Spectral Methods in Representation Theory of Algebras and Applications to the Study of Rings of Singularities (08w5060)

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September 7–September 12, 2008

1 Antecedents

The roots of representation theory go far back into the history of mathematics: the study of symmetry, starting with the Platonic solids and the development of group theory; the study of matrices and the representation theory of groups by Klein, Schur and others which led to the development of the concepts of rings, ideals and modules; the study of normal forms in analysis, in the work of Weierstrass, Jordan and Kronecker, among others; the development of Lie theory. Some of the most famous Hilberts problems relate representation theory with fundamental geometric concepts.

Starting in the middle 60's of last century, the representation theory of finite dimensional algebras had a very fast development with three main trends visible: The categorical point of view, represented by Maurice Auslander and his school, leading to the concepts of almost-split sequences and Auslander-Reiten quivers. The introduction of the concept of representations of quivers by Pierre Gabriel, which is a main tool in the analysis of the representation theory of finite dimensional algebras and their representations. The reformulation of problems from representation theory as matrix problems, associated to the Ukrainean school of A. Roiter, which lead to classification results in certain representation-infinite situations and the conceptual dichotomy of algebras according to their representation type as tame or wild.

The Representation Theory of finite dimensional algebras has by now reached a highly mature stage, as can be seen from the now well-established links to major mathematical subjects like Combinatorics, Lie theory, Group theory, Algebraic Geometry, Singularity theory, etc. Moreover, there have been important recent developments to mention just few of them: The "Representation dimension" was the topic of a very successful conference at the University of Bielefeld, Germany, in May, 2008 and another attractive subject "Cluster algebras and categories" will be in the center of a Workshop and a Conference in Mexico in December, 2008. As a direct antecedent we mention that there was a very successful Workshop on the topic at the BIRS organized earlier, viz. "Interaction of Finite Dimensional Algebras with other areas of Mathematics" 04w5501 on September 25 - 30, 2004.

The workshop **"Spectral Methods in Representation Theory of Algebras and Applications to the study of Rings of Singularities" 08w5060** on September 7 - 12, 2008 was organized as an answer to the need of interaction of mathematicians in different areas interested on a common subject. For both workshops at BIRS the response of invited speakers and then the participants has been very positive.

2 The Workshop: settings, scope and program

The aim of the Workshop was to explore recent developments in the spectral analysis of triangulated categories with Serre duality and to facilitate synergies among groups of mathematicians interested on the topic but working in different areas of mathematics (Representation Theory of Algebras, Lie Algebras, Singularity Theory, among others). Let us introduce some notation:

Let k be a fixed field. For a finite dimensional k-algebra A of finite global dimension, the *Coxeter transformation* ϕ_A is an automorphism of the Grothendieck group $K_0(A)$ such that $[X^*]\phi = [\tau_{D(A)}X^*]$ for any complex X^* in the bounded derived category $D(A)$ of finite dimensional A-modules, where $\tau_{D(A)}$ is the Auslander-Reiten translation in $D(A)$ and $[X^*]$ is the class of X^* in $K_0(D(A))$ which is isomorphic to $K_0(A)$. The characteristic polynomial χ_A of ϕ_A, called the *Coxeter polynomial* of A, and the corresponding spectrum of ϕ_A control the growth behavior of $\tau_{D(A)}$. A similar approach holds for triangulated categories with Serre duality and a tilting object. In particular these methods apply to derived categories of coherent sheaves over (a possibly weighted) projective variety with a tilting complex.

Clearly, the spectrum of the Coxeter transformation is an important derived invariant of a homological nature that establishes links between representation theory and other fundamental areas: Lie algebras, C^*-algebras, singularity theory, rings of automorphic forms, spectra of graphs and the theory of links and knots, among other topics.

2.1 The topics of the Workshop

The main aim of the Workshop was to bring together researchers from representation theory, Lie theory and singularity theory using triangulated categories as a bridge and the spectral analysis of Coxeter transformations as a common language. The organizers of the Workshop hence proposed to consider the following main (but not only) directions:

2.1.1 Spectral theory of quivers

Let $A = k\Delta$ be an algebra associated to a finite connected quiver without oriented cycles. In this situation, the representation type of A is determined by properties of the spectral radius ρ_A: in fact, A is tame if and only if $\rho_A = 1$, moreover, if A is wild, then $\rho_A > 1$ is a simple root of χ_A.

