

A hybrid direct search and model-based derivative-free optimization method with dynamic decision processing

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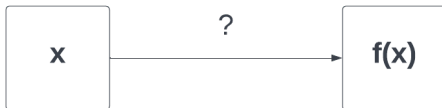
July 19, 2022

Outline

- 1 Introduction
- 2 DQL Method
- 3 SMART DQL Method
- 4 Solid Tank Design
- 5 Conclusion

Derivative-Free and Black-Box Optimization

- Derivative-Free: No derivative information is used or available.
- Black-Box Function: The evaluation process is hidden.



Motivation

- We have a lot of well-developed methods for black-box problems.
- Due to the nature of black-box problems, we do not know how to choose the appropriate method.
- Inspired by the RQLIF method [Manno et al., 2020], we combine the strengths of three kinds of search strategies into one method.
- Allow the method to choose search strategies *dynamically* and *adaptively*.

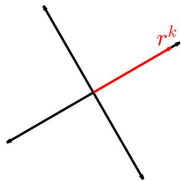
DQL Method Framework

- 1 Initialize
- 2 Direct Search Step
- 3 Quadratic Search Step
- 4 Linear Search Step
- 5 Update, Stop or Loop

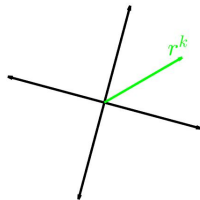
Framework of the Direct Step

Search on the directions of rotated positive and negative coordinate direction by a step length of δ^k .

- Desired Direction



- Undesired Direction



Direct Step Strategy 1: Random Rotation

The rotation directions alternates between two options:

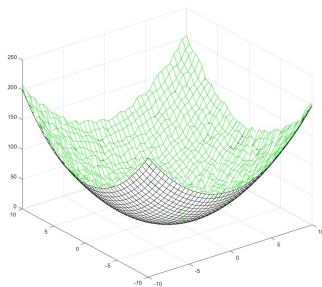
- the coordinate directions.
- a random rotation.

Framework of the Quadratic Step

Extract the quadratic information from the previously evaluated candidates within the trust region.

- Least-Squares Quadratic Model

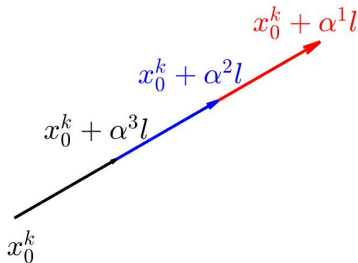
- Approximate Newton's Method



Framework of the Linear Step

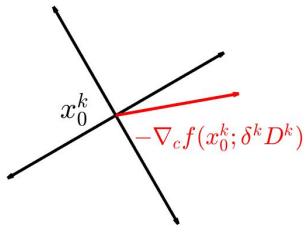
$$\mathbb{L} = \{x_0 + \alpha^j l\}$$

- Search direction $l \in \mathbb{R}^n$
- Linear search steps $\{\alpha^j \in \mathbb{R}\}$

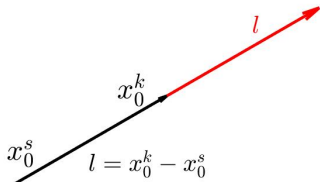


Linear Step Strategies: Determine Search Direction

- Approximate Steepest Descent
 $l = -\nabla_c f(x_0^k; \delta^k D^k)$

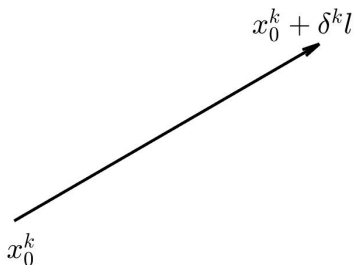


- Last descent $l = x_0^k - x_0^s$

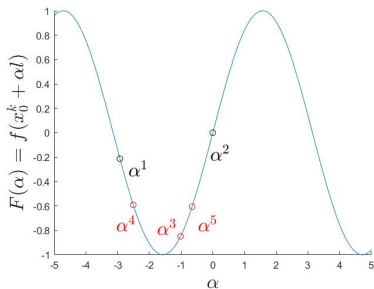


Linear Step Strategies: Determine Search Step Length

- Step Length δ^k



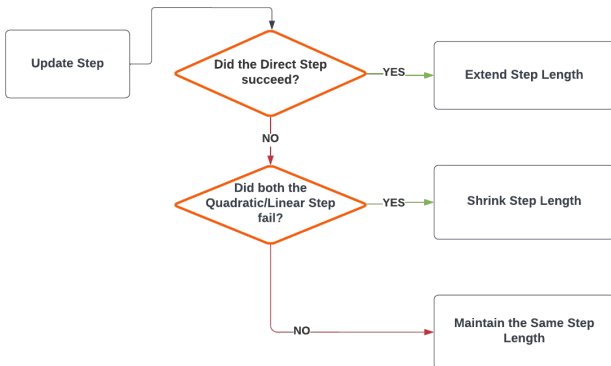
- Safeguarded Bracket Search [Mifflin and Strodiot, 1989]



