CMO-BIRS Workshop Quantum Markov Semigroups in Analysis, Physics and Probability 15w5086

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1 Overview of the Field

Quantum Markov semigroups (QMSs), also called quantum dynamical semigroups in the physics literature, were introduced in the seventies by a number of physicists to model the non-unitary evolution of quantum systems interacting with the external environment. From the mathematical point of view, they are semigroups of positive (or completely positive) maps on an operator algebra, continuous in the time variable with respect to some natural topology. They have been also studied as mathematical generalization of classical Markov semigroups, namely semigroups on commutative algebras of continuous functions. Since its introduction, the notion of a quantum Markov semigroup has been intensively studied by both physicists and mathematicians. Nowadays they play a fundamental role in models of open quantum systems, non-equilibrium phenomena (entropy methods), decoherence, quantization and orthogonal polynomials, (interacting) Fock spaces and other related topics.

Problems on QMSs often constitute a difficult mathematical challenge because several methods developed for classical Markov semigroups (e.g. conditioning, coupling) do not work in this framework and the development of new ones is now beginning.

The main outcomes of these investigations will be a deeper understanding of the underlying non-commutative mathematics and new results on mathematical models for open quantum systems.

The workshop gathered researches from many countries in the world working on this research topic, at the crossroad between functional analysis, quantum mechanics, probability and operator theory, with different points of views and goals but very close techniques.

2 Recent Developments and Open Problems

The main research lines at present time concern:

1. general structural and analytical properties,

2. the study of relevant classes, for instance QMSs emerging from weak coupling or low density limits, QMSs describing non-equilibrium phenomena, QMSs modelling effects such as decoherence, QMSs of systems with special symmetries (e.g. circulant QMSs, [7]),

3. QMSs and quantum master equations in special models of open quantum systems,
4. the study of QMSs on interacting Fock spaces (e.g. by second quantization of contractive semigroups on their n-particle spaces) and QMSs on quantum groups (especially as Markov semigroups of quantum Lévy processes).

The generator of a uniformly continuous QMS on the algebra $\mathcal{B}(\mathsf{h})$ of all bounded operators on a Hilbert space h has the well-known Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) form. This result has been extended to weak*-continuous QMSs under certain domain assumptions. It is known, however, that the existence of a generalized GKSL representation of the generator is closely related with the existence of units in the associated product system (see [3, 4, 5]); as a consequence, QMSs with generators that are not of the GKSL form exist. The GKSL form of the generator, however, is very useful because it allows one to reduce problems at the operator algebra level to the Hilbert space level. Research aiming at finding classes of QMSs with generalized GKSL generators is still in progress and some results were reported at this meeting (see e.g. Androulakis' talk).

The set of fixed points and the so-called decoherence free subalgebra (the algebra where a QMS acts as a semigroup of homomorphism) play a key role in the study of properties of the QMS itself (as it also happens for the set of fixed points of a classical Markov semigroup). It has been recently shown ([8]) that, if either the decoerence free subalgebra or the set of fixed points are purely atomic von Neumann algebras, the GKSL structure of the generator QMS can be further specialized giving insight on the action of the semigroup and physical properties of the open system evolution it describes.

This feature appears more clearly in the study of relevant classes of QMSs as those arising from weak coupling or low density limits (see Accardi's talks).

Models for quantum systems undergoing random collisions (Carlen's talk), mean field laser quantum master equations (Mora's talk) and other models arising in quantum physics have begun to be investigated. Several fundamental properties have been established but the full picture is not yet clear. A short list of open problems include: the complete description of invariant states of QMSs of weak-coupling limit type, the classification of product systems of QMSs with unbounded generator modelling physically important quantum open systems, the study of the fine structure and properties (e.g. hypercontractivity) of QMSs in interacting Fock spaces (see Bozejko-Speicher for q-Fock spaces), computing (or at least estimating) the spectral gap and the logarithmic Sobolev constant of remarkable QMSs, the development of a perturbation theory paralleling the classical perturbation theory for self-adjoint operators, the study of relationships between algebraic and analytic properties.

