New evidence that cohomological invariants might determine Albert algebras/groups of type F_4 uniquely up to isomorphism:

To Andrei Rapinchuk

on the occasion of his 60th birthday

(joint work with A. Lourdeaux , A. Pianzola)

Vladimir Chernousov
University of Alberta

BIRS workshop June 16, 2022

1 Albert Algebras

2 Cohomological invariants

3 The main result and strategy of the proof

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endows B with a Jordan algebra structure, which we denote by B^+.

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One then writes $A = J(D, \mu)$ and says that A arises from D and μ via the first Tits construction.

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3. Rost Theorem: assume $\xi_1, \xi_2 \in H^1(F, G_0)$ have the same cohomological invariants. Then there exist extensions K/F of degree dividing 3 and L/F of degree prime to 3 such that $\xi_{1,K} = \xi_{2,K}$ and $\xi_{1,L} = \xi_{2,L}$.

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