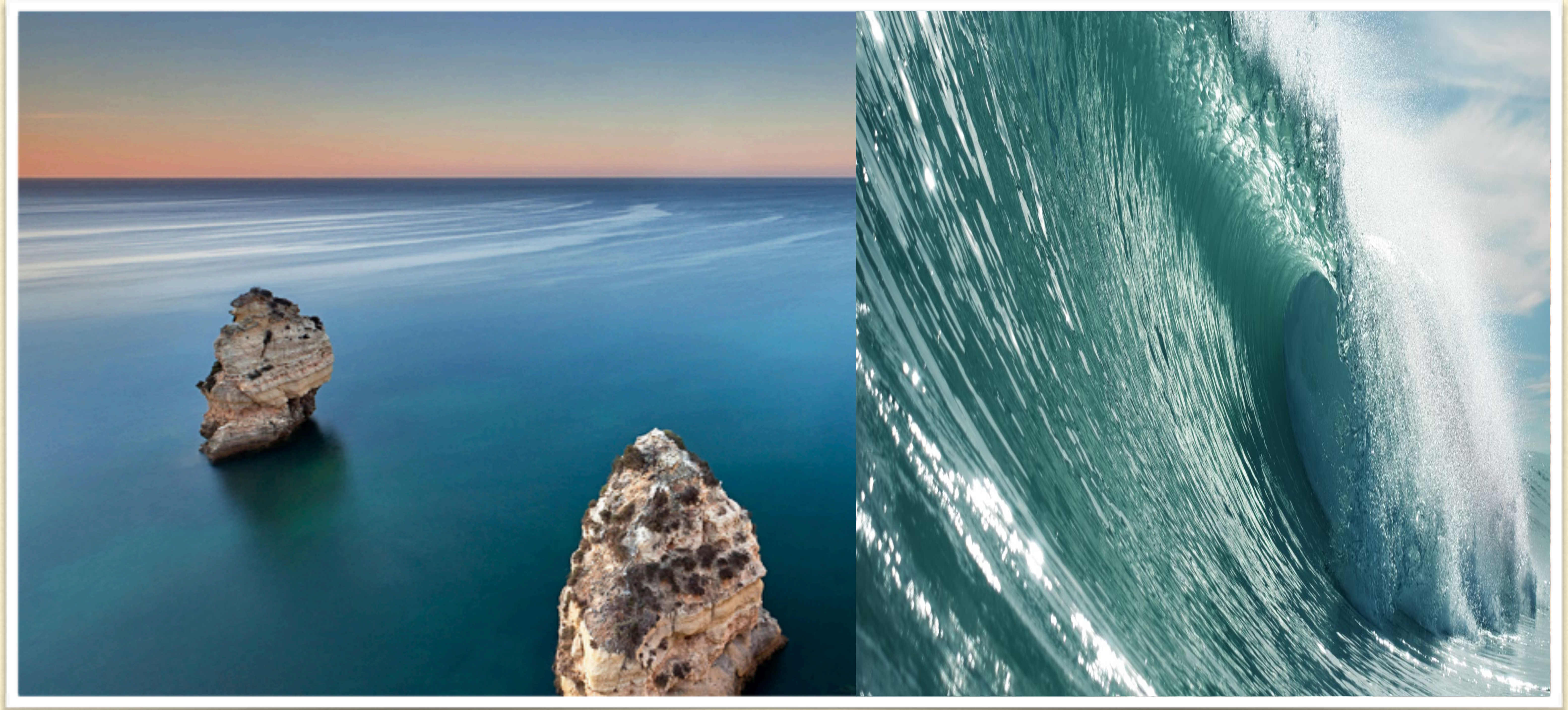


CLOSING ENERGY BUDGETS IN OCEAN MODELS

Rémi Tailleux - University of Reading – Banff Breakout Session - 14 October 2019

MAIN ISSUES

- Energy errors in mechanical energy and heat budgets don’t have same implications
- Constructing energy conserving Boussinesq equations is easy. Still can be crap though.
- Importance of averaging operators. Parameterization of advection versus forces. Thermodynamics of mean variables.
- Linking energetics of the resolved scales with that of the unresolved scales
- Partial thermodynamic quantities. Isolating the freshwater from the seawater