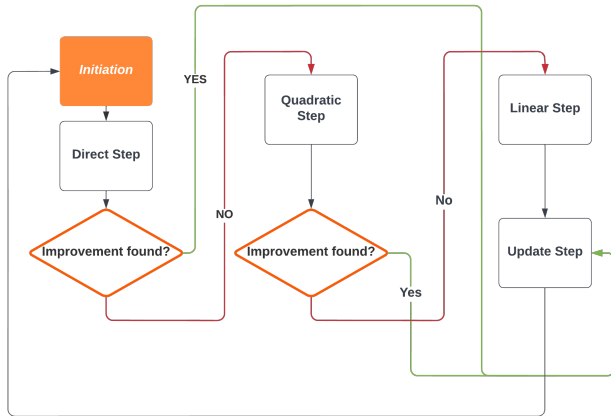
Linear Step Strategies

Label	Search Direction /	Search Step α
Strategy 1	Steepest Descent	One Step (δ^k)
Strategy 2	Steepest Descent	Bracket Search
Strategy 3	Last Descent	One Step (δ^k)
Strategy 4	Last Descent	Bracket Search

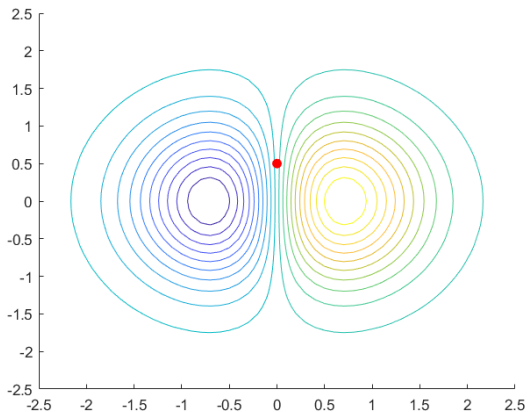
Framework of the Update Step



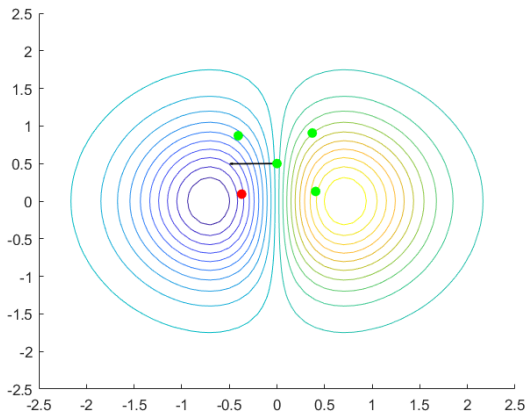
Flow Diagram of the DQL method



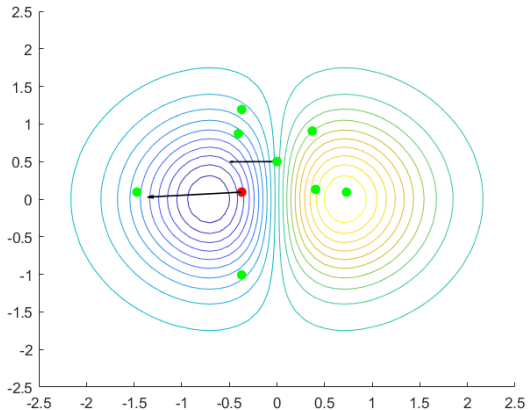
Demo of the DQL method



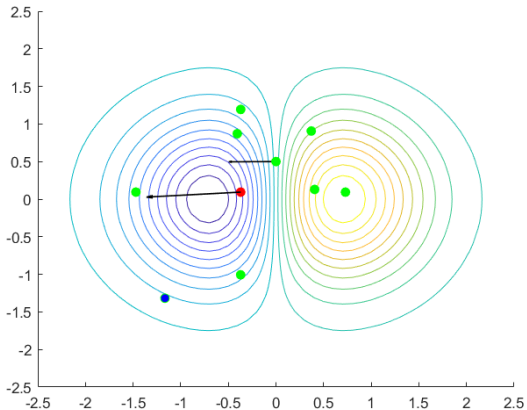
Demo of the DQL method



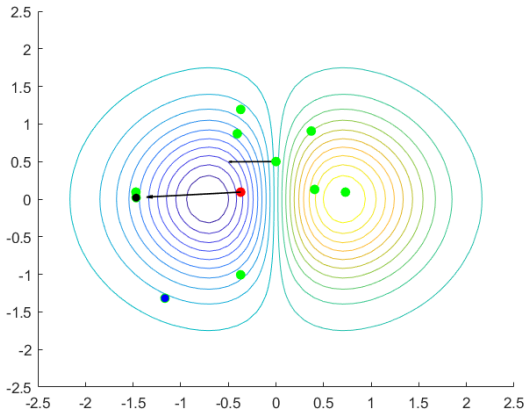
Demo of the DQL method



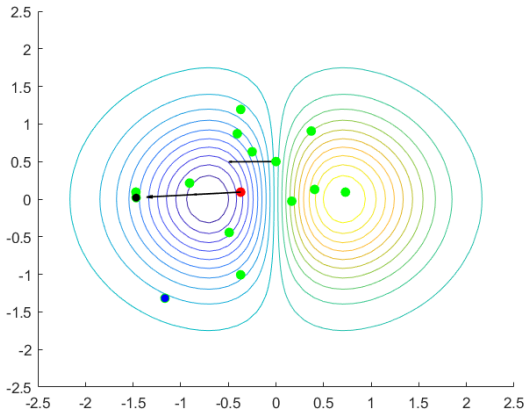
Demo of the DQL method



Demo of the DQL method



Demo of the DQL method



Convergence Analysis

Theorem 1