3 Presentation Highlights

B. V. Rajarama Bhat explored the various implications of existence of a unit for a quantum dynamical semigroup. The concept of *unit* plays an important role in classifying product systems and semigroups of endomorphisms (E_0-semigroups). A one parameter semigroup of the form $X \mapsto C_t X C_t^*$ is said to be a unit of a given quantum dynamical semigroup τ, if it is dominated by τ up to scaling, that is, there exists a positive scalar q such that

$$X \mapsto \tau_t(X) - e^{-qt} C_t X C_t^*$$

is completely positive for all t.

Recent results on models of quantum systems of N *particles undergoing binary collisions*, focusing on propagation of chaos and the rate of convergence to equilibrium, where presented by **M. Carvalho** and **E. Carlen**. These questions arise from the work of Mark Kac and his investigation into the probabilistic structure underlying the Boltzmann equation. Recently, the quantum mechanical variation on Kac's question has begun to be investigated. In this case, the Kac Master equation becomes an evolution equation of Lindblad type, while the corresponding Boltzmann equation is a novel sort of non-linear evolution equation for a density matrix.

A lower bound to the spectral gap of the Davies generator for general *N qubit commuting Pauli Hamiltonians*, was presented by **K. Temme**. It is expected that this bound will provide the correct asymptotic scaling of the gap with the systems size up to a factor of 1/N in the low temperature regime. He derived rigorous thermalization time bounds, also called mixing time bounds, for the Davies generators of these Hamiltonians.

Davies generators are given in the form of a Lindblad equation and the corresponding Markov semigroups are known to converge to the Gibbs distribution of the particular Hamiltonian for which they are derived. The bound on the spectral gap essentially depends on a single number μ referred to as the generalized energy barrier. When any local defect can be grown into a logical operator of a stabilizer code S by applying single qubit Pauli operators and in turn any Pauli operator can be decomposed into a product of the clusters of such excitations, μ corresponds to the largest energy barrier of the canonical logical operators. The main conclusion that can be drawn from the result is, that the presence of an energy barrier for the logical operators is in fact, although not sufficient, a necessary condition for a thermally stable quantum memory when we assume the full Davies dynamics as noise model.

Using *hypocoercivity*, **Lukas Newmann** studied the decay to the stationary solution of a kinetic relaxation-model describing a generation-recombination reaction of two species.

G. Androulakis and **M. Ziemke**, reviewed their recent joint work showing that without assuming *uniform continuity*, the generator of a quantum Markov semigroup can be written in a form similar to Lindblad's form. Compared with others results in this direction, their result has different and minimal assumptions.

A theory of *perturbation* of a quantum dynamical semigroup by another was developed by **K.B. Sinha**, constructing the total semigroup and proving interesting results on the conservativity (Markov property) of the total semigroup.

F. Fagnola presented recent results on the structure of norm-continuous quantum Markov semigroups with *atomic* fixed point algebra or atomic decoherence-free subalgebra, providing a natural decomposition of a Markovian quantum open system into its *irreducible* components and *noiseless* components. He also discussed new characterisations of the structure of invariant states and *decoherence-free* subsystems with applications to quantum Markov semigroups arising from the stochastic limit, see [6, 8, 9, 10].

Stochastic limit type semigroups, which an interesting class of semigroups possessing non-equilibrium stationary states [1], where discussed by **L. Accardi**. The Gorini-Kossakowski-Sudarshan and Lindblad (GKSL) generators of these semigroups are deduced from realistic Hamiltonian interactions. The relations between the generator and the non-equilibrium stationary states are more subtle than in the equilibrium case since they involve *microcurrents*, that are identically zero in the equilibrium case. The deductive basis of the connections, between states and generators, is the *principle of similarity*: one of the basic principles of the stochastic limit theory. The Markov generators emerging from this theory have a very special structure which gives, contrarily to the more general but abstract GKSL form, an immediately intuitive picture of the microinteractions that build up the whole dynamics. In particular to each one of such generators, is associated its *interaction graph*, whose topology is strongly related with the structure of the invariant states.

M.J. Kastoryano provided an overview of the recent results involving the *mixing time analysis* of quantum dynamical semigroups, and their application to problems in quantum information theory and manybody physics. These results include quantum *functional inequalities*, such as Poincare, Nash and LogSobolev inequalities. The applications include: analysis of quantum memories, quantum algorithms for state preparation and sampling, and quantum shannon theory.

