

BIRS WORKSHOP ON  
“**LINEAR OPERATORS: THEORY,  
APPLICATIONS, AND COMPUTATION**”

August 12-14, 2004.

**Background:** The theory of matrices and linear operators is going through a highly productive phase driven largely by a great variety of applications. These include magnetohydrodynamics, vibrations of continua, systems theory, signal processing, for example. They frequently concern the spectral properties of operators on Krein or Pontryagin spaces, and also require modern techniques in perturbation theory and differential equations. The workshop will provide an opportunity for informal discussion and presentation of current research projects in these areas.

**Organising Committee:** Paul Binding and Peter Lancaster,  
Dept. of Mathematics and Statistics, University of Calgary.

MEALS

Continental breakfast: 7:00-9:00, 2nd floor lounge, Corbett Hall, Friday and Saturday (no charge).

Lunch (buffet) available in Donald Cameron Hall at \$12.50+tax. 11.30-13.30.

Dinner (buffet) available in Donald Cameron Hall at \$23 + tax. 17.30-19.30.

Coffee breaks in 2nd floor lounge, Corbett Hall, as scheduled below.

For other light meal options there are the “Gooseberry’s Deli” in the Sally Borden Building and the Kiln Cafe near Donald Cameron Hall. The town of Banff has a wide selection of restaurants, etc., and is about fifteen minutes walk from Corbett Hall.

MEETING ROOMS

**All lectures will be held in Max Bell 159.** Accessible across the bridge from the second floor of Corbett Hall.

Rooms 155-159 are all available, in the basement of the Max Bell building. Other space is contracted to other Banff Centre guests - including any food and beverage in those areas.

## PROGRAM

THURSDAY, 16.00: Check-in begins at the front desk of the Professional Development Centre.

20.00: Informal gathering in the second floor lounge of Corbett Hall. Beverages and snacks are available on a cash honour-system basis.

Day	Time	Speaker	Title
Thu.	20.00-	-	Informal gathering in 2nd floor lounge, Corbett Hall.
Fri.	09.00-09.50	Langer	Normal operators in Pontryagin space
Fri.	09.55-10.35	Markus	On the connection between the indices of a Fredholm block operator matrix and its determinant
Fri.	10.40-11.00	-	Coffee
Fri.	11.00-11.30	Tretter	Operators associated with the Klein-Gordon equation
Fri.	11.35-12.25	Choi	Normal dilations
Fri.	12.25-14.00	-	LUNCH
Fri.	14.00-14.50	Lancaster	Inverse eigenvalue problems for vibrating systems
Fri.	14.55-15.25	Zhou	An $H$ -orthogonalization process and its applications
Fri.	15.30-15.50	-	Coffee
Fri.	15.50-16.40	Watkins	Solving Hamiltonian quadratic eigenvalue problems
Fri.	16.45-17.25	Xu	A structured staircase algorithm for the Kronecker canonical form of skew-symmetric/symmetric pencils.
Fri.	TBA	Allegretto	Eigenvalue estimates in lagoon ecology problems
Sat.	09.00-09.50	Rodman	Classes of normal linear transformations with respect to an indefinite inner product.
Sat.	09.55-10.25	Boulton	Approximation of eigenvalues of selfadjoint operators in a gap of the essential spectrum
Sat.	10.30-10.50	-	Coffee
Sat.	10.50-11.20	Farenick	A triangular representation of quasitriangular operators
Sat.	11.25-12.15	Churchill	The Galois theory of linear differential equations

Please note that it is necessary to check-out of the accomodation in Corbett Hall on Saturday morning, although other space remains available until 16.00. Please vacate your room by 10.50. (Luggage can be left in the second floor lounge.) Keys must be returned by 12.45.



## The Galois theory of linear differential equations

R. C. Churchill,

This will be an introductory talk on the Galois theory of linear homogeneous ordinary differential equations. The group of such an equation will be defined, the type of information packaged within will be discussed, and explicit examples will be considered, including the Bessel and hypergeometric equations. A more geometric approach to the group will be indicated.

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## A triangular representation of quasitriangular operators

D. Farenick, University of Regina

It is well known that (bounded, linear) operators on infinite-dimensional Hilbert spaces need not possess eigenvalues. However, if  $N$  is a normal operator on a separable Hilbert space, then Weyl-von Neumann-Berg theorem asserts that there are operators  $D$  and  $K$  such that  $K$  is compact,  $D$  is diagonal, and  $N=D+K$ . Moreover, there is a  $*$ -representation  $f$  of the  $C^*$ -algebra generated by  $N$  such that  $f(N)$  is a diagonal operator.

