# Second Northwest Functional Analysis Seminar March 17-19, 2005 


#### Abstract

MEALS Breakfast (Continental): 7:00-9:00 am, 2nd floor lounge, Corbett Hall, Friday \& Saturday (included in workshop) Lunch (Buffet): 11:30 am-1:30 pm, Donald Cameron Hall (included in workshop) Dinner (Buffet): 5:30-7:30 pm, Donald Cameron Hall (included in workshop) Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall (included in workshop) **For other lighter meal options at the Banff Centre, there are two other options: Gooseberry's Deli, located in the Sally Borden Building, and The Kiln Cafe, located beside Donald Cameron Hall. There are also plenty of restaurants and cafes in the town of Banff, a 10-15 minute walk from Corbett Hall.**


## MEETING ROOMS

All lectures are held in Max Bell 159. Hours: 6 am- 12 midnight. LCD projector, overhead projectors and blackboards are available for presentations Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

## SCHEDULE

## Thursday

16:00 Check-in begins (Front Desk, Professional Development Centre, open 24 hours)
Friday
7:00-9:00 Breakfast, 2nd floor lounge, Corbett Hall
9:00-9:10 Opening remarks
9:10-10:00 Karoly Bezdek, Hadwiger's illumination conjecture
10:10-10:40 Coffee Break, 2nd floor lounge, Corbett Hall
10:40-11:10 Ebrahim Samei, Local operators and hyper-Tauberian algebras
11:20-11:50 Bahram Rangipour, From motivation to application of Hopf cyclic cohomology
12:00-1:30 Lunch
1:30-2:10 Juliana Erlijman, Subfactor constructions with braided categories
2:20-2:50 Adi Tcaciuc, Stabilization and asymptotic structure of Banach spaces
3:00-3:30 Coffee break, 2nd floor lounge, Corbett Hall
3:30-4:00 S. Mahmoud Manjeghani, Young's inequality for the trace of operators
4:10-4:50 Alex Brudnyi, Metric spaces with linear extensions preserving Lipschitz conditions
5:00-5:30 Ali-Amir Husain, On the cohomology of operator algebras
5:40-7:30 Dinner

## Saturday

| 7:00-9:00 | Breakfast, 2nd floor lounge, Corbett Hall |
| :--- | :--- |
| 9:00-9:30 | Sarah Reznikoff, The generalized annular Temperley-Lieb algebra |
| 9:40-10:20 | Martin Argerami, Joint majorization in finite factors |
| 10:30-11:00 | Coffee break, 2nd floor lounge, Corbett Hall |
| 11:00-11:50 | John Phillips, An Introduction to spectral flow and index formulas |
| 12:00-1:30 | Lunch |
| 1:30-2:00 | Daniel Goncalves, C ${ }^{*}$-algebras from substitution tilings: a new construction |
| $\mathbf{2 : 1 0 - \mathbf { 2 : 4 0 }}$ | Peter Pivovarov, An alternate proof of the existence of a convex body lacking |
|  | symmetric projections |
| $\mathbf{2 : 5 0 - 3 : 2 0}$ | Jiři Spurny, Affine Baire-one functions on compact convex sets |

Checkout by 12 noon.
** 2-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 16:00 on Saturday, although participants are still required to checkout of the guest rooms by 12 noon. There is no coffee break on Saturday afternoon, but self-serve coffee and tea are always available in the 2nd floor lounge, Corbett Hall.

# Second Northwest Functional Analysis Seminar March 17-19, 2005 

ABSTRACTS<br>(in alphabetic order by speaker surname)

## Speaker: Martin Argerami (University of Regina)

Title: Joint majorization in finite factors
Majorization of selfadjoint matrices was introduced by T. Ando and it has played an important role in linear algebra by providing a link between certain algebraic and geometric properties of matrices. Later, these notions were generalized to operator algebras by F. Hiai and others. In joint work with P. Massey, we extend the notion of majorization between commuting families of selfadjoint operators in a finite factor. As a byproduct, we obtain a local characterization of unital positive maps commuting with a given state.