MECHANICAL VERSUS THERMODYNAMIC SOURCES OF ENERGY IN THE OCEAN

Heat Transport: Peak about 2 PW

Wind Power Input: About 2 TW

Buoyancy Power Input: About 0.5 TW

Mechanical sources of energy about 3 orders of magnitude smaller than Heat sources of energy

An energy error of 0.1 TW is probably inconsequential if it affects the Heat Budget, but likely to be a key source of error (Wrong forces in momentum equations) if it affects the mechanical energy budget (Kinetic Energy +Available Potential Energy)

SOURCES OF ENERGY ERRORS

$$\frac{D\vec{v}}{Dt} + f\vec{k}\times\vec{v} + \frac{1}{\rho}\nabla p = D_v + \vec{F}_{missing}$$

Affect KE budget

$$\frac{\partial\theta}{\partial t} + \left(\vec{v} + \vec{v}_{eddy}\right)\cdot\nabla\theta = \nabla\cdot(K\nabla\theta) + \dot{\theta}_{nonconservative}$$

Affect APE and Heat budgets

All parameterizations of subgrid-scale effects have energetics implications

How do you go from a 1-D energy error back to a missing 3D force?

ENERGY FORMS

Standard Energy per unit mass

$$E_{ocean} = \frac{\vec{v}^2}{2} + gz + h(\eta, S, p) - \frac{p}{\rho}$$

Kinetic Energy

Gravitational Potential Energy

Internal Energy = Specific Enthalpy – Pressure x Specific Volume

EXACT PARTITIONING OF REDUCED POTENTIAL ENERGY (TAILLEUX 2018)

$$h(\eta, S, p) + gz + \frac{p_r(z) - p}{\rho} = h + gz - \frac{p'}{\rho} = \Pi_1 + \Pi_2 + B_r$$

$$\Pi_1 = h(\eta, S, p) - h\big(\eta, S, p_r(z)\big) + \frac{p_r(z) - p}{\rho} \approx \frac{p'^2}{2\rho c_s^2} > 0$$

Available Acoustic Energy (AAE)

$$\Pi_2 = h\big(\eta, S, p_r(z)\big) - h\big(\eta, S, p_r(z_r)\big) + g(z - z_r) \approx \frac{N_r^2 (z - z_r)^2}{2} > 0$$

Available Potential Energy (APE)

$$B_r = h\big(\eta, S, p_r(z_r)\big) + g z_r$$

Background Potential Energy (BPE) = Heat

AVAILABLE ACOUSTIC ENERGY (AEE)

$$\Pi_1 = h(\eta, S, p) - h\big(\eta, S, p_r(z)\big) + \frac{p_r(z) - p}{\rho} \approx \frac{p'^2}{2\,\rho\, c_s^2} \geq 0$$

Generally neglected in the Boussinesq and Anelastic approximation

AVAILABLE POTENTIAL ENERGY (APE) DENSITY

$$\Pi_2 = h\big(\eta, S, p_r(z)\big) - h\big(\eta, S, p_r(z_r)\big) + g(z - z_r) \approx \frac{N_r^2 (z - z_r)^2}{2} > 0$$

Local version of Lorenz globally-defined APE

$$\iiint_V \Pi_2 \rho dV \geq APE_{Lorenz}$$

Π_2 can be constructed from any reasonable reference state, e.g., horizontally averaged density field. Does not absolutely require adiabatic re-arrangement.

DEFINITIONS OF HEAT

Based on potential temperature, still very much in use

$$H = c_{p0}\theta$$ (Bryan, 1962)

Based on potential enthalpy and Conservative Temperature, recommended by TEOS10

$$H = h(\eta, S, p_{00}) = c_{p0}\Theta$$ (McDougall, 2003)

New definition, based on partitioning of PE into APE+AAE and BPE (Tailleux, 2018)

$$H = h(\eta, S, p_r(z_r)) + g z_r = c_{p0}\Theta_T$$ `Salty' Static Energy estimated in state of rest

DEFINITIONS OF HEAT

Should we worry that oceanographers and atmosphericists use incompatible definitions of heat?

Based on potential temperature, still very much in use

$H = c_{p0}\theta$ (Bryan, 1962)

Should we worry that moist static energy keeps being converted with KE, no true heat?

