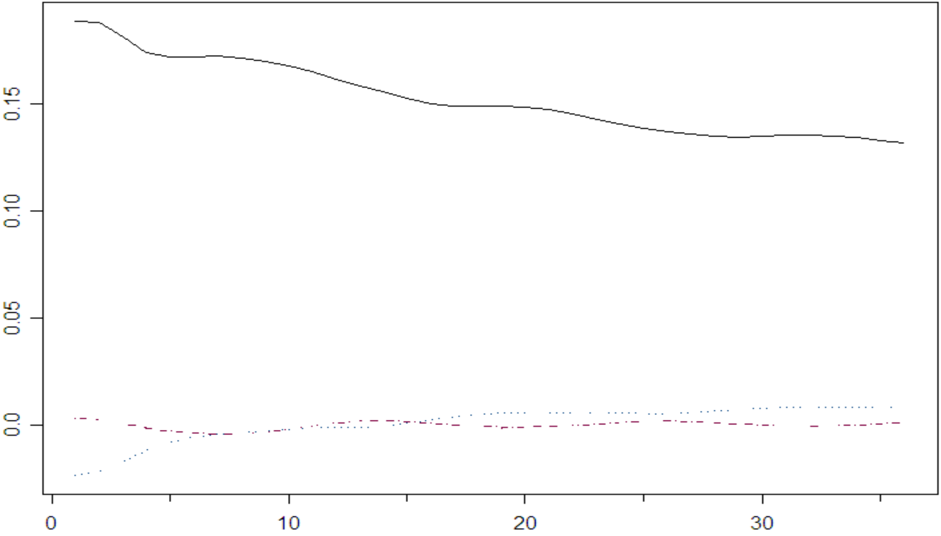
PCA of Henry Hub Natural Gas Forward Prices



HH PCA Loadings



HH Loadings on their Absolute Importance Scale



Changing Variables

$$\textbf{time-of-maturity } T \quad \rightrightarrows \quad \textbf{time-to-maturity } \tau$$

changes dependence upon t

$$t \hookrightarrow F(t, T) = F(t, t+\tau) = \tilde{F}(t, \tau)$$

For **pricing purposes**

- For T fixed, $\{F(t, T)\}_{0 \le t \le T}$ **is a martingale**
- For τ fixed, $\{\tilde{F}(t, \tau)\}_{0 \le t}$ **is NOT a martingale**

$\tilde{F}(t,\tau) = F(t, t+\tau)$, $\quad \tilde{\mu}(t,\tau) = \mu(t, t+\tau)$, and $\tilde{\sigma}_k(t,\tau) = \sigma_k(t, t+\tau)$,

In general dynamics become

$$d\tilde{F}(t,\tau) = \tilde{F}(t,\tau)\left[\left(\tilde{\mu}(t,\tau) + \frac{\partial}{\partial \tau}\log \tilde{F}(t,\tau)\right)dt + \sum_{k=1}^{n} \tilde{\sigma}_k(t,\tau)dW_k(t)\right], \qquad \tau$$

PCA with Seasonality

Fundamental Assumption

$$\sigma_k(t,T) = \sigma(t)\sigma_k(T-t) = \sigma(t)\sigma_k(\tau)$$

for some function $t \hookrightarrow \sigma(t)$

Notice

$$\sigma_S(t) = \tilde{\sigma}(0)\sigma(t)$$

provided we set:

$$\tilde{\sigma}(\tau) = \sqrt{\sum_{k=1}^{n} \sigma_k(\tau)^2}.$$

Conclusion

$t \hookrightarrow \sigma(t)$ is (up to a constant) the **instantaneous spot volatility**

Rationale for a New PCA

- Fix times-to-maturity $\tau_1, \tau_2, \ldots, \tau_N$
- Assume on each day t, quotes for the forward prices with times-of-maturity $T_1 = t + \tau_1$, $T_2 = t + \tau_2$, ..., $T_N = t + \tau_N$ are available

$$\frac{d\tilde{F}(t,\tau_i)}{\tilde{F}(t,\tau_i)} = \left(\tilde{\mu}(t,\tau_i) + \frac{\partial}{\partial\tau}\log\tilde{F}(t,\tau_i)\right)dt + \sigma(t)\sum_{k=1}^{n}\sigma_k(\tau_i)dW_k(t) \qquad i = 1,\ldots,N$$

Define $\mathbf{F} = [\sigma_k(\tau_i)]_{i=1,\ldots,N,\ k=1,\ldots,n}$.

$$d\log\tilde{F}(t,\tau_i) = \left(\tilde{\mu}(t,\tau_i) + \frac{\partial}{\partial\tau_i}\log\tilde{F}(t,\tau_i) - \frac{1}{2}\sigma(t)^2\tilde{\sigma}(\tau_i)^2\right)dt + \sigma(t)\sum_{k=1}^{n}\sigma_k(\tau_i)dW_k(t),$$

