Naked Firewalls

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Introduction

In the firewall proposal, it is assumed that the firewall lies near the event horizon and should not be observable except by infalling observers, who are presumably terminated at the firewall. However, if the firewall is located near where the horizon would have been, based on the spacetime evolution up to that time, later quantum fluctuations of the Hawking emission rate can cause the 'teleological' event horizon to have migrated to the inside of the firewall location, rendering the firewall naked. In principle, the firewall can be arbitrarily far outside the horizon. This casts doubt about the notion that a firewall is the 'most conservative' solution to the information loss paradox.

The AMPS Argument

Almheiri, Marolf, Polchinski and Sully (AMPS) argued that local quantum field theory, unitarity, and no-drama (the assumption that infalling observers should not experience anything unusual at the event horizon if the black hole is sufficiently large) cannot all be consistent with each other for the Hawking evaporation of a black hole with a finite number of quantum states given by the Bekenstein-Hawking entropy.

AMPS suggested that the 'most conservative' resolution to this inherent inconsistency between the various assumptions is to give up no-drama. Instead, an infalling observer would be terminated once he or she hits the so-called firewall. This seems rather surprising, because the curvature is negligibly small at the event horizon of a sufficiently large black hole, and thus one would expect nothing special but low energy physics.

The Heart of the AMPS Argument

Assuming unitarity, the information contained inside a black hole should eventually be recovered from the Hawking radiation. The late time radiation purifies the earlier radiation, so the late time radiation should be maximally entangled with the earlier radiation.

By the monogamy of quantum entanglement, the late time radiation cannot also be maximally entangled with the interior of the black hole. This means that the field configuration across the event horizon is generically not continuous, which leads to a divergent local energy density. More explicitly, the quantum field Hamiltonian contains terms like $(\partial_r \varphi)^2$. The derivative is divergent at some $r = R$ if the field configuration is not continuous across R. This is the firewall.

The Location of a Firewall

Usually it is thought that a firewall lies on or just inside an old black hole event horizon. Then it would be completely invisible to observers outside. For a firewall that is not too far outside the event horizons, it is still doubtful that it would be perceptible to far-away observers, since it would seem that such a firewall is well hidden inside the Planckian region of the *local* thermal atmosphere.

Causality Assumption for Firewalls

Here we make the assumption that a firewall, if it exists, has a location determined by the past history of the Hawking evaporating black hole spacetime and is near where the event horizon would be if the evaporation rate were smooth, without quantum fluctuations. Then we show that quantum fluctuations of the evaporation rate in the future can move the event horizon to the inside of the firewall location, rendering it naked.

Vaidya Metric Assumption

For simplicity, we shall approximate the metric near the horizon of an evaporating black hole by the Vaidya metric with a negative energy influx:

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2. \tag{1}$$

Here $M(v)$ is the mass of the black hole, which is decreasing as a function of the advanced time v. For a smooth evaporation rate of a spherical black hole emitting mainly photons and gravitons, we shall take (in Planck units)

$$\dot{M} \equiv \frac{dM}{dv} = -\frac{\alpha}{M^2}, \tag{2}$$

where α is a constant which, in my Ph.D. thesis of forty years ago, I numerically evaluated to be about 3.7474×10^{-5}.

Location of the Unperturbed Event Horizon

The apparent horizon is located at $r_{\rm ApH} = 2M(v)$, whereas the event horizon is generated by radially outgoing null geodesics,

$$\dot{r} \equiv \frac{dr}{dv} = \frac{1}{2}\left(1 - \frac{2M(v)}{r}\right), \tag{3}$$

on the boundary of such null geodesics reaching out to future null infinity, instead of falling in to the singularity that is believed to be inside the black hole. For an unperturbed smooth evaporation rate $\dot{M} \equiv dM/dv = -\alpha/M^2$, the event horizon is given by the solution to Eq. (3) such that it does not diverge exponentially far away from the apparent horizon in the future, giving

$$r_{\rm EH} = 2M[1 - 4\alpha/M^2 + O(\alpha^3/M^6)]. \tag{4}$$

Location of the Firewall near the Unperturbed Horizon

We shall assume that the firewall, if it exists, is close to where the event horizon would be if the black hole evolves smoothly and adiabatically according to $dM/dv = -\alpha/M^2$, which we shall call the unperturbed horizon. However, the actual event horizon depends on the future evolution of the spacetime, and not just on that of its past. Therefore, quantum fluctuations in the future spacetime can lead the event horizon to deviate significantly from the unperturbed horizon. If the mass loss rate exceeds the adiabatic formula, then the event horizon will be inside the unperturbed horizon. As a result a firewall located at the unperturbed horizon would become naked, visible from future null infinity.