Let A be a unital $*$-algebra and ε a unital $*$-homorphism from A to the complex numbers. A functional ψ on A is called *conditionally positive* (or a *generating functional*) on (A, ε) if (a) $\psi(1) = 0$, (b) ψ is hermitian, (c) ψ is positive on the kernel of ε. Examples are the linear extensions to the group algebra of negative type functions on groups, positive functions classify Lévy processes on groups or involutive bialgebras. **U. Franz** presented new results about the relation between conditionally positive functions and 1cocycles, and about the existence of a decomposition of such functions into a maximal Gaussian part and a Gauss-free remainder.

Unlike Riemann integral, the stochastic integral depends on the way the sample points are chosen in the subintervals of a partition. Thus for example, in Itô integral the sample points are the leftend points of the subintervals of a partition, while in Stratonovich integral they are the midpoints. One can imagine a stochastic integral in which the sample points are always chosen one quarter away from the leftend points and three quarters from the right endpoints of the subintervals of a partition. To each of these stochastic integrals it corresponds a particular generalized Wick product. This generalized Wick product links the integrand to the stochastic infinitesimal dBt. **A. Stan** presented a Hölder inequality for the norms of these *generalized Wick products*. In order to bound the bilinear generalized Wick product, it is necessary to use the second quantization operators.

A Bargmann representation of a probability measure on the real numbers was discussed by **N. Asai**. Focussing on the representation associated with the (α, q)-Fock space introduced recently by Bozejko-Ejsmont-

Hasebe, which is a generalization of the q-Fock space by Bożejko-Speicher. This result generalizes that of van Leeuwen-Maassen.

Quantum white noise calculus provides a framework of operator theory on (Boson) Fock space by means of infinite dimensional distribution theory based on a Gelfand triple $\mathcal{W} \subset \Gamma(H) \subset \mathcal{W}^*$. It is natural to think of the derivatives of a white noise operator $\Xi = \Xi(a_s, a_t^*; s, t \in T)$ with respect to the pointwise annihilation and creation operators a_s, a_t^*, formally denoted by

$$D_t^+ = \frac{\delta\Xi}{\delta a_t^*}, \qquad D_t^- = \frac{\delta\Xi}{\delta a_t}.$$

In fact, the quantum white noise derivatives are defined by

$$D_\zeta^+\Xi = [a(\zeta), \Xi] = a(\zeta)\Xi - \Xi a(\zeta), \qquad D_\zeta^-\Xi = -[a^*(\zeta), \Xi] = \Xi a^*(\zeta) - a^*(\zeta)\Xi,$$

for any white noise operator $\Xi \in \mathcal{L}(\mathcal{W}, \mathcal{W}^*)$ and $\zeta \in E$. Then $D_\zeta^\pm$ are Wick derivations on $\mathcal{L}(\mathcal{W}, \mathcal{W}^*)$ and white noise operators are characterized by differential equations of Wick type. **N. Obata** reviewed the fundamentals on quantum white noise derivatives and showed their applications: (i) the implementation problem for the canonical commutation relation and derivation of Bogoliubov transform; (ii) calculating the normal-ordered form of the composition of the exponential ofa quadratic annihilation operator and its adjoint. This work is based on a long-term collaboration with **Un Cig Ji** who formulated a quantum extension of the Girsanov theorem as an implementation problem for admissible white noise operators. For a systematic study of implementation problems, they first start with a detailed study of admissible white noise operators with constructions of Wick algebras. Secondly, they develop the concept of quantum white noise derivatives of admissible operators and as applications, they study general forms of implementation problem for admissible white noise operators. Finally, an implementation problem with solution as a quantum martingale is applied to quantum Girsanov theorem.

