Banff International Research Station
for Mathematical Innovation and Discovery

PIMS/UNAM Algebra Summer School
July 1 - July 6, 2006

SCHEDULE

	Sunday July 2 [1]	Monday July 3 [2,3]	Tuesday July 4	Wednesday July 5
9:00-10:00	Pianzola	Zhang	Goodearl	Huisgen-Zimmermann
10:00-10:30	C O F F E E B R E A K			
10:30-11:30	de la Pe~na	Adem	Geiss	Smith
11:30-12:15	*	Michael Barot	Bautista	*
	L U N C H			
14:00-14:45	Dugas	Mendoza	XXX	Martinez
14:45-15:30	Bruin	Elizondo	XXX	Behrend
15:30-16:00	C O F F E E B R E A K			
16:00-16:45	Assem	Bell	XXX	Valencia
16:45-17:30	Cunningham	Cliff	XXX	Lewis

Note:
1. Welcome/introduction to BIRS at 8.45 am on Sunday
2. Group photo to be taken at 12.15 pm on Monday
3. Tour of The Banff Centre on Monday at 1.15-2.00 pm

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ABSTRACTS

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Birge Huisgen-Zimmermann

TITLE: Moduli Spaces as a Classification Tool in the Representation Theory of Finite Dimensional Algebras

ABSTRACT: We explain -- first in an intuitive fashion, then more formally -- what it means for a collection of finite dimensional representations to be classifiable by way of a fine or coarse moduli space (in the sense of Mumford). Then we will describe two different roads towards deciding classifiability of representations singled out by various choices of discrete isomorphism invariants; these two lines are parallelled by two different access roads to the pertinent moduli spaces. One was opened up by A. King, who adapted techniques of Mumford to the representation theory of algebras, the other is illustrated by the speaker's recent research into parametrizations of finite dimensional modules by projective varieties. Several classification theorems and concrete examples will show the potential and the relative bonuses of the two viewpoints.

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Alejandro Adem

Introduction to Cohomology of Finite Groups

In this talk I will discuss basic topics from the cohomology of finite groups, explaining the main techniques and calculational methods. We will also discuss open problems and connections to other areas of mathematics.

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Christof Geiss

Preprojective Algebras and Cluster Algebras

This is a report on a joint project with B. Leclerc (Caen) and J. Schr"oer (Bonn).

Cluster algebras where interoduced by Fomin and Zelevinsky around 2000 as a combinatorial device to study questions about dual canonical bases and total positivity in representation theory. Since then there has been a lot of activity and cluster structures have been discovered in many areas of mathematics.

Let N be a a maximal unipotent subgroup of complex simply connected simple Lie group, and $\mathfrak{n}$ its Lie algebra. It is well-known that the restricted dual of the envelopping algebra $U(\mathfrak{n})$ may be identified with the coordinate ring $\mathbb{C}[N]$. This ring has a

canonical cluster algebra structure by work of Berenstein-Fomin-Zelevinsky.

Let Λ be the preprojective algebra of the corresponding Dynkin type. This is a finite-dimensional self-injective algebra. Using the representation varieties of $\Lam$ for all dimension vectors, Lusztig constructed geometrically the semicanonical basis of $U(\mathfrak{g})$.
We manage to show that the elements of the dual of this basis, which correspond to open orbits in the representation variety, form the cluster monomials for $\mathbb{C}[N]$. In this way, indecomposable $\Lam$-modules without selfextension correspond to cluster variables.

In fact, more generally each $\Lam$-module yields an interesting function on N, and we can describe their multiplication in representation theoretical terms.

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Arturo Pianzola

The Language of Forms
(A journey from the Moebius strip to affine Kac-Moody
algebras)

Abstract: One of the most recurrent themes in Geometry, is the study of objects that locally look the same. I will explain, mainly via examples and in non-technical terms, how the concept of ``locally look the same'' has evolved through time (mostly through some beautiful ideas of Serre and of Grothendieck).

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Jose A de la Pe~na

Decomposition of Modules over Rings with Several Objects.

We consider rings R with enough idempotents (in a set E) and which are locally bounded, meaningthat for any two idempotents e,f in E the module eRf is of finite length over eRe and for each e there is only a finite number of f in E with eRf not zero or fRe not zero. We study decompositions of right modules over R as direct sums of indecomposable modules, in particular for modules X such that the lenght of Xe is finite over eRe for any e in E. The discussion includes theorems in classical ring theory and representation theory of categories over fields. We consider properties of indecomposable modules and their endomorphism rings.

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Ken Goodearl

Quantized Coordinate Rings

Abstract: The development of the subject called "Quantum Groups" over the past two decades has introduced a lot of interesting new objects for algebraists to study, including the algebras indicated in the title. These are noncommutative versions of the classical rings of polynomial functions on affine algebraic varieties studied in algebraic geometry, and these "quantized" coordinate rings have surprisingly many properties in common with the classical ones. In fact, a guiding principle is to treat them as if they were rings of functions on some nonexistent geometric object. The aim of the talk will be to introduce a sampling of these algebras and to describe some of what is known about them, emphasizing the relations with classical coordinate rings.