For a one-point extension $A = B[P]$ with P an indecomposable projective B-module associated to a source b of Δ, the Coxeter polynomials are related by $\chi_A(T) = (1+T)\chi_B(T) - T\chi_C(T)$, where C is the quotient of B obtained by killing the extension vertex b. Such formulas allow the inductive calculation of Coxeter polynomials of tree algebras, for example.

2.1.2 Representation theory of algebras and sheaves

Canonical algebras $C = C(p, \lambda)$ depending on a weight sequence $p = (p_1, ...p_t)$ of positive integers and a parameter sequence $\lambda = (\lambda_3, ..., \lambda_t)$ of pairwise distinct non-zero elements from the base field k. The finite dimensional representation theory of mod(C) is completely controlled by the category $\operatorname{coh}\mathbb{X})$ of coherent sheaves on a non-singular weighted projective line $\mathbb{X} = \mathbb{X}(p, \lambda)$, since the bounded derived categories of $\operatorname{coh}\mathbb{X}$ and of mod(C) are equivalent.as triangulated categories. The representation type of C is determined by the genus $g_{\mathbb{X}} = 1 + \frac{1}{2}((t-2)p - \sum p/p_i)$, where $p = l.c.m.(p_1, ..., p_t)$. For $g_{\mathbb{X}} = 1$, the algebra C is of tubular type, and therefore the classification problem for $\operatorname{coh}\mathbb{X}$ relates to Atiyah's classification of vector bundles over an elliptic curve.

2.1.3 Surface and Fuchsian singularities

The links between tame hereditary algebras and simple surface singularities has been known for some time. The link is formally established through the analysis of the behavior of the Auslander-Reiten translation of hereditary algebras. More generally:

Consider the Auslander-Reiten translation τ_A in the category of finite dimensional A-modules and P an indecomposable projective A-module. In case A is not representation-finite, we get a well-defined graded algebra

$$R(A,P) = \bigoplus_{n=0}^{\infty} \mathrm{Hom}_A(P, \tau_A^{-n} P)\,.$$

For $A = \mathbb{C}\tilde{\Delta}$ a tame hereditary algebra, the algebra $R(A,P)$ is isomorphic to the algebra of invariants $\mathbb{C}[x,y]^G$, where $G \subset SL(2,\mathbb{C})$ is a binary polyhedral group of type Δ. Accordingly the completion of the graded algebra $R(A,P)$ is isomorphic to a *surface singularity* of type Δ. Assuming that C is a canonical algebra of wild representation type a similar construction yields an algebra $R(A,P)$ that is isomorphic to the algebra of entire automorphic forms associated to the action of a suitable *Fuchsian group* of the first kind, acting on the upper half plane.

2.1.4 Strange duality and mirror symmetry

The 14 exceptional unimodal hypersurface singularities and the singularities involved in the extension of the *Arnold's strange duality* are examples of Fuchsian singularities. In all those cases, the graded rings associated to the singularities are of the form $R(C,P)$ for a canonical algebra C and an indecomposable projective C-module P, such that the one-point extension $A = C[P]$ is an extended canonical algebra with spectral radius $\rho_A = 1$. There is an interesting relationship between the bounded derived category of $C[P]$ and the triangulated category of singularities of the graded algebra $R(C[P])$. Further a duality defined by Saito between cyclotomic polynomials extends to the Coxeter polynomials of the extended canonical algebras yielding Arnold's strange duality between the corresponding singularities. It is well-known that Arnold's strange duality is related to the *mirror symmetry* of K3 surfaces.

2.2 Invited speakers

In order to build the right interactions between different groups of mathematicians, the organizers invited experts from different areas:

Representation Theory of Algebras: F. Chapoton (U. Lyon, France), D. Happel (U. Chemnitz, Germany), L. Hille (U. Münster, Germany), D. Kussin (U. Paderborn, Germany), H. Krause (U. Paderborn, Germany), H. Meltzer (U. Szczecin, Poland) I. Reiten (U. Trondheim, Norway), C. M. Ringel (U. Bielefeld, Germany).

Triangulated and Calabi-Yau Categories: A. Buan (U. Trondheim, Norway), M. Barot (UNAM, Mexico-City), C. Geiß (Mexico-City), O. Iyama (U. Nagoya, Japan), B. Keller (U. Paris, France).