In a similar vein, an operator  $S$  on a separable Hilbert space is said to be quasitriangular if there are operators  $T$  and  $K$  such that  $K$  is compact,  $T$  is triangular, and  $A=T+K$ . In joint work with Brian Forrest and Laurent Marcoux of the University of Waterloo, we show that a nonselfadjoint version of the Weyl-von Neumann-Berg theorem holds. Namely, there is an isometric representation  $g$  of the algebra generated by  $A$  for which  $g(A)$  is triangular.

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## Inverse eigenvalue problems for vibrating systems

P. Lancaster, University of Calgary

A vibrating system is defined by three  $n \times n$  matrices which form the coefficients of a quadratic matrix polynomial. Eigenvalues and eigenvectors are determined by such a system in a natural way. We consider the problem: What are the defining properties of the eigenvalue and eigenvector data? Under what conditions do they determine real systems? Hermitian systems? Real symmetric systems? Real symmetric systems with positive definite coefficients?

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## Normal operators in Pontryagin space

H. Langer, Technical University of Vienna

According to a theorem of M.A. Neumark, for a bounded normal operator  $N$  ( $NN^+ = N^+N$ ) in a Pontryagin space  $\Pi_\kappa$  there exists a  $\kappa$ -dimensional nonpositive subspace which is invariant under  $N$  and  $N^+$ . After some reduction by a finite dimensional subspace, a spectral function of  $N$  is defined using the spectral functions of the operators  $A = \frac{N+N^+}{2}$  and  $B = \frac{N-N^+}{2i}$ . For the case  $\kappa = 1$  a complete description of the spectral structure of  $N$  is given.

**On the connection between the indices of a Fredholm block operator matrix and its determinant**

A.S.Markus, Ben Gurion University

It is known that the index of a Fredholm block operator matrix is equal to the index of its determinant provided that all, entries commute modulo the trace class. It will be proved that the trace class can be replaced in this statement by a larger operator ideal.

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**Classes of normal linear transformations with respect to an indefinite inner product.**

L. Rodman. College of William and Mary

The talk will focus on recent results and open problems concerning linear transformations on a complex or real vector space that are normal with respect to an indefinite inner product. In particular finite dimensional vector spaces will be emphasized. The following topics will be discussed:

- 1.Indecomposable normals
- 2. Canonical forms (if available) for some classes of normals
- 3. Shells - sets in the three dimensional real vector space that are obtained using sesquilinear forms associated with linear transformations - and various properties of shells of normal transformations.
- 4. Various conditions for normality that may or may not be necessary or sufficient, but are necessary and sufficient if the indefinite inner product is actually positive definite
- 5. Invariant semidefinite subspaces
- 6. Polar decompositions

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**Operators associated with the Klein-Gordon equation**

Christiane Tretter, University of Bremen

In this talk an abstract model for the Klein-Gordon equation is considered. Three operators acting in three different spaces are associated with this abstract model. In order to study their spectral properties, indefinite inner products are introduced. The structure of the spectra of these operators and their relations are investigated.

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## Solving Hamiltonian quadratic eigenvalue problems

D. S. Watkins, Washington State University

Quadratic eigenvalue problems with Hamiltonian structure arise in several contexts, including the study of corner singularities in anisotropic elastic solids. We show how to solve these by transforming the original problem to an eigenvalue problem for a large Hamiltonian matrix and then applying structure-preserving Krylov subspace methods to the transformed problem. A variety of methods are considered, including some that attack the Hamiltonian problem directly and others that make a further transformation to either skew-Hamiltonian or symplectic form. All of the structure-preserving methods are more accurate than a comparable method that ignores the structure. The fastest of the structure-preserving methods is more efficient than the method that ignores the structure.

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## A structured staircase algorithm for the Kronecker canonical form of skew-symmetric/symmetric pencils

H. Xu, Kansas State University

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## An $H$ -orthogonalization process and its applications

F. Zhou, University of Regina

For an invertible Hermitian matrix  $H$ , an algorithm to give an  $H$ -orthonormal basis in  $\mathcal{C}^n(\mathcal{R}^n)$  is provided. The algorithm can be used to give a random solution to matrix equation  $X^*H_1X = H_2$ , where  $H_1$  and  $H_2$  are two fixed Hermitian matrices with the same inertia. The talk will also show how to use a solution of  $X^*HX = \hat{P}$  to give a random  $H$ -self-adjoint matrix  $A$ .