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James Zhang

Quantum Projective 3-spaces

One of the main questions in non-commutative projective geometry is to classify all quantum projective 3-spaces. We will outline an algebraic approach to this problem and study some examples.

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Paul Smith

Non-commutative Surfaces

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Ibrahim Assem
Partie de gauche et composantes d'Auslander-Reiten d'une algbre d'Artin

Ceci est un travail conjoint avec J. A. Cappa, M. I. Platzeck et S. Trepode

Soient A une algbre d'Artin et modA la catgorie des A-modules droite de type fini. La partie de gauche L_A est la sous-catgorie pleine de modA qui contient les objets indcomposables M de modA tels que chaque prdcesseur de M est de dimension projective 0 ou 1. Nous tudions les composantes du carquois d'Auslander-Reiten de A (ou de son support gauche A_lambda) qui coupent L_A, et en particulier celles qui contiennent les Ext-injectifs de L_A.

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Kai Behrend

Title: Donaldson-Thomas Type Invariants via Microlocal Geometry

Abstract:
We prove that Donaldson-Thomas type invariants (such as holomorphic Casson invariants) are equal to weighted Euler characteristics of their moduli spaces. In particular, such invariants depend only on the scheme structure of the moduli space, not the symmetric obstruction theory used to define them. We also introduce new invariants generalizing Donaldson-Thomas type invariants to mod- uli problems with open moduli space. These are useful for computing Donaldson-Thomas type invariants over stratifications.

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Javier Elizondo

The total coordinate ring of a normal projectiva variety

Abstract: In this talk I will give a short survey of the total coordinate ring (Cox ring) of a normal projective variety.

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Clifton Cunningham

TITLE: Some examples of automorphic representations

ABSTRACT: In this talk I will describe a strategy for producing examples of automorphic representations of some classical groups which yields explicit information about the corresponding Galois representations and L-functions. This is joint work with Lassina Dembl.

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Gerald Cliff

Schur algebras for classical groups

Abstract. Let K be an infinite field. The classical Schur algebra S(n,r), for the general linear group, consists of endomorphisms of the r-fold tensor product of an n-dimensional vector space over K, which commute with the action of the symmetric group on r symbols. This is dual to the coalgebra of polynomials of degree r in n-squared variables. It is quasi-hereditary, and has a natural basis dual the monomials of degree r. There are analogues for the symplectic and orthogonal groups, due to Donkin and Doty, which I will discuss. One replaces A(n,r) by the coalgebra of polynomials restricted to the group of symplectic (respectively, orthogonal) similitudes. Bases are harder to find, and in particular for the orthogonal case are not known.

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Alex Dugas

Relative Gorenstein dimensions and stable equivalence

The notion of Gorenstein dimension was originally developed by Auslander and Bridger for modulesover commutative noetherian rings, but has since been studied in various more general contexts. For a ring R we shall introduce the notion of relatively Gorenstein projective moduleswith respect to a subcategory $\mathcal{C}$ of Mod-R, and use them to define relative Gorenstein dimensions of R-modules. When R is an Artin algebra, these relative dimensions satisfy analogues of several nice properties of standard Gorenstein dimensions. Moreover, we will show that these relative Gorenstein dimensions are invariant under any stable equivalence between Artin algebras without nodes.

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Raymundo Bautista (UNAM-Morelia)

Derived Endo-Discrete Artin Algebras.

Let A be an Artin algebra and X an object in D the bounded derived category of the category modA, the category of the finitely generated left A-modules. Denote by E(X) the endomorphism ring of X in D.

The homology endo-dimension of X is given by a sequence of non-negative integers (d(i), i in the set of the integers) almost all of them zero, where d(i) is the length as left E(X)-module of the i-th homology group of X.

The algebra A is called derived endo-discrete if for any given sequence of non-negative integers (d(i), i in the set of the integers), with almost all d(i) equal to zero, there are only finitely many isomorphism

classes of objects X in D having homology endo-dimension (d(i)). In this talk the following result is proved:

A is derived endo-discrete if and only if H, the bounded derived category of Mod A the category of all left A-modules, has not generic complexes.

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Carlos Valencia

Monomial Rees algebras and the Mengerian property.

Joint work with I. Gitler and R.H. Villareal.

Abstract:

In this talk we will present some relations between the Mengerian property of a clutter and the normality of the Rees algebra of the square free monomial ideal associated to this clutter. In particular we will give a characterization of the Mengerian property in terms of the normality of the Rees algebra and the integrality of some polyhedrons.

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Jason Bell

Primitivity in twisted homogeneous coordinate rings.
(Joint with Dan Rogalski.)