Singularity Theory: V. Batyrev (U. Tübingen, Germany), W. Ebeling (U. Hannover, Germany), D. Murfet (U. Canberra, Australia), A.Takahashi (U. Kyoto, Japan), K. Ueda (U. Osaka, Japan).

Lie algebras: K. Saito (U. Kyoto, Japan), Rafael Stekolshchik (Bar Illan University, Israel).

In addition to this list of participants we invited a number of students and young talents.

2.3 Preparation for the Workshop

As part for the preparation of the Workshop, two of the authors prepared a survey presenting the position on the topic coming from the Representation Theory of Algebras. The paper *Spectral Analysis of finite dimensional algebras and singularities* by H. Lenzing and J.A. de la Peña was made available through the arXiv (`arXiv:0805.1018`); it is meanwhile published, see [28].

The Workshop did also profit from the recently published book by R. Stekolshchik [42] covering, in particular, the Lie theoretic aspects of spectral analysis.

2.4 The program

Each day there were a set of Survey talks and Key Note talks programmed, among them:

Monday: *Stekolshchik*: Coxeter transformations, the McKay correspondence and the Slodowy correspondence; *De la Peña*: Coxeter transformation of distinguished classes of finite dimensional algebras; *Happel*: On the coefficients of the Coxeter polynomial

Tuesday: *Ringel*: Kronecker modules; *Iyama*: Tilting and cluster tilting objects for quotient singularities; *Ebeling*: McKay correspondence for the Poincaré series of Kleinian and Fuchsian singularities.

Wednesday: *Meltzer*: Stable categories of vector bundles: Generalities and the domestic case; *Takahashi*: Homological mirror symmetry for cusp singularities.

Thursday: *Saito*: Highest weight integrable representations of elliptic algebras and cuspidal algebras; *Lenzing*: Spectral analysis of singularities; *Kussin*: Stable categories of vector bundles: The tubular case; *Keller*: The periodicity conjecture via 2-Calabi-Yau categories; *Ueda*: Dimers, quivers and toric Calabi-Yau 3-folds.

Friday: *Krause*: Ext-orthogonal pairs for hereditary noetherian rings; *Batyrev*: McKay correspondence in higher dimensions and mirror symmetry.

3 Scientific Progress Made

One highlight of the Workshop was the report of B. Keller on his solution of the general case of the periodicity conjecture from mathematical physics from the beginning of the 1990s, dealing with discrete dynamical systems of pairs of Dynkin diagrams. Solutions for special cases by Frenkel-Szenes, Gliozzi-Tateo and Fomin-Zelevinsky have been known before. The proof is based on cluster category techniques and involves an analysis of Coxeter transformations. For a preliminary version compare [16].

In the opinion of the organizers further significant progress has been made in the following subjects, where substantial impact on the further development of representation theory and its relations to singularity theory is expected.

3.1 Sequences of algebras

Many classes of finite dimensional algebras have been extensively studied. For instance this applies to self-injective algebras, hereditary algebras, canonical algebras, tilted and quasitilted algebras, piecewise hereditary algebras, etc. On the other hand, several researchers consider whole *sequences of algebras* $(A_n)_n$, where all A_n follow the 'same' building principle and with the number of simple A_n-modules growing.

This procedure was recently formalized in the construction of sequences of accessible algebras. Starting from the k-algebra k, accessible algebras are obtained by successive one-point extensions with exceptional modules. Each such *accessible tower* A_n of algebras consists of algebras of finite global dimension and leads to a tower of triangulated categories $\mathcal{T}_1 \subset \mathcal{T}_2 \subset \cdots$ of triangulated categories, where $\mathcal{T}_n$ is the bounded derived category of finite dimensional A_n-modules.

There are many reasons to investigate such algebras and their associated derived categories: (i) By design accessible algebras have a combinatorial flavor, inherited from the largely combinatorial character of exceptional modules; (ii) Many interesting algebras are derived equivalent to accessible algebras; (iii) Accessible algebras have a strong affinity to spectral analysis; (iv) Derived accessible algebras seem to be those finite dimensional algebras with the closest connection to singularity theory.