Let function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ has compact level set $L(x^0)$. In addition, let ∇f be Lipschitz continuous in an open set containing $L(x^0)$. Then the DQL method results in

$$\liminf_{k \rightarrow +\infty} \left\| \nabla f(x^k) \right\| = 0,$$

and $\{x^k\}$ has a limit point x^ for which $\nabla f(x^*) = 0$.*

Proof.

The proof can be found in the thesis [Huang, 2022, Thm 3.5]. \square

Performance Benchmark

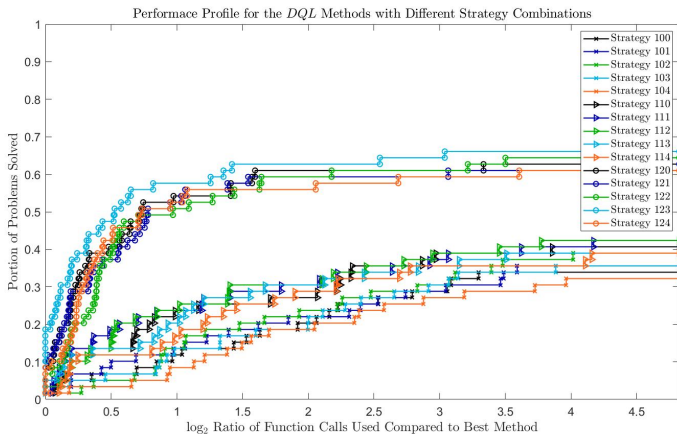
- Direct Step
1 option: Strategy 1
- Quadratic Step
3 options: Disable, Strategy 1-2
- Linear Step
5 options: Disable, Strategy 1-4

Is there a winner among 15 combinations?

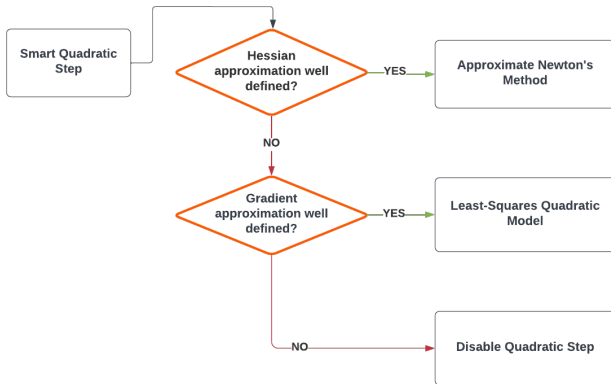
Performance Benchmark: Stopping Conditions

Parameter	Value
ϵ_{∇}	10^{-6}
$\epsilon_{\text{MAX_STEP}}$	10^{-3}
$\epsilon_{\text{MIN_STEP}}$	10^{-12}
MAX_SEARCH	10000

