Project Title: Modeling stream ecosystems

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The goal of this project is to derive and analyze mathematical models for stream ecosystems, to explore different mechanisms and biological hypotheses that lead to spatial patterning of the stream population distribution and composition, and to compare the model outcome with field experiments.

Biological background

Data show that species composition, distribution, and abundance vary along the length of a stream ecosystem, even where there are no major changes in river morphology or seasonal patterns in flow rates among stream sections. The major ecological question is what controls the upstream distribution of individuals and species? More specifically, can these distributions be explained from individual behavior and interaction?

Processes governing changes in species composition along the length of a river involve the interaction of food web processes, competition, stream flow, and (species and stage specific) dispersal. Understanding this interplay is a key factor in predicting effects of natural or human changes such as disruption of the food web (through pollution or introduction of exotic species) or changes in stream flow (dam building). Observed impacts for the stream ecosystem are extinctions, species replacement, or invasion. The linkage between human impacts and the response of aquatic systems in Southern Alberta is being investigated in a major project funded by NSERC Networks of Centres of Excellence in Clean Water.

Abstract for the presentation: Solutions for the drift paradox

In this talk, I will present model and possible solutions for the "drift paradox" as a first step in modeling and understanding stream ecosystems. This is joint work with M.A. Lewis (Alberta) and E. Pachepsky (UCSB). Individuals in streams are constantly subject to predominantly unidirectional flow. The question of how these populations can persist in upper stream reaches is known as the "drift paradox". We employ a general mechanistic movement-model framework and derive dispersal kernels for this situation. Thin- as well as fat-tailed kernels are derived. Then we introduce population dynamics and analyze the resulting integrodifferential equation. In particular, we study how the critical domain size and the invasion speed (see below) depend on the velocity of the stream flow. We give exact conditions under which a population can persist in a finite domain in the presence of stream flow, as well as conditions under which a population can spread against the direction of the flow. We find a critical stream velocity above which a population cannot persist in an arbitratily large domain. At exactly the same stream velocity, the invasion speed against the flow becomes zero; for larger velocities, the population retreats with the flow.

Mathematical background

Mathematical models for spatially distributed populations have a rich and diverse history. Spatially explicit models have been formulated as reaction diffusion equations, integro-difference equations, cellular automata, and related systems. The model we derive and study falls in the category of integrodifferential equations (reaction-dispersal equations) and is of the form

$$\frac{\partial}{\partial t}u(t,x) = f(u(t,x)) - \mu u(t,x) + \mu \int k(x,y)u(t,y)dy.$$

In this equation, $u(t,x)$ denotes the density of individuals at time t at location x. Population growth and death are modeled by the function f. Individuals leave their site x with rate μ and settle at x according to the probability of moving from y to x, given by the dispersal kernel $k(x,y)$.

We derive two classes of dispersal kernels from mechanistic movement models. These dispersal kernels are not symmetric due to the advection velocity of the stream. As this velocity increases, the skewness of the kernel increases.

The critical domain size and the invasion speed are two mathematical quantities closely related to ecological indicators. The critical domain size stems from the fact that the total reproductive rate of the population scales with habitat area, the dispersal loss scales with boundary length. Since surface area scales with the square of the linear dimension, boundary loss dominates dynamics of small patches but plays a diminished role in the dynamics of larger patches. The assumption that a small population grows in the absence of boundary loss leads to the existence of a critical size for the habitat, below which the population cannot persist, and above which populations can grow. Mathematically, one linearizes the governing equation around the zero solution and analyzes conditions for which this solution is unstable so that small perturbations grow. For the above equation, this amounts to finding the leading eigenvalue of the integral operator. For a certain class of kernels, we are able to do that explicitly, for other kernels, we use numerics.

The invasion speed is one of several measures of how fast a population spreads into previously uninhabited territory. Mathematically, one assumes that there is a solution of the govering equation in the form of a traveling wave and computes the minimal speed of that wave. For the equation above, we need to know the moment generating function of the dispersal kernel to compute the invasion speed.

A slightly different measure is the asymptotic spreading speed. Its definition is that an observer who travels slightly faster will eventually see no individuals whereas an observer traveling slightly slower will see the population at its carrying capacity. Mathematically, the asymptotic spreading speed involves the construction of appropriate sub- and supersolutions to the govering equation.