A. Barhoumi proved that each probability measure on real numbers, with all moments, is canonically associated with (i) a $*$–Lie algebra; (ii) a *complexity index* labeled by pairs of natural integers. The measures with complexity index $(0, K)$ consist of two disjoint classes: that of all measures with finite support and the semi–circle–arcsine class. The class $(1, 0)$ includes the Gaussian and Poisson measures and the associated $*$–Lie algebra is the Heisenberg algebra. The class $(2, 0)$ includes the non standard (i.e. neither Gaussian nor Poisson) Meixner distributions and the associated $*$–Lie algebra is a central extension of $sl(2, R)$. Starting from $n = 3$, the $*$–Lie algebra associated to the class $(n, 0)$ is infinite dimensional. This produces a new class of infinite dimensional $*$–Lie algebras, canonically associated to probability measures, which does not seem to be present in the literature.

J.A. Agredo presented a full characterisation of *decoherence free* subspaces of a given quantum Markov semigroup with generator in a generalised Lindbald form which is valid also for infinite dimensional systems. These results, extending those available in the literature concerning finite dimensional systems, were illustrated by some examples.

Based on a joint work with Franco Fagnola, **C. Mora** examined the dynamics of the solution of a *non-linear* quantum master equation describing a quantum system formed by two level atoms and a single mode of an optical cavity that are coupled to two reservoirs. First, he established the well-posedness of the non-linear quantum master equation under study. Then, he present a *bifurcation* analysis of the solution structure.

O. Arizmendi presented recent results obtained in collaboration with T. Hasebe. They considered the Belinschi-Nica semigroup of homomorphisms and realized it as a *free multiplicative subordination*. This realization allows to define more general semigroups which are also homomorphism with respect to multiplicative subordination. For this semigroups they show that a differential equation holds generalizing the complex Burgers equation.

By using the standard techniques of operator algebras, **F. Fidaleo** studied the general structure of the stochastic processes on the two-sides chain (whose parameter is then an integer) in the quantum setting, together with their natural symmetries such as the invariance under the shift and the permutations. The *ergodic properties* of stationary and exchangeable processes are investigated for many interesting cases arising from physics, and including those arising from free probability. Among that, he treats in detail the Bose/Fermi alternative (by describing the quantum generalisation of the De Finetti and Hewitt-Savage Theorems), and more generally the q-canonical commutation relations $-1 < q < 1$, where $q = 0$ corresponds to the free (or

Boltzmann) case. In the last situation, he showed that the set of shift-invariant states (corresponding to the stationary processes) is made of a singleton. The universal situation arising from free probability and corresponding to free product C^*-algebra (with or without an identity) is also treated in detail. More precisely, he connects some natural algebraic properties of the stochastic process, like that to being a *product state* or a *block-singleton* one, to natural ergodic properties of the state (on the free product C^*-algebra) naturally associated to the process under consideration, like the weak clustering, and the property of *convergence to the equilibrium*, respectively. The ergodic properties of the class of the so-called Haagerup states on the group C^*-algebra of the free group on infinitely many generators are investigated in full generality. Finally, he also specialise his investigation to the study of ergodic properties of stationary processes arising from the monotone (firstly defined and investigated by Lu and Muraki), and Boolean processes. He applies the results for achieving the structure of stationary states, and in the Boolean case, he presented a De Finetti theorem too.

The symmetric operator *amenability* of operator algebras was discussed by **J. Heo**. For completely contractive Banach algebras, he gave equivalent conditions to symmetric operator amenability for operator algebras and proved some structural properties of symmetric operator amenability. He also discussed about the existence of complete bounded Jordan derivation from symmetrically operator amenable Banach algebra into operator bi-module.

H. J. Yoo reported his results on the investigation of *multi-dimensional* orthogonal polynomials. Based on the previous work by Accardi-Barhoumi-Dhahri he construct creation, annihilation, and preservation operators (CAP operators) on an interacting Fock space for a given orthogonal system. He also considered the reconstruction theorem. Given CAP operators on an interacting Fock space, he gives sufficient conditions for the operators so that they define a probability measure and moreover they are reconstructed from the measure. He discussed also the deficiency rank of the Jacobi operators, its relation to the support of the measure and considered some examples.

Convex structures arising from quantum information theory, were presented by **S.-H. Kye**. In order to distinguish entanglement from separability, one may have to understand the boundary of the convex set of all separable states, which consists of *faces*. As the first step, one looks for faces which are affinely isomorphic to simplices, in various levels. In the course of discussion, one needs some techniques from various fields of mathematics; combinatorics, algebraic geometry as well as functional analysis. He explored simplex-like convex sets arising in this way.