Performance Benchmark: Numerical Result



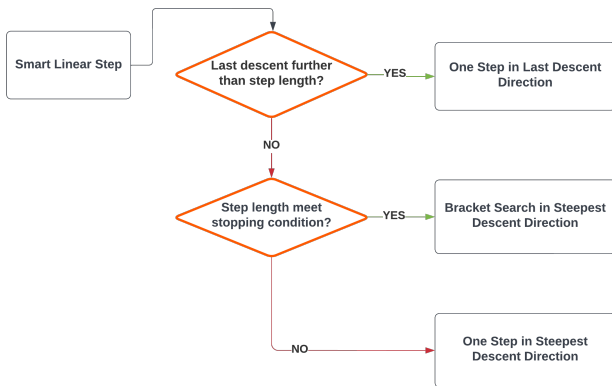
Smart Quadratic Step



Smart Linear Step

- One Step in Last Descent Direction
 - Best Exploration Ability
- Bracket Search in Steepest Descent Direction
 - Best Exploitation Ability
- One Step in Steepest Descent Direction
 - Simple and Efficient

Smart Linear Step

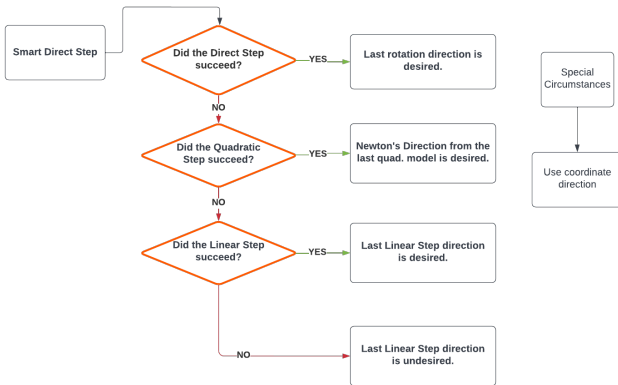


Smart Direct Step

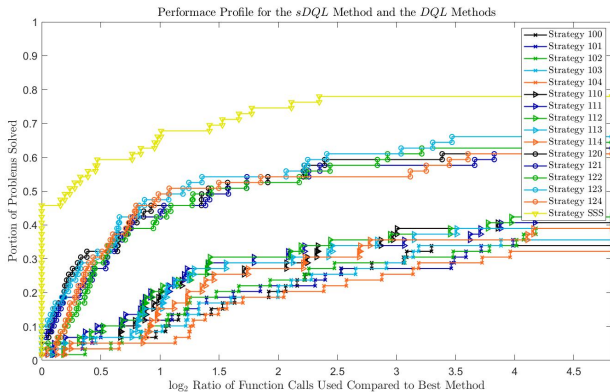
What information can we extract from the last iteration?

- Direct Step
Is r^{k-1} a good rotation direction?
- Quadratic Step
Is m^{k-1} a good quadratic model?
- Linear Step
Is l^{k-1} a good linear search direction?

Smart Direct Step



Performance of SMART DQL Method



Background

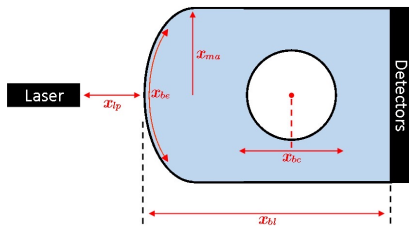
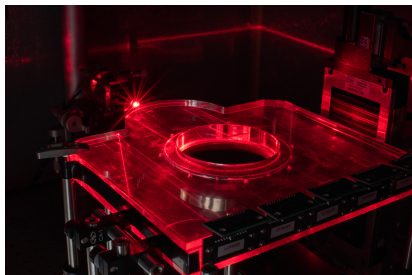


Figure: Solid Tank Design (Picture by Andy Oglivy).

Background

$$x = [x_{bl} \quad x_{bc} \quad x_{lp} \quad x_{ma} \quad x_{be}]^T \in \mathbb{R}^5$$
$$x_{bl} \in [200, 400]$$
$$x_{bc} \in [-30, 30]$$
$$x_{lp} \in [40, 100]$$
$$x_{ma} \in [40, 80]$$
$$x_{be} \in [0, 1]$$

$$\max\{F(x) | x \in C\}$$

Experiment Results

Table: Experimental Results for Solid Tank Design Problem

	Water	FlexDos3D	ClearView TM
SMART DQL Method	2.7676	2.9360	2.9522
Grid Search Method	2.5611	2.9105	2.8694

Conclusion

DQL method

- is a local DFO method.
- is able to combine multiple search strategies.
- is converging to local optima for some functions.

SMART DQL method

- is built under the framework of DQL method.
- is able to choose search strategies dynamically and adaptively.
- is faster and more robust than any simple combinations from our DQL method study.
- is more reliable and efficient in real-world application as compared to the Grid Search Method

Future Development

- Develop the global search ability.
- Integrate more search strategies.
- Design a more sophisticated decision tree.
- Specialize the decision making mechanism for different real-world applications.

Reference

Thank you!



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A hybrid direct search and model-based derivative-free optimization method with dynamic decision processing.



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