## Speaker: Karoly Bezdek (University of Calgary) <br> Title: On Hadwiger's illumination conjecture

Abstract: Let $\mathbf{K}$ be a convex body (i.e. a compact convex set with nonempty interior) in $d$-dimensional Euclidean space $\mathbb{E}^{d}$ with $d \geq 2$. According to Hadwiger, an exterior point $\mathbf{p} \in \mathbb{E}^{d} \backslash \mathbf{K}$ of $\mathbf{K}$ illuminates the boundary point $\mathbf{q}$ of $\mathbf{K}$ if the half line emanating from $\mathbf{p}$ passing through $\mathbf{q}$ intersects the interior of $\mathbf{K}$. Furthermore, a family of exterior points of $\mathbf{K}$ say, $\mathbf{p}_{1}, \mathbf{p}_{2}, \ldots, \mathbf{p}_{n}$ illuminates $\mathbf{K}$ if each boundary point of $\mathbf{K}$ is illuminated by at least one of the point sources $\mathbf{p}_{1}, \mathbf{p}_{2}, \ldots, \mathbf{p}_{n}$. Finally, the smallest $n$ for which there exist $n$ exterior points of $\mathbf{K}$ that illuminate $\mathbf{K}$ is called the illumination number of $\mathbf{K}$ denoted by $I(\mathbf{K})$. In 1960, Hadwiger raised the following amazingly elementary but, very fundamental question, which is usually quoted as Hadwiger's illumination conjecture:

The illumination number $I(\mathbf{K})$ of any convex body $\mathbf{K}$ in $\mathbb{E}^{d}, d \geq 3$ is at most $2^{d}$ and $I(\mathbf{K})=2^{d}$ if and only if $\mathbf{K}$ is an affine $d$-cube.

This conjecture is very easy to prove in the plane. However, it is still open in $\mathbb{E}^{d}$ for all $d \geq 3$. In the talk we survey the major results known about this problem.

## Speaker: Alex Brudnyi (University of Calgary)

Title: Metric spaces with linear extensions preserving Lipschitz conditions
Abstract: We study a new bi-Lipschitz invariant $\lambda(M)$ of a metric space $M$; its finiteness means that Lipschitz functions on an arbitrary subset of $M$ can be linearly extended to functions on $M$ whose Lipschitz constants are enlarged by a factor controlled by $\lambda(M)$. We show that $\lambda(M)$ is finite for several important classes of metric spaces. These include metric trees of arbitrary cardinality, groups of polynomial growth, Gromov-hyperbolic groups, certain classes of Riemannian manifolds of bounded geometry and finite direct sums of arbitrary combinations of these objects. On the other hand, we construct an example of a twodimensional Riemannian manifold $M$ of bounded geometry for which $\lambda(M)=\infty$. Also, we prove that $\lambda(M)=\infty$ for any infinite-dimensional Banach space $M$.

Speaker: Juliana Erlijman (University of Regina)
Title: Subfactor constructions with braided categories
Abstract: We review and extend certain constructions of subfactors to the context of braided tensor categories and discuss the computation of important invariants.

Speaker: Daniel Goncalves (University of Victoria)
Title: $C^{*}$-algebras from substitution tilings: a new construction
Abstract: Given a substitution tiling that satisfies some standard conditions, we introduce an equivalence relation on $\mathbf{R}^{d}$ by saying that two points, $x$ and $y$, are equivalent if the patch defined by $y$ on the tiling matches the patch defined by $x$ translated by $y-x$. We then associate a $C^{*}$-algebra to this equivalence relation trough the groupoid approach and use the inflation map on the tiling to get an inductive limit of $C^{*}$-algebras. Finally, we compute the $K$-theory of these $C^{*}$-algebras for some examples.