Instantaneous variance/covariance matrix $\{M(t); t \geq 0\}$ defined by:

$$d[\log\tilde{F}(\,\cdot\,,\tau_i), \log\tilde{F}(\,\cdot\,,\tau_j)]_t = M_{i,j}(t)dt$$

satisfies

$$M(t) = \sigma(t)^2\left(\sum_{k=1}^{n}\sigma_k(\tau_i)\sigma_k(\tau_j)\right)$$

or equivalently

$$M(t) = \sigma(t)^2\mathbf{F}\mathbf{F}^*$$

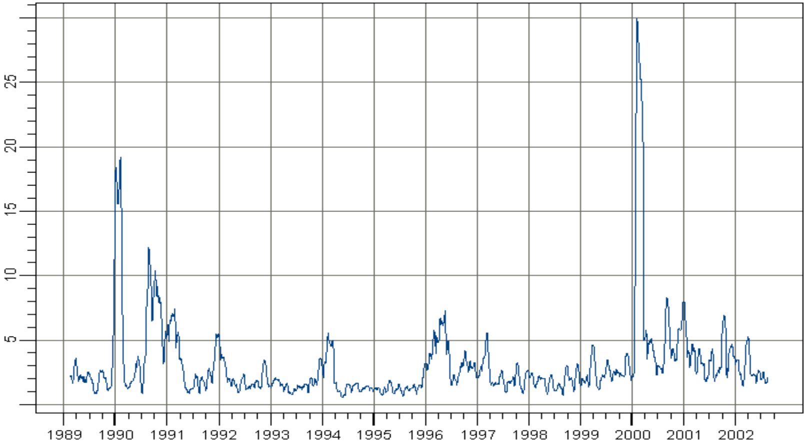
Strategy Summary

- Estimate instantaneous spot volatility $\sigma(t)$ (in a rolling window)
- Estimate $\mathbf{FF}^*$ from historical data as the empirical auto-covariance of $\ln(F(t,\cdot)) - \ln(F(t-1,\cdot))$ after normalization by $\sigma(t)$
- Instantaneous auto-covariance structure of the entire forward curve becomes time independent
- Do SVD of auto-covariance matrix and get

$$\tau \hookrightarrow \sigma_k(\tau)$$

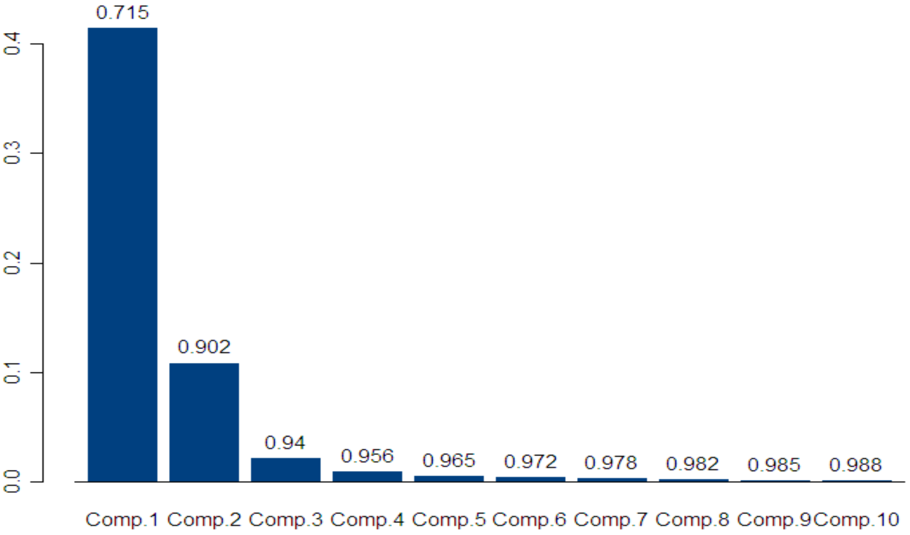
- Choose order n of the model from their relative sizes

The Case of Natural Gas

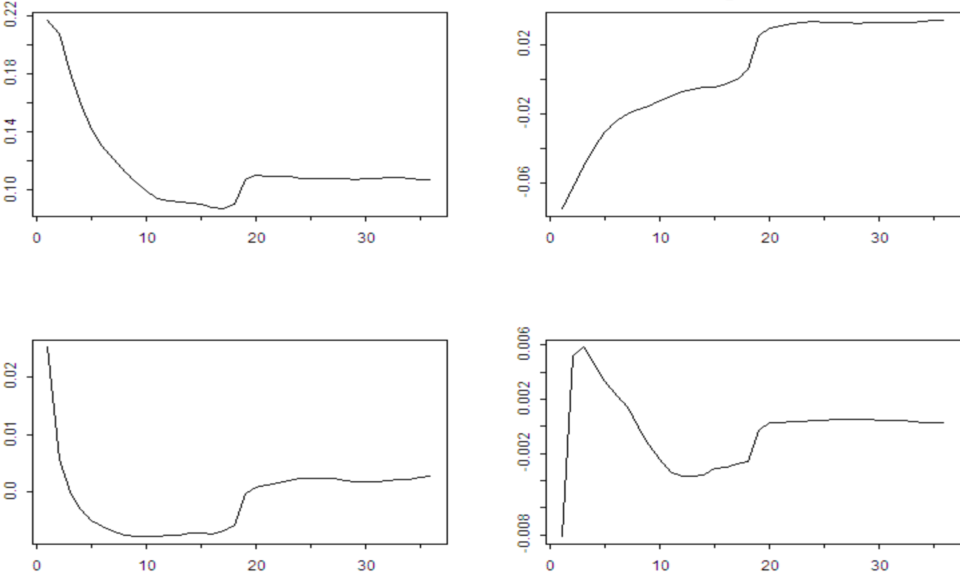


Instantaneous standard deviation of the Henry Hub natural gas spot price computed in a sliding window of length 30 days.