Abstract: Give a projective variety X, an invertible sheaf $\mathcal{L}$ and an automorphism σ of X, one can form the \emph{twisted homogeneous coordinate ring}
$$B(X,\mathcal{L},\sigma)=\bigoplus_{i=0}^{\infty} H^0(X,\mathcal{L}_n),$$ where $$\mathcal{L}_n = \mathcal{L}\otimes \sigma^*\mathcal{L}\otimes \cdots \otimes (\sigma^{n-1})^*\mathcal{L}.$$
Our main result is an analogue of the Dixmier-Moeglin equivalence for twisted homogeneous coordinate rings over uncountable fields. Specifically, we show that for a prime ideal P in $B(X,\mathcal{L},\sigma)$, the following are equivalent.
\begin{enumerate}
\item{P is primitive.}
\item{ $B(X,\mathcal{L},\sigma)/P$ has finitely many height one primes.}
\item{ The centre of the quotient division algebra of $B(X,\mathcal{L},\sigma)$ is algebraic over the base field.}
\end{enumerate}
It is unknown whether this equivalence holds over countable fields, but we present reasons why we do not expect it to hold.

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James D. Lewis

Title: Arithmetical Invariants of Algebraic Cycles

Abstract: In some recent joint work with Shuji Saito, I will discuss some new arithmetical invariants associated to algebraic cycles on a projective algebraic manifold. In keeping with the general theme of this summer school, many of the techniques arise from homological algebra, differential equations (regularity issues), differential operators (arising from D-modules), and arithmetical variations of Hodge structures (related to moduli).

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Title: Deciding the existence of rational points on curves

Speaker: Nils Bruin (SFU)

Abstract: While it is known that Hilbert's 10th problem - deciding whether a polynomial equation has integral solutions - has no automatic solution, one can still hope that for subclasses of polynomial equations and for rational solutions, such an algorithm might exist.

Recently, experiments and theoretical work inspired by these experiments have provided some quite convincing evidence that for rational points on projective curves, such an algorithm does indeed exist and that we in fact already know the algorithm. I will outline this algorithm and indicate the heuristics that indicate it is correct.

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Michael Barot

Lie algebras associated to unit forms

Abstract: The talk will present results from a joint work with D. Kussin and H. Lenzing: to each unit form q (that is an integral quadratic form whose square-coefficients equal one) we associate a Lie algebra A(q) in a way that generalizes simply laced Kac-Moody algebras and elliptic Lie algebras of type D and E. We show that if q,q' are connected, positive semidefinite and equivalent unit forms then A(q) and A(q') are isomorphic.

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Roberto Martinez Villa

Artin-Schelter Regular algebras and Categories

[see below]

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Octavio Mendoza

An approach to the finitistic dimension conjecture

[see below]
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Artin-Schelter Regular algebras and Categories

R. Martínez-Villa and Oeyvind Solberg
Instituto de Matemáticas de la UNAM,
Morelia México
Trondheim University, Trondheim Norway

May 29, 2006

Abstract

The aim of this talk is to show how to extend the notion of an Artin-Schelter regular algebra to non connected algebras and to categories. An example of a connected non noetherian algebra is given to show that the noetherian condition is not needed.

We prove the following theorem:

Theorem: Let C be an Artin Schelter regular $K-$category. Denote by S the set of representatives of the simple functors in $Mod(C)$and by $E(C)$ the Ext- category. Then $Mod(E(C))$ is Frobenius of finite length.

In the Koszul case the converse is true:

Theorem: Let C be a Koszul category of finite length. Assume C is Frobenius. Then $E(C)$is Artin Schelter regular.

Applications to the representations of a finite dimensional algebra will be given.

AN APPROACH TO THE FINITISTIC DIMENSION CONJECTURE

OCTAVIO MENDOZA
JOINT WORK WITH: FRANÇOIS HUARD AND MARCELO LANZILOTTA

Let R be a finite dimensional k-algebra over an algebraically closed field k. Denote by $\operatorname{mod} R$ the category of all finitely generated left R-modules. For a given full subcategory $\mathcal{X}$ of $\operatorname{mod} R$, we denote by $\operatorname{pfd} \mathcal{X}$ the projective finitistic dimension of $\mathcal{X}$. That is, $\operatorname{pfd} \mathcal{X} := \sup \{\operatorname{pd} X \ : \ X \in \mathcal{X} \text{ and } \operatorname{pd} X < \infty\}$.

It was conjectured, by M. Auslander, in the 60's that $\operatorname{pfd}(R) := \operatorname{pfd} \operatorname{mod} R$ has to be finite. Since then, many people have been working to get a proof of this conjecture. Some time ago, K. Igusa and J. Todorov defined a function $\Psi : \operatorname{mod} R \to \mathbb{N}$ which has been very useful to prove that $\operatorname{pfd}(R)$ is finite for some class of algebras. In order to have a different approach to the finitistic dimension conjecture, we propose to consider a class of full subcategories of $\operatorname{mod} R$ instead of a class of algebras, namely to take the class of categories $\mathcal{F}(\theta)$ of θ-filtered R-modules for all stratifying systems $(\theta, \leq)$ in $\operatorname{mod} R$. Some partial results will be discussed.