Speaker: Ali-Amir Husain (University of Alberta)
Title: On the cohomology of operator algebras
Abstract: By analogy with the join in topology, the join $\mathcal{A} * \mathcal{B}$ for operator algebras $\mathcal{A}$ and $\mathcal{B}$ acting on Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$, respectively, was defined by Gilfeather and Smith. Assuming that $\mathcal{K}$ is finitedimensional, they calculated the Hochschild cohomology groups for $\mathcal{A} * \mathcal{B}$ with coefficients in $\mathcal{L}(\mathcal{K} \oplus \mathcal{H})$. We assume that $\mathfrak{A}$ is a maximal abelian von Neumann algebra acting on $\mathcal{H}, \mathcal{A}$ is a subalgebra of $\mathfrak{A} \bar{\otimes} \mathcal{L}(\mathcal{K})$, and $\mathcal{B}$ is an ultraweakly closed subalgebra of $M_{n}(\mathfrak{A})$ containing $\mathfrak{A} \otimes 1_{n}$. We show that $\mathcal{B}$ may be decomposed into a finite sum of free modules. In this context, we redefine the join of $\mathcal{A}$ and $\mathcal{B}$, generalize the calculations of Gilfeather and Smith, and calculate $H^{m}(\mathcal{A} * \mathcal{B}, \mathfrak{A} \bar{\otimes} \mathcal{L}(\mathcal{K} \oplus \mathcal{H}))$, for all $m \geq 0$.

Speaker: S. Mahmoud Manjeghani (University of Regina)
Title: Young's inequality for the trace of operators
Abstract: In this talk, I will establish a tracial version of Young's classical inequality that applies to positive operators in any semifinite von Neumann algebra. The case of equality is characterised under the assumption that the trace is finite and faithful.

Speaker: John Phillips (University of Victoria)
Title: An Introduction to spectral flow and index formulas
Abstract: I will explain the notion of spectral flow in the context of two symmetries $F_{0}=2 P-1$ and $F_{1}=2 Q-1$, where $P$ and $Q$ are two projections on a Hilbert space, $H$ with $P-Q$ compact. I will show explicitly why the straight line path from $F_{0}$ to $F_{1}$ has spectral flow and why it is the index of the (Fredholm) operator $Q P: P(H) \rightarrow Q(H)$ : that is, $\operatorname{sf}\left(F_{0}, F_{1}\right)=\operatorname{Index}(Q P)$. (Most of these ideas go back to BDF, Bojarski, and Wojciechowski). When $P-Q$ satisfies a summability condition such as $\operatorname{Tr}\left(|P-Q|^{q}\right)<\infty$ for some real $q \geq 1$, I will prove a trace formula for spectral flow:

$$
\operatorname{sf}\left(F_{0}, F_{1}\right)=\operatorname{Index}(Q P)=\operatorname{Tr}\left((P-Q)|P-Q|^{q-1}\right)
$$

Actually I will prove a much more general result at no extra cost which forms the basis of the following theorem:
Theorem. Let $\left(H, D_{0}\right)$ be an odd p-summable Fredholm module for the unital $*$-algebra $\mathcal{A}$, and let $P=$ $\chi_{[0, \infty)}\left(D_{0}\right)$. Then for each $u \in U(\mathcal{A})$ with $\left[D_{0}, u\right]$ bounded, PuP is a Fredholm operator in $P \mathcal{B}(H) P$ and if $\left\{D_{t}^{u}\right\}$ is any piecewise $C^{1}$ path in $D_{0}+\mathcal{B}(H)_{\mathrm{sa}}$ from $D_{0}$ to $u D_{0} u^{*}$, then:

$$
\operatorname{Index}(P u P)=\operatorname{sf}\left(\left\{D_{t}^{u}\right\}\right)=\frac{1}{C_{p / 2}} \int_{0}^{1} \operatorname{Tr}\left(\frac{d}{d t}\left(D_{t}^{u}\right)\left(1+\left(D_{t}^{u}\right) 2\right)^{-p / 2}\right) d t
$$

where $C_{p / 2}=\int_{-\infty}^{\infty}(1+x 2)^{-p / 2} d x$.
To this point, this is joint work with Alan Carey. Applications of these spectral flow formulas to a proof of the Connes-Moscovici local index theorem are joint work with Alan Carey, Adam Rennie and Fyodor Sukochev.