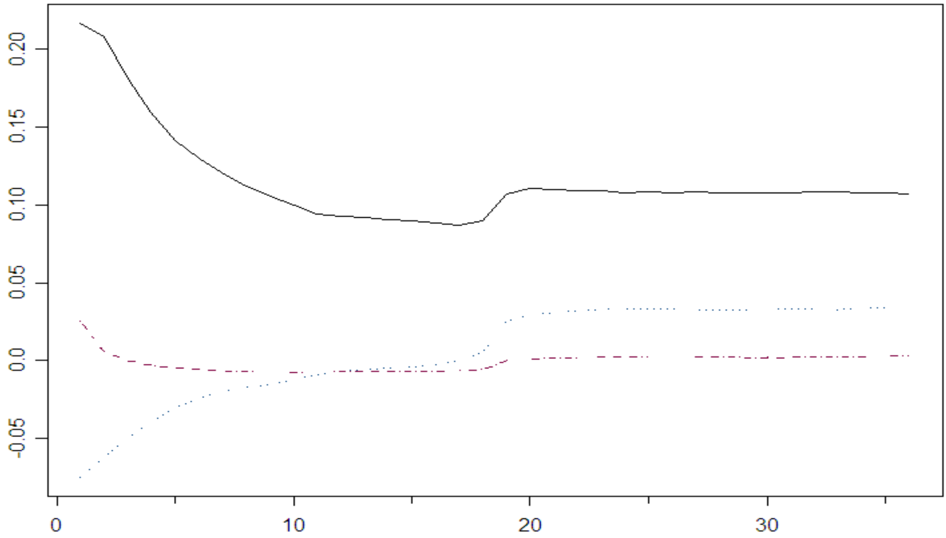
PCA of Henry Hub Natural Gas De-Seasonalized Forward Prices



HH De-Seasonalized PCA Loadings



HH De-Seasonalized Loadings on their Absolute Importance Scal



Mean Reversion toward the cost of production

The example of the power prices

- **Reduced Form Models**
  - Nonlinear effects (exponential OU^2)

Supply/Demand & Price Formation

Mean Reversion toward the cost of production
The example of the power prices

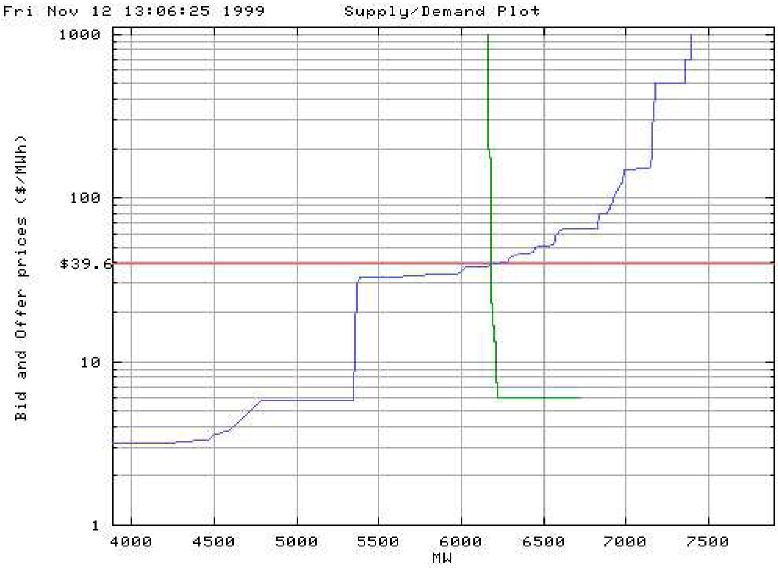
- **Reduced Form Models**
  - Nonlinear effects (exponential OU^2)
  - Jumps (Geman-Roncoroni)
- **Structural Models**
  - Inelastic Demand
  - The Supply Stack

Barlow (based on **merit** order graph)

- $s_t(x)$ supply at time t when power price is x
- $d_t(x)$ demand at time t when power price is x

Power price at time t is number S_t such that

$$s(S_t) = d_t(S_t)$$



Example of a merit graph (Alberta Power Pool, courtesy M. Barlow)