Quantum Fluctuation for the Mass Evaporation

From $\dot{r} \equiv dr/dv = (1/2) - M/r$, one can write the mass $M = M(v)$ in the Vaidya metric in terms of the event horizon radius $r = r(v) \equiv r_{\rm EH}(v)$ as $M = \frac{1}{2}r - r\dot{r}$. Let M_1, r_1 and M_2, r_2 be the mass and radius of the unperturbed black hole and of its fluctuations, respectively, with the total mass $M = M_1 + M_2$ and the event horizon radius $r = r_1 + r_2$.

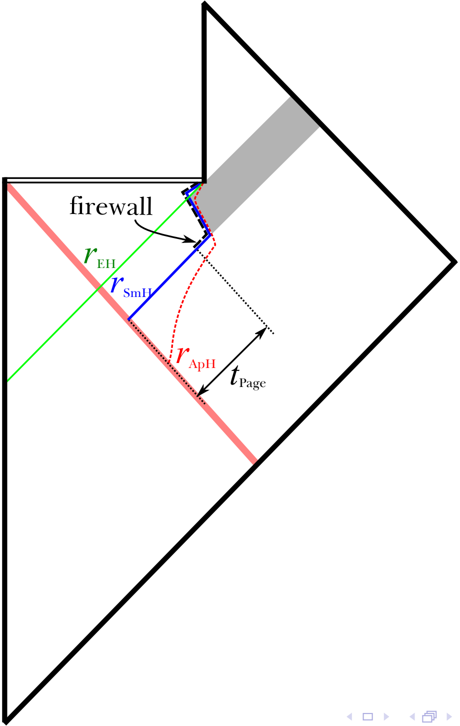
Now suppose that the unperturbed mass loss would give $M = M_1 = M_1(v) = (1/2)r_1 - r_1\dot{r}_1$, such that $\dot{M}_1 \approx -\alpha/M_1^2$, and that quantum fluctuations $M_2 = M_2(v)$ and $r_2 = r_2(v)$ are small compared with the total mass and the event horizon radius, respectively. Then $M = M_1 + M_2 = (1/2)r - r\dot{r} = (1/2)(r_1 + r_2) - (r_1 + r_2)(\dot{r}_1 + \dot{r}_2) \approx M_1 + (1/2)r_2 - r_1\dot{r}_2$.

For simplicity we are making the highly idealized assumption that even with quantum fluctuations, the metric remains spherically symmetric and Vaidya near the event horizon.

Departure from the Unperturbed Event Horizon

Now for some particular advanced time $v = v_0$, let us ignore quantum fluctuations before this time, so that $M_2(v) = 0$ for $v < v_0$, and let us define the constant $M_0 = M(v_0) = M_1(v_0)$. To leading order in $M_0 \gg 1$ and $|v - v_0| \ll M_0^3$, the fractional decay of the black hole over the advanced time $v - v_0$ is small, and the negative of the coefficient of $\dot{r}_2$ in $M \approx M_1 + \frac{1}{2} r_2 - r_1 \dot{r}_2$ may be written as $r_1 \approx 2M_1 \approx 2M_0$. Then one gets $(1/2) r_2 - 2M_0 \dot{r}_2 \approx M_2(v)$. The solution of this differential equation that has no exponentially growing departure of the event horizon $r(v) = r_1 + r_2$ from the unperturbed horizon $r_1(v)$ at late times is

$$r_2 \approx \exp\left(\frac{v - v_0}{4M_0}\right) \int_v^\infty dv' \frac{M_2(v')}{2M_0} \exp\left(\frac{v_0 - v'}{4M_0}\right). \qquad (5)$$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A Particular Fluctuation for the Evaporation Rate