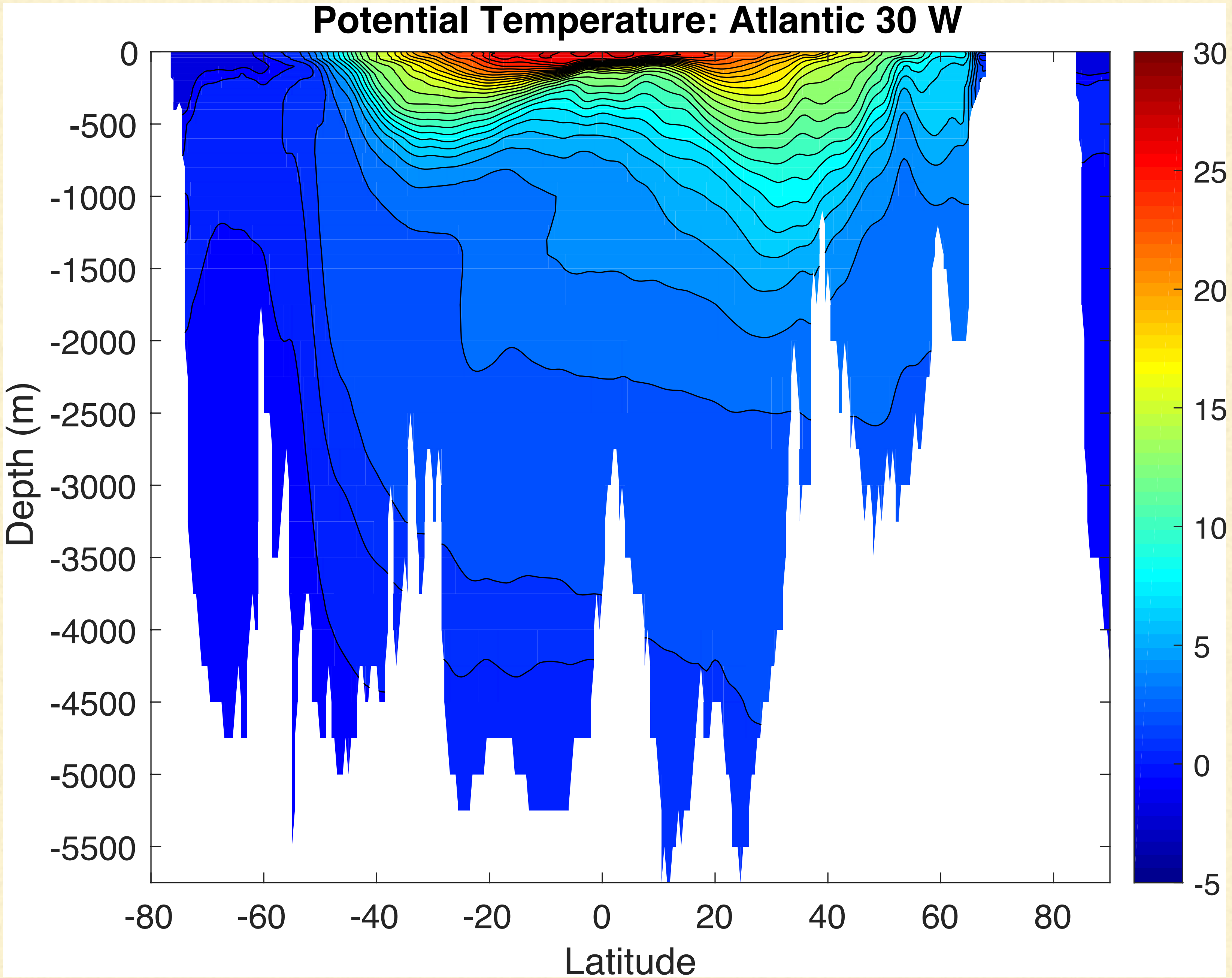
Based on potential enthalpy and Conservative Temperature, recommended by TEOS10