Barlow's Proposal

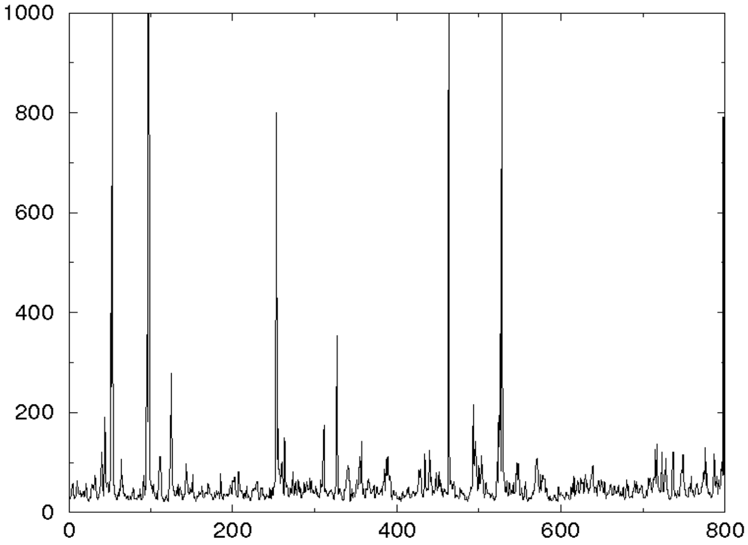
$$S(t) = \begin{cases} f_\alpha(X_t) & 1+\alpha X_t > \epsilon_0 \\ \epsilon_0^{1/\alpha} & 1+\alpha X_t \le \epsilon_0 \end{cases}$$

for the **non-linear** function

$$f_\alpha(x) = \begin{cases} (1+\alpha x)^{1/\alpha}, & \alpha \neq 0 \\ e^x & \alpha = 0 \end{cases}$$

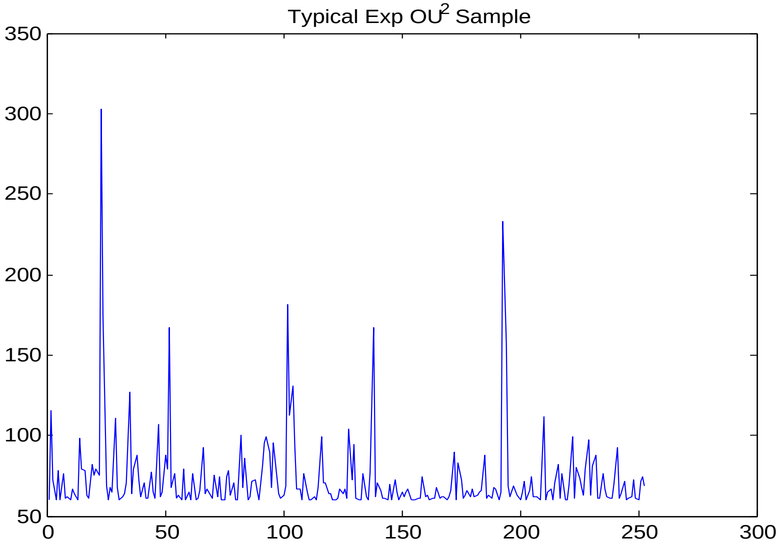
of an **OU** diffusion

$$dX_t = -\lambda(X_t - \overline{x})dt + \sigma dW_t$$



Monte Carlo Sample from Barlow's Spot Model (courtesy M. Barlow)

Cheap Alternative



Example of a Monte Carlo Sample from the Exponential of an OU^2

Negative Prices

Consider the case of **PJM**
(Pennsylvania - New Jersey - Maryland)

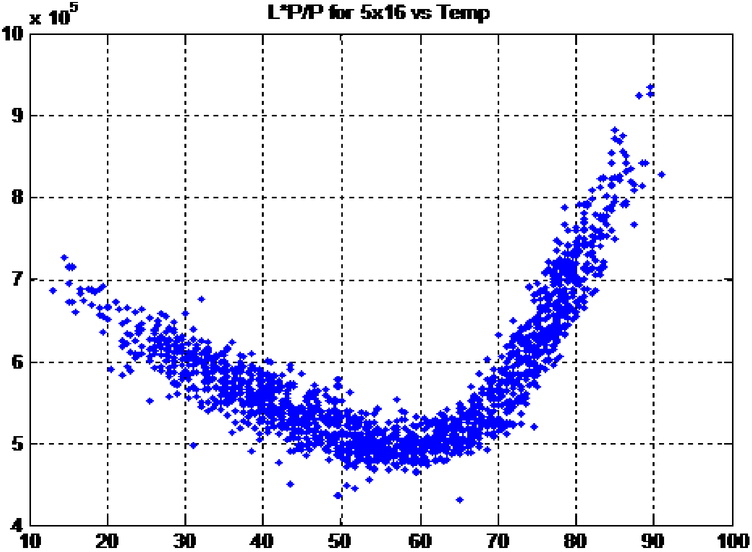
- Over $3,000$ nodes in the transmission network
- Each day, and for each node
  - Real time prices
  - Day-ahead prices
  - Hour by hour load prediction for the following day
- **Historical prices**
- In 2003 over $100,000$ instances of **NEGATIVE PRICES**
  - Geographic clusters
  - Time of the year (**shoulder months**)
  - Time of the day (**night**)
- **Possible Explanations**
  - Load miss-predicted
  - High temperature volatility

Other Statistical Issues: Modelling Demand

For many contracts, delivery needs to match demand

- **Demand** for energy highly correlated with **temperature**
  - Heating Season (winter) HDD
  - Cooling Season (summer) CDD
- **Stylized Facts** and **First (naive) Models**
  - Electricity demand = β * weather + α

Load / Temperature



Daily Load versus Daily Temperature (PJM)