Since the adiabatic evolution gives $\dot{M}_1 \approx -\alpha/M_0^2$ for $M_0 \gg 1$ and $|v - v_0| \ll M_0^3$, let us consider a quantum mass fluctuation that gives, with $\theta(v - v_0)$ the Heaviside step function,

$$\dot{M}_2 = -\theta(v - v_0)\frac{\alpha}{M_0^2} f \exp\left(-\frac{\beta(v - v_0)}{4M_0}\right), \tag{6}$$

which has two constant parameters, namely f for how large the quantum fluctuation in the energy emission rate is relative to the adiabatic emission rate $-\alpha/M^2$ (with f assumed to be positive so that the quantum fluctuation increases the emission rate above the adiabatic value), and β for how fast the quantum fluctuation in the energy emission rate decays over an advanced time of $4M_0$ (the inverse of the surface gravity κ of the black hole).

Results for the Mass and Horizon Fluctuation

Then with $M_2(v) = 0$ for $v < v_0$, one gets

$$M_2 \approx -\theta(v - v_0)\frac{4\alpha}{\beta M_0} f \left[1 - \exp\left(-\frac{\beta(v - v_0)}{4M_0}\right)\right]. \tag{7}$$

Plugging this back into Eq. (5) then gives

$$\begin{aligned} r_2 \quad &\approx -\theta(v_0 - v)\frac{8\alpha f}{(1+\beta)M_0} \exp\left(\frac{v - v_0}{4M_0}\right) \\ &- \theta\,(v - v_0)\frac{8\alpha f}{\beta(1+\beta)M_0}\left[1+\beta-\exp\left(\frac{-\beta(v-v_0)}{4M_0}\right)\right]. \end{aligned} \tag{8}$$

Consequences of This Fluctuation

This particular form of the emission rate fluctuation implies that the total mass fluctuation away from the unperturbed evolution is $M_2(\infty) = -4\alpha f/(\beta M_0)$. Then the radial fluctuation in the event horizon radius at the advanced time $v = v_0$, when $-r_2(v)$ has its maximum value, is $r_2(v_0) \approx [2\beta/(1+\beta)]M_2(\infty)$. This means that if the quantum fluctuation in the energy emission rate is very short compared with $4M_0$ (decaying rapidly in comparison with the surface gravity of the black hole), so that $\beta \gg 1$, then $r_2(v_0) \approx 2M_2(\infty)$, twice the total mass fluctuation. However, we shall just assume that β is of the order of unity and hence get $r_2(v_0) \sim M_2(\infty)$ as an order-of-magnitude relation. Note that the reduction in the radius of the event horizon at $v = v_0$, where the fluctuation in the mass emission rate starts, occurs *before* there is any decrease in the mass below the adiabatic value $M_1(v)$, because the location of the 'teleological' event horizon is defined by the future evolution of the spacetime.

Quantum Fluctuations Can Render a Firewall Naked

Therefore, if the putative firewall occurs at a location determined purely causally by the past behavior of the spacetime, and is sufficiently near where the event horizon would be under unperturbed smooth adiabatic emission thereafter, then quantum fluctuations, at later advanced times that reduce the mass of the hole below that given by the unperturbed adiabatic evolution, would move the actual event horizon inward (even before quantum fluctuations in the mass emission rate begin), so that the event horizon becomes inside the location of the putative firewall. That is, quantum fluctuations that increase the mass emission rate render such a firewall naked, visible to the external universe.

Conclusion: Firewalls Are Not Conservative

More specifically, being in the exterior of the event horizon means that the firewall could potentially influence the exterior spacetime, so that even observers who do not fall into the black hole could have a fiery experience. In addition, the presence of a firewall well outside the event horizon could affect the spectrum of the Hawking radiation, which means that the presence of a firewall could be inferred even by asymptotic observers. Such a 'naked firewall,' i.e., a firewall far outside the event horizon, is therefore problematic, and giving up the no-drama assumption no longer seems like a palatable 'most conservative' option.