By way of example, let A_n be the path algebra of the linear quiver $1 \xrightarrow{x} 2 \xrightarrow{x} 3 \xrightarrow{x} \cdots \xrightarrow{x} n-1 \xrightarrow{x} n$ with n vertices modulo all relations of the form $x^r = 0$ (for a fixed integer r). Other such sequences arise from poset algebras (= algebras of fully commutative quivers) following a common building principle (concatenation of commutative squares, diamonds, etc.) as follows:

$$\begin{array}{cccccccccccc} B_n & \circ & \to & \circ & \to & \circ & \to & \cdots & \to & \circ & \to & \circ \\ & \downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow \\ & \circ & \to & \circ & \to & \circ & \to & \cdots & \to & \circ & \to & \circ \end{array} \tag{1}$$

which is an accessible algebra. This example allows variations like

$$\begin{array}{cccccccccccccc} C_n & \circ & \to & \circ & \to & \circ & \to & \cdots & \to & \circ & \to & \circ & \to & \circ \\ & \downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow & & \\ & \circ & \to & \circ & \to & \circ & \to & \cdots & \to & \circ & \to & \circ & & \end{array} \tag{2}$$

or

$$\begin{array}{ccccccccccccccc} D_n & \circ & \to & \circ & \to & \circ & \to & \cdots & \to & \circ & \to & \circ & & \\ & \downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow & & \\ & \circ & \to & \circ & \to & \circ & \to & \cdots & \to & \circ & \to & \circ & \to & \circ \end{array} \tag{3}$$

mixing commutativity and zero relations. In each case the subscript n denotes the number of vertices. A typical result in this context is the following, see [28] for further details. Note that such diagrams occur as Coxeter-Dynkin diagrams of singularities, see [10], [9].

Proposition.*Let A_n be the linear quiver of n vertices with the relations $x^3 = 0$. Then A_{11} is derived-equivalent to the canonical algebra of type $(2,3,7)$ and A_{12} is derived-equivalent to the corresponding extended canonical algebra. Moreover, the algebra A_{12} occurs as the endomorphism ring of a tilting object in the triangulated category of singularities of the (graded) surface singularity $x^2 + y^3 + z^7$.*

The above reports the status of knowledge during the Workshop. Meanwhile is it known how to extend the result to study the derived categories of A_n for an arbitrary n, yielding results of a similar flavor. Note that for $n \geq 7$ the derived category of A_n-modules has wild representation type.

3.2 Singularities associated to a weighted projective line, Orlov's theorem and stable categories of vector bundles

The issue deals with one of the lucky instances in mathematics where one has very different descriptions of the same mathematical object, allowing to merge the knowledge from the various perspectives. To discuss these objects properly, we briefly review some properties of weighted projective lines and their associated graded singularities.

For a given weight sequence $\underline{p} = (p_1, \ldots, p_t)$, $t \geq 0$, of integers $p_i \geq 2$ we form the rank one abelian group $\mathrm{L}(\underline{p})$ on generators $\vec{x}_1, \ldots, \vec{x}_t$ subject to the relations $\vec{c} := p_1\vec{x}_1 = \cdots = p_t\vec{x}_t$. If additionally $\underline{\lambda} = (\lambda_3, \ldots, \lambda_t)$ is a parameter sequence of pairwise distinct non-zero elements from k, we form the commutative affine k-algebra $\mathrm{S}(\underline{p}, \underline{\lambda})$ on generators $x_1, \ldots, x_t$ subject to the $(t-2)$ relations $x_i^{p_i} = x_2^{p_2} - \lambda_i x_1^{p_1}$, $i = 3, \ldots, t$. Attaching x_i degree $\vec{x}_i$, the algebra $S = \mathrm{S}(\underline{p}, \underline{\lambda})$ becomes an $\mathrm{L}(\underline{p})$-graded algebra with homogeneous components $S_{\vec{x}}$, $\vec{x} \in \mathrm{L}(\underline{p})$, which are finite dimensional k-vector spaces. If $\mathcal{O}$ denotes the structure sheaf of $\mathbb{X}$, then naturally $\mathrm{Hom}(\mathcal{O}(\vec{x}), \mathcal{O}(\vec{y})) = S_{\vec{y}-\vec{x}}$. If the (orbifold) *Euler characteristic* $\chi = 2 - \sum_{i=1}^{t}(1 - 1/p_i)$ of $\mathbb{X}$ is different from zero, then there exists an alternative descriptions of $\mathrm{coh}\,\mathbb{X}$ by a positively $\mathbb{Z}$-graded Gorenstein algebra as follows. Restricting the grading of S to the subgroup of $\mathrm{L}(\underline{p})$ generated by the dualizing element $\vec{\omega} = (t-2)\vec{c} - \sum_{i=1} 1^t \vec{x}_i$ yields a positively $\mathbb{Z}$-graded algebra $R = \mathrm{R}(\underline{p}, \underline{\lambda})$, $R = \bigoplus_{n=0}^{\infty} R_n$ where