Other Statistical Issues: Modelling Demand

For many contracts, delivery needs to match demand

- **Demand** for energy highly correlated with **temperature**
  - Heating Season (winter) HDD
  - Cooling Season (summer) CDD
- **Stylized Facts** and **First (naive) Models**
  - Electricity demand = β * weather + α
    - Not true all the time
    - Time dependent β by filtering !
  - From the stack: Correlation (Gas,Power) = f(weather)
    - No significance, too unstable
    - Could it be because of heavy tails?
- **Weather dynamics** need to be included
  - **Another Source of Incompleteness**

First Faculty Meeting of New PU President

Princeton University Electricity Budget

2.8 M $ over (PU is small)

- The University has its own Power Plant
- Gas Turbine for Electricity & Steam
- Major Exposures
  - Hot Summer (air conditioning) Spikes in Demand, Gas & Electricity Prices
  - Cold Winter (heating) Spikes in Gas Prices

Risk Management Solution

- Never Again such a Short Fall !!!
- Student (Greg Larkin) Senior Thesis
- **Hedging Volume Risk**
  - Protection against the Weather Exposure
  - **Temperature Options** on CDDs (Extreme Load)
- **Hedging Volume & Basis Risk**
  - Protection against Gas & Electricity Price Spikes
  - Gas purchase with **Swing Options**

Mitigating Volume Risk with Swing Options

Exposure to spikes in prices of

- Natural Gas (used to fuel the plant)
- Electricity Spot (in case of overload)

Proposed Solution

- Forward Contracts
- Swing Options

Pretty standard

Mitigating Volume Risk

- Use **Swing Options**
- Multiple Rights to deviate (within bounds) from base load contract level
- **Pricing & Hedging** quite involved!
  - Tree/Forest Based Methods
    - Direct Backward Dynamic Programing Induction (à la Jaillet-Ronn-Tompaidis)
  - **New Monte Carlo Methods**
    - Nonparametric Regression (à la Longstaff-Schwarz) Backward Dynamic Programing Induction

Review: **Classical Optimal Stopping Problem: American Option**

- $X_0, X_1, X_2, \cdots, X_n, \cdots$ rewards
- Right to ONE Exercise
- Mathematical Problem

$$\sup_{0 \le \tau \le T} \mathbb{E}\{X_\tau\}$$

Mathematical Solution

- Snell's Envelop
- Backward Dynamic Programming Induction in Markovian Case

Standard, Well Understood

New Mathematical Challenges

In its simplest form the problem of **Swing/Recall** option pricing is an

Optimal Multiple Stopping Problem

- $X_0, X_1, X_2, \cdots, X_n, \cdots$ rewards
- Right to N Exercises
- Mathematical Problem

$$\sup_{0 \le \tau_1 < \tau_2 < \cdots < \tau_N \le T} \mathbb{E}\{X_{\tau_1} + X_{\tau_2} + \cdots + X_{\tau_N}\}$$

- **Refraction** period θ

$$\tau_1 + \theta < \tau_2 < \tau_2 + \theta < \tau_3 < \cdots < \tau_{N-1} + \theta < \tau_N$$

Part of recall contracts & crucial for continuous time models

Instruments with Multiple American Exercises

- **Ubiquitous in Energy Sector**
  - Swing / Recall contracts
  - End user contracts (EDF)
- **Present in other contexts**
  - Fixed income markets (e.g. chooser swaps)
  - Executive option programs
    Reload $\longrightarrow$ Multiple exercise, Vesting $\longrightarrow$ Refraction, $\cdots$
  - Fleet Purchase (airplanes, cars, $\cdots$)
- **Challenges**
  - Valuation
  - Optimal exercise policies
  - Hedging

Some Mathematical Problems

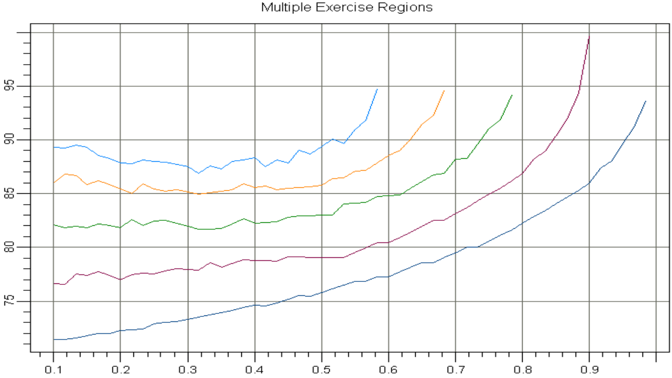
Recursive re-formulation into a hierarchy of classical optimal stopping problems

- Development of a theory of *Generalized Snell's Envelop* in continuous time setting
- Find a form of Backward Dynamic Programing Induction in Markovian Case
- Design & implement efficient numerical algorithms for finite horizon case

Results

- Perpetual case: abstract nonsense
  R.C.& S.Dayanik (diffusion), R.C.& N.Touzi (GBM)
- Perpetual case: Characterization of the optimal policies
  R.C.& S.Dayanik (diffusion), R.C.& N.Touzi (GBM)
- Finite horizon case
  Jaillet - Ronn - Tomapidis (Tree) R.C. N.Touzi (GBM) B.Hambly (chooser swap)

R.C.-Touzi, (Bouchard)



Exercise regions for $N = 5$ rights and finite maturity computed by Malliavin-Monte-Carlo.