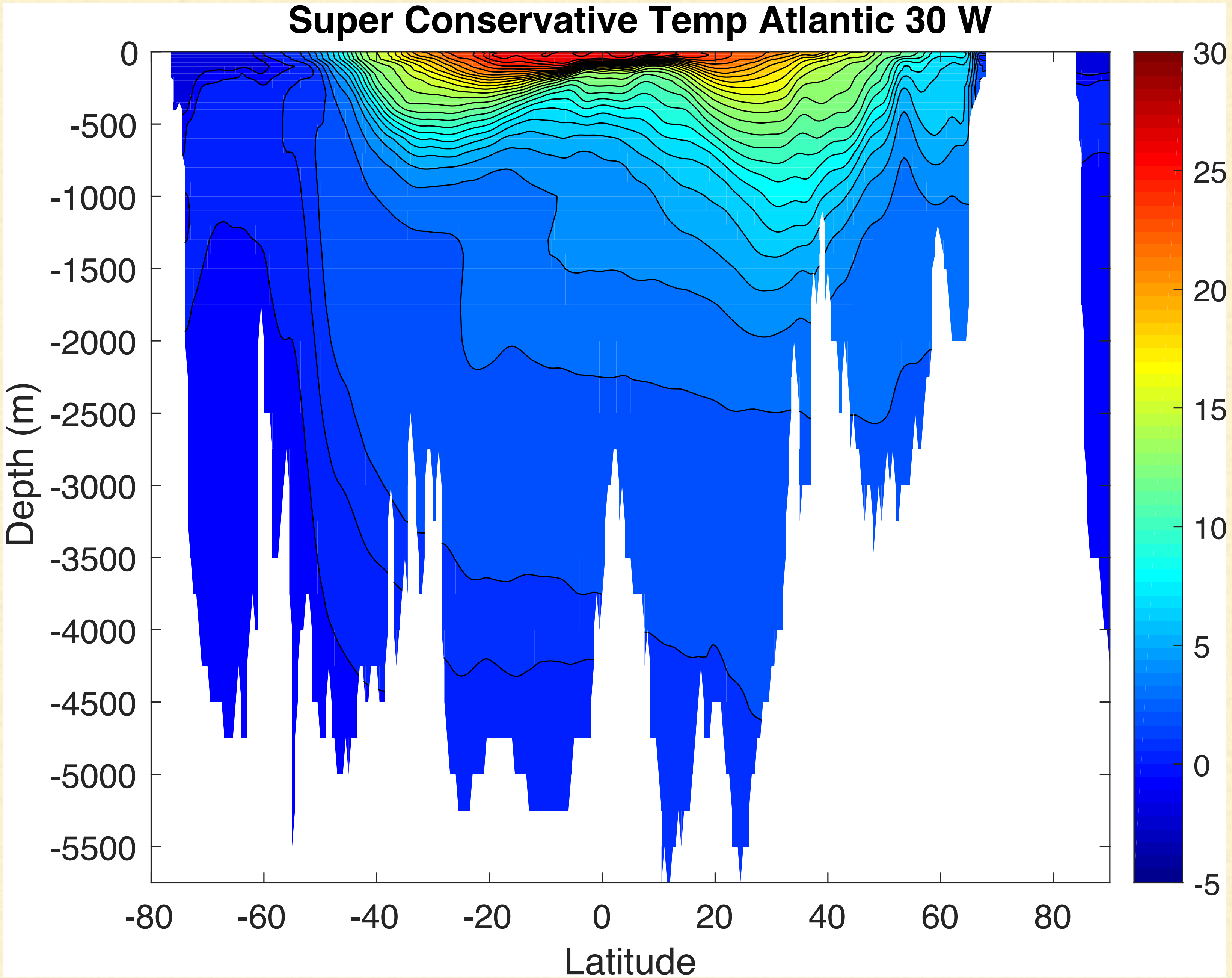
$H = h(\eta, S, p_{00}) = c_{p0}\Theta$ (McDougall, 2003)

New definition, based on partitioning of PE into APE+AAE and BPE (Tailleux, 2018)