$$R_n = \begin{cases} S_{-n\vec{\omega}} & \text{if} \quad \chi > 0 \\ S_{n\vec{\omega}} & \text{if} \quad \chi < 0. \end{cases}$$

The category $\mathrm{vect}\,\mathbb{X}$ of vector bundles over $\mathbb{X}$ relates to suitable categories of graded maximal Cohen-Macaulay modules. Indeed, for example, in case $\chi > 0$ then $R = \mathrm{R}(\underline{p}, \underline{\lambda})$ is a positively $\mathbb{Z}$-graded algebra with three generators and a single relation. Accordingly R is complete intersection, in particular Gorenstein. Moreover, sheafification $M \mapsto \widetilde{M}$ induces a natural equivalence $\mathrm{CM}^{\mathbb{Z}}\text{-}R \to \mathrm{vect}\,\mathbb{X}$. In case $\chi < 0$ then $R = \mathrm{R}(\underline{p}, \underline{\lambda})$ is $\mathbb{Z}$-graded Gorenstein, in general not complete intersection. Moreover, sheafification $M \mapsto \widetilde{M}$ induces a natural equivalence $\mathrm{CM}^{\mathbb{Z}}\text{-}R \to \mathrm{vect}\,\mathbb{X}$.

For a variety X Orlov investigated the triangulated category $\mathrm{D}_{\mathrm{Sg}}(X)$ of the singularities of X defined as the quotient of the bounded derived category $\mathrm{D}^b\,\mathrm{coh}\,(X)$of coherent sheaves modulo the full subcategory of perfect complexes. If X is affine with coordinate algebra R, then this category $\mathrm{D}_{\mathrm{Sg}}(R)$ is just the quotient $\mathrm{D}^b\,\mathrm{mod}\text{-}R/\,\mathrm{D}^b\,\mathrm{proj}\text{-}R$, where $\mathrm{proj}\text{-}R$ is the category of finitely generated projective R-modules. Orlov further introduced a graded variant

$$\mathrm{D}^{\mathbb{Z}}_{\mathrm{Sg}}(R) = \mathrm{D}^b\,\mathrm{mod}^{\mathbb{Z}}\text{-}R/\,\mathrm{D}^b\,\mathrm{proj}^{\mathbb{Z}}\,R$$

called the *triangulated category of the graded singularity* R which will play a central role in this section.

Under the name *stabilized derived category of R* the categories $\mathrm{D_{Sg}}(R)$ were already introduced by Buchweitz. His results easily extend to the graded case and yields for an R that is graded Gorenstein an alternative description of $\mathrm{D}^{\mathbb{Z}}_{\mathrm{Sg}}(R)$ as the *stable category of graded maximal Cohen-Macaulay modules* $\underline{\mathrm{CM}}^{\mathbb{Z}}\text{-}R$. More precisely, he showed that the category $\mathrm{CM}^{\mathbb{Z}}\text{-}R$ of maximal graded Cohen-Macaulay R-modules is a Frobenius-category, hence inducing on the attached stable category $\mathrm{CM}^{\mathbb{Z}}\text{-}R$ of graded maximal Cohen-Macaulay modules modulo projectives, the structure of an *algebraic* triangulated category.

Stable categories of vector bundles are another class of triangulated categories, whose properties are dominated by the hereditary abelian category of coherent sheaves on a weighted projective line. By definition, the *stable category of vector bundles* $\underline{\mathrm{vect}}\mathbb{X}$ is the triangulated category obtained from $\mathrm{vect}\,\mathbb{X}$ as the factor category $\mathrm{vect}\,\mathbb{X}/[\mathcal{L}]$ of $\mathrm{vect}\,\mathbb{X}$ modulo the two-sided ideal of all morphisms factoring through a finite direct sum of members from $\mathcal{L}$, where $\mathcal{L}$ denotes a τ-orbit of line bundles on $\mathbb{X}$.