Mitigation of Volume Risk with Temperature Options

- Rigorous Analysis of the Dependence between the **Budget Shortfall** and **Temperature** in Princeton
- Use of Historical Data (**sparse**) & Define of a **Temperature Protection**
  - Period of the Coverage
  - Form of the Coverage
- Search for the **Nearest Weather Stations** with HDD/CDD Trades
  - La Guardia Airport (LGA)
  - Philadelphia (PHL)
- Define a Portfolio of LGA & PHL forward / option Contracts
- Construct a **LGA / PHL basket**

Pricing: How Much is it Worth to PU?

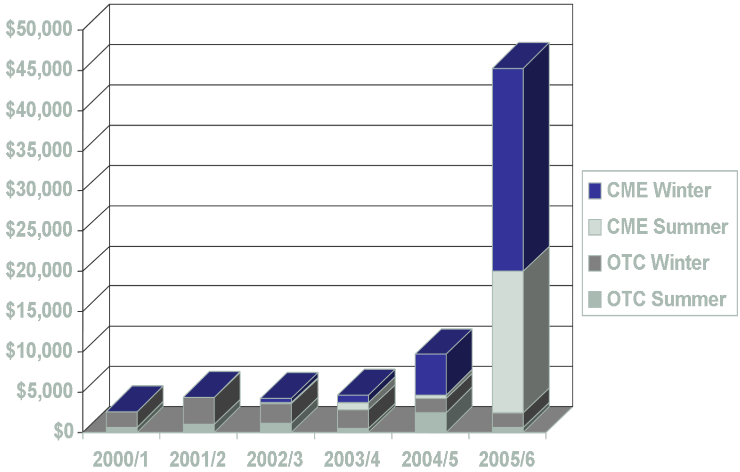
- **Actuarial / Historical Approach**
  - Burn Analysis
  - Temperature Modeling & Monte Carlo VaR Computations
  - Not Enough Reliable Load Data
- **Expected (Exponential) Utility Maximization** (**A. Danilova**)
  - Use Gas & Power Contracts
  - Hedging in Incomplete Models
  - Indifference Pricing
  - Very Difficult Numerics (whether PDE's or Monte Carlo)

Weather is an essential economic factor

- *'Weather is not just an environmental issue; it is a major economic factor. At least 1 trillion USD of our economy is weather-sensitive'* (William Daley, 1998, US Commerce Secretary)
- **20% of the world economy** is estimated to be affected by weather
- Energy and other industrial sectors, Entertainment and Tourism Industry, ...
- **WRMA**

Weather Derivatives as a **Risk Transfer** Mechanism (**El Karoui - Barrieu**)

Size of the Weather Market



Total Notional Value of weather contracts: (in million USD) Price Waterhouse Coopers market survey).

Weather Derivatives

- **OTC** Customer tailored transactions
  - Temperature, Precipitation, Wind, Snow Fall, ...
- **CME** ($\approx$ 50%) (Tempreature - Launched in 1999)
  - 18 American cities
  - 9 European cities (London, Paris, Amsterdam, Berlin, Essen, Stockholm, Rome, Madrid and Barcelona)
  - 2 Japanese cities (Tokyo and Osaka)

An Example of Precipitation Contract

- Physical Underlying Daily Index:
  - Precipitation in Paris
  - A day is a rainy day if precipitation exceeds 2mm
- Season
  - 2000: April thru August + September weekends
  - 2001: April thru August + September weekends
  - 2002: April thru August + September weekends
- Aggregate Index
  - Total Number of Rainy Days in the Season
- Pay- Off
  - Strike, Cap, Rate

RainFall Option Continued

- **Who Wanted this Deal?**
  - A Natural Trying to Hedge RainFall Exposure (Asterix Amusement Park)
- **Who was willing to take the other side?**
  - Speculators
  - Insurance Companies
  - Re-insurance Companies
  - Statistical Arbitrageurs
  - Investment Banks
  - Hedge Funds
  - Endowment Funds
  - ....

Other Example: Precipitation / Snow Pack

- **City of Sacramento**
  - HydroPower Electricity
- Who was on the other side?
  - Large Energy Companies (**Aquila, Enron**)

Who is covering for them?

Jargon of Temperature Options

For a given **location**, on any given day t

$$CDD_t = \max\{T_t - 65, 0\} \qquad HDD_t = \max\{65 - T_t, 0\}$$

Season

- One Month (CME Contracts)
- May 1st September 30 (CDD season)
- November 1st March 31st (HDD season)

Index

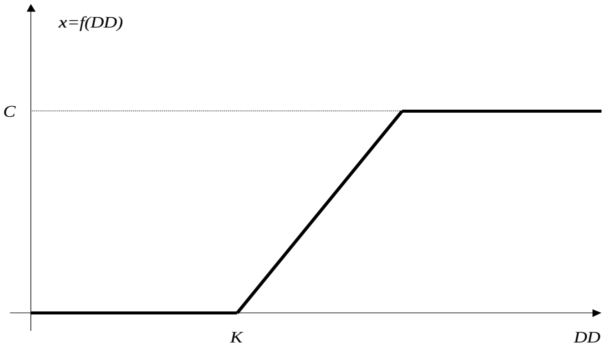
- Aggregate number of DD in the season

$$I = \sum_{t \in \text{Season}} CDD_t \quad \text{or} \quad I = \sum_{t \in \text{Season}} HDD_t$$