4 Scientific Progress Made

We think that this workshop provided invaluable contribution to the outline of main streams and research lines on the subject through discussion of results and open problems and creation of connections based on common scientific interests.

QMSs and, more generally, Quantum Probability, cast in the framework of theories under construction in which they would constitute relevant contributions. Several problems were born from concrete problems in Physics or Quantum Information. The problem of finding non trivial and concretely applicable results in this area is of great importance in several applications.

Awareness of the state of the art and open problems outlined in Section 2 of the present report will provide very good guidelines for several researchers. There are good reasons to believe that methods and ideas discussed at the workshop, together with some new collaboration, will stimulate investigation and the discovery of new results in the forthcoming years.

We know from e-mail contacts we had after the meeting that, in particular:

- Abdessatar Barhoumi is planning to organize a school and workshop on quantum Markov semigroups and interacting Fock spaces in Tunisia in October 2016,
- Luigi Accardi and Michael J. Kastoryano will exchange views on QMSs of stochastic limit type,
- Nobuaki Obata and Un Cig Ji had the opportunity to discuss the connection of their work on the representation of remarkable operators in Fock spaces with QMSs techniques,

- Franco Fagnola and Jorge Bolaños finished a joint paper on the range of the generator of a QMS that will appear in Infin. Dimens. Anal. Quantum Probab. and Related Topics,
- Julián Agredo had the opportunity to discuss with Carlos Mora and Luigi Accardi of his program of studying a quantum analogue of classical couplings and Wasserstein distances,
- R. Quezada and J. Bolaños continued their discussion on a lower bound of the spectral gap and simple necessary and sufficient conditions for irreducibility of circulant QMS.
- Fernando Guerrero could count on L. Accardi's advice in his research on non-symmetric exclusion processes,
- George Androulakis, Matthew Ziemke and Franco Fagnola discussed on the extension of a result of Davies on the GKSL form to the case of QMSs with an arbitrary invariant state,
- J.C. García, F. Guerrero and R. Quezada started to work on the construction of non-equilibrium stationary states of infinite dimensional GKSL generators of the stochastic limit type.
- Kalyan B. Sinha and Carlos Mora discussed on possible perturbation results for QMSs arising from the interplay of classical perturbation theory and representations by means of stochastic Schrödinger equations,
- Francesco Fidaleo and Jorge Bolaños, who's now visiting Fidaleo in Rome, continued their discussions on models for Bose-Einstein condensation,
- Carlos Mora and F. Fagnola discussed convergence to the unique invariant state for the mean field laser equation.

5 Outcome of the Meeting

This meeting laid the groundwork for several future collaborations on QMSs and, more generally, on "non-commutative methods" in the study of evolutions of open quantum systems.

Papers are now in preparation based on progress made during the meeting. It seems reasonable to expect others in the nearest future.

Several young Ph.D.s and post-docs from Mexico and United States (we counted at least ten) had the opportunity to interact and establish contacts with researchers based in Europe and Asia whom they could hardly meet without this workshop.

Finally, we would like to quote the feedback we got from two colleagues in two letters addressed to all participants.

Luigi Accardi: *"I think I am interpreting the feelings of most of participants by expressing my heartiest thanks to the organizers of the Oaxaca meeting for having organized one of the most interesting scientific workshops I have attended in past years.*

What I most appreciated in this workshop could be termed biodiversity: in an academic world that tends to be more and more 'club' or 'clan' oriented, the organizers were wise and courageous enough to invite groups of scientists who deal, from different points of view, with problems which have deep common roots and intersections.

The most exciting thing for me was to see several young people, from different parts of the world and with different scientific experience, attacking problems that were studied for many years in quantum probability, and not only obtaining new results, but introducing new powerful ideas and techniques to deal with these problems.

I am sure that this will create a lot of fruitful interactions and, for the little that I can do, I will work in this direction."

Eric Carlen: *"I too would like to thank the organizers – and participants – for a great conference. The subject range was quite broad, but there was not a single talk I did not find interesting, and I found very many of them very interesting. There were an unusually high number for me in this category, and I am sure this meeting will lead to new collaboration for many..."*

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