Theorem. *Let $\mathbb{X}$ be a weighted projective line of weight type $\underline{p} = (p_1, \ldots, p_t)$. Then the category $\underline{\mathrm{vect}}\mathbb{X}$ is a triangulated category with Serre duality which has a tilting object. Moreover a tilting object T can be chosen in such a way such that:*

(i) $\mathrm{End}(T)$ *is the path algebra of the Dynkin quiver* $[p_1, \ldots, p_t]$ *if* $\chi > 0$.

(ii) $\mathrm{End}(T)$ *is the canonical algebra of (tubular) weight type* $\underline{p}$ *if* $\chi = 0$.

(iii) $\mathrm{End}(T)$ *is an extended canonical algebra of type* $\langle p_1, \ldots, p_t \rangle$ *if* $\chi < 0$.

The following issues are in process to be addressed:

- stable categories of vector bundles yield a categorification of the Milnor lattice of large classes of singularities
- it is surprising, and needs further investigation, that this is largely possible through categories, being derived hereditary or stable hereditary!
- a detailed understanding of the stable categories of vector bundles for weighted projective curves (and their noncommutative generalizations) is desirable.

3.3 The representation theory of wild Kronecker algebras

The r-Kronecker algebra K_r is the path algebra of the quiver $1 \overset{\rightrightarrows}{\underset{\to}{\vdots}} 2$ (r arrows). For each r the algebra K_r is hereditary, thus belongs to that class of algebras whose representation theory is best understood. For $r = 1$ the representation type of K_r is finite, for $r = 2$ it is tame (and derived equivalent to the classification of coherent sheaves on the projective line), while for $r \geq 3$ the classification problem is wild, and generally thought to be hopeless.

Ringel proposes to attack this problem by means of *covering theory*. By now it is a standard tool to investigate tame algebras and their representations, but curiously a serious application of covering methods for wild type does not exist. It is expected that attacking the representation theory of wild Kronecker algebras with the techniques of covering theory will yield substantial new insight in the study of representations for algebras of wild type in general. Because of the appearance of representation theory of the Kronecker quiver in many situations in mathematics, new lines of interactions to neighboring subjects should be a consequence of these investigations.

4 Outcome of the Meeting

The response of participants of the Workshop was very positive. Important new results were presented and new unexpected links were discovered, many of them still mysterious. The organizers were happy with the intense discussions among participants, resulting in fresh connections between representation theory, the theory of triangulated category and singularity theory. We are convinced that the synergies provided by the

Workshop will continue and provide a sharper focus on the problems of central importance. Some problems emerging at the Workshop even qualify as test problems for the area, that is, core problems that stretch the capabilities of the available methods to their limits or well beyond. This, no doubt, will mean a strong stimulus to further developments in the subject.

We take the opportunity to point to a direct offspring of our BIRS Workshop. On the initiative of C.M. Ringel it was agreed at the end of the Workshop to follow one of the links emerging at the Workshop (between stable categories of vector bundles and a certain class of representation-finite Nakayama algebras) more closely through lectures by Ebeling, Happel and Lenzing at Bielefeld University end of October 2008. This event grew to the ADE-chain Workshop [37] yielding a confirmation for the expected relationship above but also producing a completely unexpected apparent relationship between the stable categories of vector bundles over Brieskorn singularities of the form $x^2 + y^3 + z^p$ and the subspace problem(s) for nilpotent operators, studied intensively by Ringel and Schmidmeier in recent years, see [38], [39], [40]. Since setting and techniques are quite different for both problems with apparently no former relationship, confirmation and investigation of this link offers another challenge originating in the outcome of the BIRS Workshop.

Finally the organizers want to point out that a majority of participants showed interest in a new BIRS Workshop of a similar format where the interaction of mathematicians working in different areas is in the focus. Taking the many experiences of the present BIRS Workshop int account, a new application has been prepared by the organizers together with C.M. Ringel (Bielefeld) for a future BIRS Workshop focussing on the most pressing questions of the present research in the topic.

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