Pay-Off

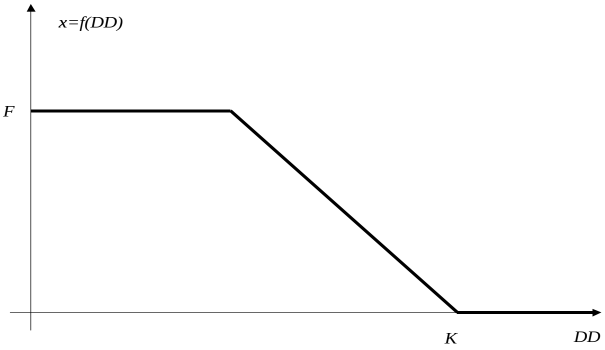
- Strike K, Cap C, Rate α

Call with Cap



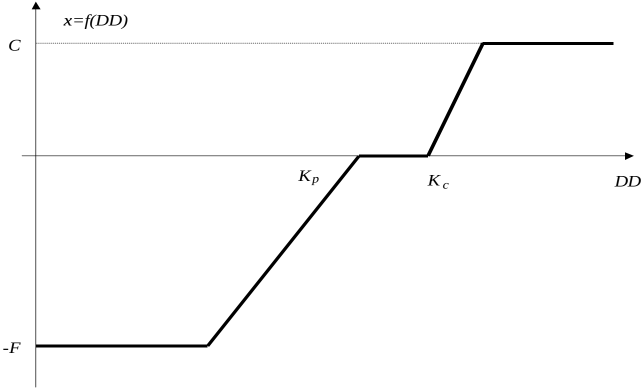
Pay-off $= \min\{\max\{\alpha * (I - K), 0\}, C\}$

Put with a Floor

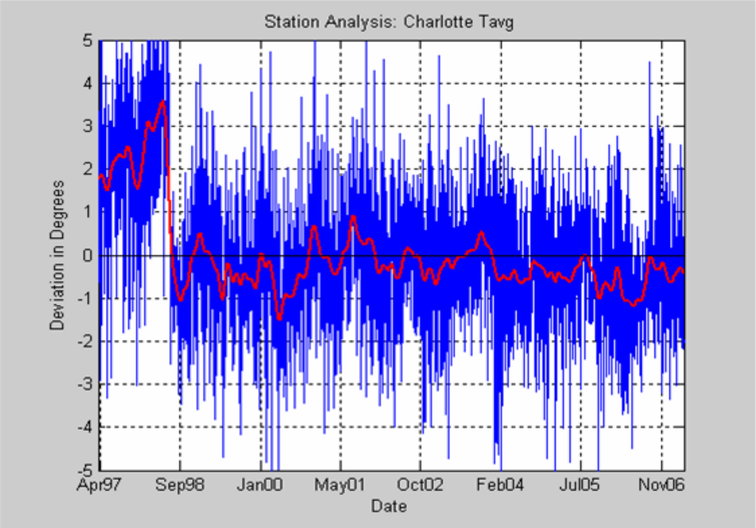


Pay-off $= \min\{\max\{\alpha * (K - I), 0\}, C\}$

Collar



Folklore of Data Reliability



Famous Example of Weather Station Change in Charlotte (NC).

Stylized Spreadsheet of a Basket Option

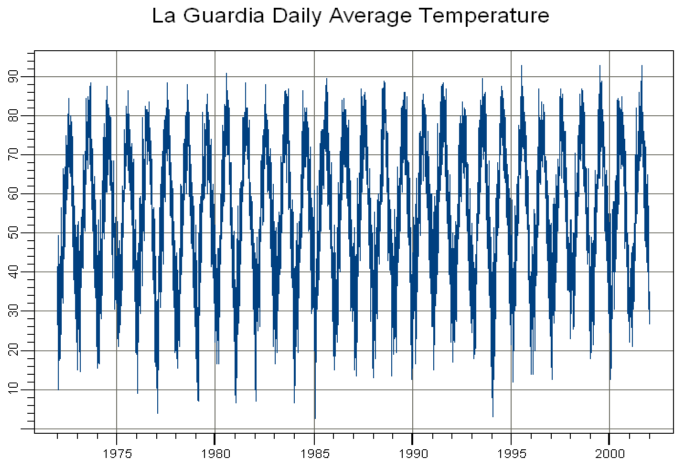
- **Structure**: Heating Degree Day (HDD) Floor (Put)
- **Index**: Cumulative HDDs
- **Term**: November 1, 2007 February 28, 2008
- **Stations**:
  - New York, LaGuardia 57.20%
  - Boston, MA 24.5%
  - Philadelphia, PA 12.00%
  - Baltimore, MD 6.30%
- **Floor Strike**: 3130 HDDs
- **Payout**: USD 35,000/HDD
- **Limit**: USD 12,500,000
- **Premium**: USD 2,925,000

