New Mechanisms for Regularity, Singularity, and Long Time Dynamics in Fluid Equations

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The 5-day workshop, held online due to the COVID-19 pandemic, attracted more than 120 registered participants from all around the world. 23 speakers presented lectures covering some of the most exciting recent results in the area of PDE analysis of mathematical fluid dynamics. Most of the talks are recorded, and are publicly available on the website https://www.birs.ca/events/2021/5-day-workshops/21w5110, and thus generating lasting impact on the field.

1 Overview of the Field

Partial differential equations inspired by dynamics of fluid flows are a classic subject studied since the 1800s, and are still of great interest to this day. The central questions from the PDE point of view are global regularity and the long-time, generic dynamics of solutions to the fundamental Euler and Navier Stokes equations (NSE). While tremendous progresses have been made over the last several decades, some of the most fundamental problems, such as the global regularity of NSE, remain open. Inspired by numerous applications in science and engineering, the study of fluid dynamics and rigorous mathematical theory continues to attract the attention of many mathematicians.

2 Recent Developments and Open Problems

Recently there have been several significant breakthroughs in this area, including plausible scenarios proposed by Hou-Luo for singularity formation in three dimensional Euler equations, the proofs of asymptotic stability of coherent structures in 2d incompressible flows, new mechanisms of creation of small scales in inviscid flows, and recent constructions of singularity formation for strong (non-smooth) solutions to 3d Euler equations and related 1d models, to name a few.

These developments have solved some long standing problems, offered fresh insights and raised important new conjectures. It now seems promising to settle several major problems in the Euler and Navier Stokes equations. Let us only mention among them the possible finite time blow up for smooth solutions to 3d Euler equation near boundary, asymptotic stability of two dimensional shear flows and vortices, justification of motion of vortex filaments in 3d and more.

We believe that new techniques developed over the last decade in the study of fluid equations are powerful and flexible, and with new input of ideas, it is an exciting time to make significant further progress in Euler and Navier Stokes equations.

3 Presentation Highlights

All the talks are very well attended and received, with fruitful discussions and follow-up questions both during and after the talks. For example, Terence Tao presented a possible strategy for producing finite singularity in the three dimensional Navier Stokes equations, the existence of which is a central open problem in fluid dynamics; Nader Masmoudi lectured on recent results on Prandtl equation and the issue of boundary layer separation; Vladimir Sverak discussed a new result on the long distance asymptotics for the steady Navier Stokes equations. We note in particular that Tom Hou outlined an approach to construct blow up solution for the three dimensional incompressible Euler equations near the boundary. Tom Hou, together with Jiajie Chen recently successfully implemented the approach in an important work. From young researchers, we mention in particular the talk by Yu Deng on the kinetic wave equations, in which he discussed his solution (with Zaher Hani) on the rigorous derivation of the kinetic wave equation from nonlinear Schrödinger equations.

4 Scientific Progress and Outcome of the Meeting

Due to the sudden impact of the COVID-19 pandemic, many planned scientific conferences and meetings were cancelled or postponed. The 5-day workshop at BIRS filled a significant gap of scientific activities and provided a precious opportunity for both senior mathematicians and young researchers working in the area of mathematical fluid dynamics to exchange the latest developments in the field. Several talks in the meetings addressed important recent breakthroughs in the field, which helped the larger community to absorb their results and ideas more quickly. The discussion between the sessions were always lively, and it is quite possible new collaborations had formed as a result of the workshop.

In addition, thanks to the online format, we were able to get many very distinguished speakers, who may be too busy to travel and participate in the week long event. There was also a larger than usual audience due to the high level of the lectures and the ease of attending. Moreover, most of the talks were recorded and published online, providing a resource that the community can use permanently. Overall, we believe the workshop is a huge success, despite the many challenges of the pandemic.

References

[1] Y. Deng and Z. Hani, Full derivation of the wave kinetic equation, arXiv 2104.11204.

[2] J. Chen and T. Hou, Stable nearly self-similar blowup of the 2D Boussinesq and 3D Euler equations with smooth data, arXiv:2210.07191