Speaker: Peter Pivovarov (University of Alberta)
Title: An alternate proof of the existence of a convex body lacking symmetric projections
Abstract: In 2001, Gluskin, Litvak, and Tomczak-Jaegermann proved the existence of a convex body $K \subset \mathbb{R}^{n}$ such that for any projection $P$ with suitably large rank, $P K$ is far from being symmetric. I will sketch an alternate proof that makes use of a recent probabilistic decoupling technique of Szarek and Tomczak-Jaegermann.

## Speaker: Bahram Rangipour (University of Victoria)

Title: From motivation to application of Hopf cyclic cohomology
Abstract: Calculating the index of transversal elliptic operators by Connes and Moscovici opened a new way to the cyclic cohomology under the name of cyclic cohomology of Hopf algebras. In the recent years the latter has been developed and generalized to Hopf cyclic cohomology. In this talk we trace the origin of this theory, define it and give a couple of its applications.

## Speaker: Sarah Reznikoff (Reed College)

Title: The generalized annular Temperley-Lieb algebra
Abstract: The ordinary annular Temperley-Lieb planar algebra is generated by the planar tangles with a single interior disk (one "input") and no homologically trivial closed loops. By definition, such tangles have an even number of distinguished points on each boundary circle. We relax this restriction and investigate the structure of the algebroid generated by annular Temperley-Lieb diagrams with odd numbers of marked boundary points.

## Speaker: Ebrahim Samei (University of Manitoba)

Title: Local operators and hyper-Tauberian algebras
Abstract: We investigate a certain class of commutative Banach algebras for which we can characterize the bounded local operators from them. We call them hyper-Tauberian algebras. We show that they are a proper subclass of the weakly amenable Tauberian algebras. Moreover, we show that, if $A$ is a hyperTauberian algebra, then the linear space of bounded derivations from $A$ into any Banach $A$-bimodule is reflexive. We apply these results to the Figà-Talamanca-Herz algebra $A_{p}(G)$ of a locally compact group $G$ for $p \in(1, \infty)$. We show that $A_{p}(G)$ is hyper-Tauberian if the principal component of $G$ is abelian. Finally, by considering the quantization of these results, we show that for any locally compact group $G, A_{p}(G)$, equipped with an appropriate operator space structure, is a quantized hyper-Tauberian algebra. This, in particular, implies that every finite subset of $G$ is a set of synthesis for $A_{p}(G)$ and $A_{p}(G)$ is operator weakly amenable.

Speaker: Jiři Spurny (University of Alberta)
Title: Affine Baire-one functions on compact convex sets
Abstract: Let $X$ be a compact convex set in a locally convex space and ext $X$ stand for the set of extreme points of $X$. We investigate a question under what conditions imposed on $X$ can any bounded Baire-one function defined on ext $X$ be extended to an affine Baire-one function on $X$. This problem is related to the notion of Bauer simplexes.

Speaker: Adi Tcaciuc (University of Alberta)
Title: Stabilization and asymptotic structure of Banach spaces
Abstract: The asymptotic theory of infinite-dimensional Banach spaces, developed by Maurey, Milman and Tomczak-Jaegermann, is concerned with the structure of infinite-dimensional Banach spaces manifested in the finite-dimensional subspaces that appear everywhere far away in the space. The class of spaces that have a simple asymptotic structure, in the sense that we can find a $1 \leq p \leq \infty$ such that all such finitedimensional subspaces as before are essentially $l_{p}^{n}$ 's, are of special interest and they are called asymptotic- $l_{p}$ spaces. We prove that if a Banach space is saturated with infinite-dimensional subspaces in which all special $n$-tuples of vectors are equivalent, uniformly in $n$, then the space contains asymptotic- $l_{p}$ subspaces, for some $1 \leq p \leq \infty$. The proof reflects a technique used by Maurey in the context of unconditional basic sequence problem and extends a result by Figiel, Frankiewicz, Komorowski, and Ryll-Nardzewski.