Weather and Commodity

- **Stand-alone**
  - temperature ($\approx$ 80%)
  - precipitation ($\approx$ 10%)
  - wind ($\approx$ 5%)
  - snow fall ($\approx$ 5%)
- **In-Combination**
  - natural gas
  - power
  - heating oil
  - propane
- Agricultural risk (yield, revenue, input hedges and trading)
- Power outage - contingent power price options

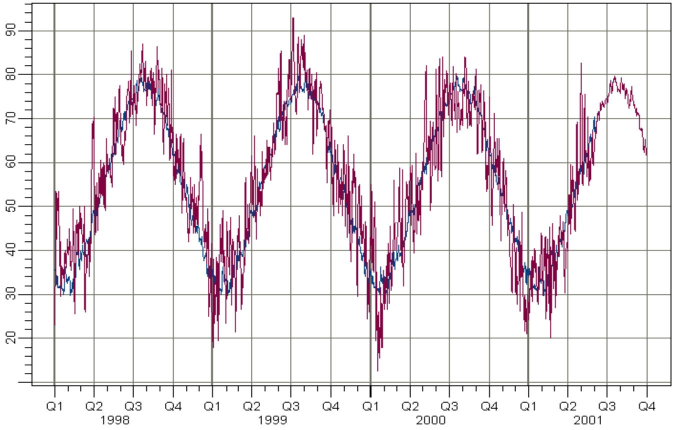
Weather (Temperatures) Derivatives

- Still Extremely **Illiquid** Markets (except for **front month**)
- **Misconception**: Weather Derivative = Insurance Contract
  - No secondary market (Except on **Enron-on-Line**!!!)
- **Mark-to-Market** (or Model)
  - Essentially never changes
  - At least, Not Until Meteorology kicks in (10-15 days before maturity)
  - Then Mark-to-Market (or Model) changes every day
  - Contracts change hands
  - That's when major losses occur and money is made
- This *hot period* is not considered in academic studies
  - Need for updates: new information coming in (temperatures, forecasts, ...)
  - Filtering is (again) the solution



Daily Average Temperature at La Guardia.

Prediction on 6/1/2001 of Summer La Guardia Average Temperature



Prediction on 6/1/2001 of daily temperature over the next four months.

The Future of the Weather Markets

- **Social function** of the weather market
  - Existence of a Market of Professionals (for weather risk transfer)
- **Under attack** from
  - (Re-)Insurance industry (but *high freuency / low cost*)
  - Utilities (trying to pass weather risk to end-customer)
    - EDF program in France
    - Weather Normalization Agreements in US
- **Cross Commodity Products**
  - Gas & Power contracts with **weather triggers/contingencies**
  - New (major) players: **Hedge Funds** provide liquidity
- **World Bank**
  - Use weather derivatives instead of insurance contracts

The Weather Market Today

- **Insurance Companies**: Swiss Re, XL, Munich Re, Ren Re
- **Financial Houses**: Goldman Sachs, Deutsche Bank, Merrill Lynch, SocGen, ABN AMRO
- **Hedge funds**: D. E. Shaw, Tudor, Susquehanna, Centaurus, Wolverine

Where is Trading Taking Place?

- Exchange: **CME** (Chicago Mercantile Exchange) 29 cites globally traded, monthly / seasonal contracts
- **OTC**
- Strong end-user demand within the **energy sector**

Incomplete Market Model & Indifference Pricing

- Temperature Options: Actuarial/Statistical Approach
- Temperature Options: Diffusion Models (Danilova)
- Precipitation Options: Markov Models (Diko)
  - *Problem:* Pricing in an Incomplete Market
  - *Solution:* Indifference Pricing à la Davis

$$
\begin{aligned}
d\theta_t &= p(t,\theta)dt + q(t,\theta)dW_t^{(\theta)} + r(t,\theta)dQ_t^{(\theta)} \\
dS_t &= S_t[\mu(t,\theta)dt + \sigma(t,\theta)dW_t^{(S)}]
\end{aligned}
$$

- θ_t **non-tradable**
- S_t **tradable**

Example: **Exponential Utility Function**

$$\tilde{p}_t = \frac{\mathbb{E}\{\tilde{\phi}(Y_T) e^{-\int_t^T V(s, Y_s) ds}\}}{\mathbb{E}\{e^{-\int_t^T V(s, Y_s) ds}\}}$$

where

- $\tilde{\phi} = e^{-\gamma(1-\rho^2)f}$
  where $f(\theta_T)$ is the pay-off function of the European call on the temperature
- $\tilde{p}_t = e^{-\gamma(1-\rho^2)p_t}$
  where p_t is price of the option at time t
- Y_t is the diffusion:

$$dY_t = [g(t, Y_t) - \frac{\mu(t, Y_t) - r}{\sigma(t, Y_t)} h(t, Y_t)]dt + h(t, Y_t) d\tilde{W}_t$$

  starting from $Y_0 = y$
- V is the time dependent potential function:

$$V(t, y) = -\frac{1-\rho^2}{2} \frac{(\mu(t, y) - r)^2}{\sigma(t, y)^2}$$

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