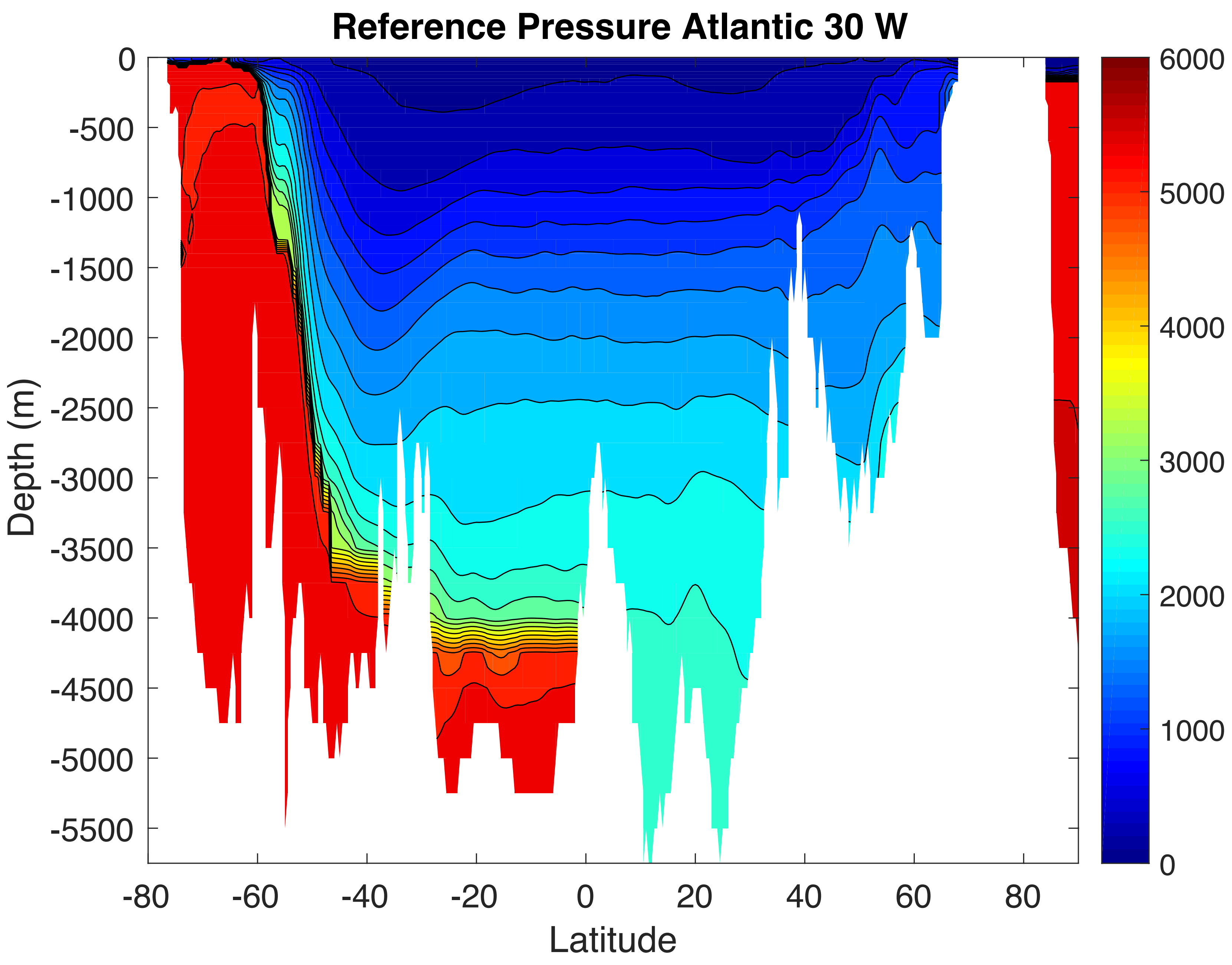
$H = h(\eta, S, p_r(z_r)) + g z_r = c_{p0}\Theta_T$ `Salty' Static Energy estimated in state of rest

COMPARISON: θ VERSUS Θ_T— DIFFER BY A FEW 10'S OF MILLI-KELVINS





Reference state pressure in the ocean – 30 W



Level of Neutral Buoyancy Equation

$$\rho(\eta, S, p_R) = \rho_R(z_R)$$

Reference state is hydrostatic

$$\frac{\partial p_R}{\partial z}(z) = -g\,\rho_R(z)$$

Reference pressure/depth are material functions of entropy and salinity

$$p_R = p_R(z_R) = p_R(\eta, S)$$

MECHANICAL VERSUS HEAT BUDGETS

Kinetic Energy Budget

$$\rho \frac{D}{Dt}\left(\frac{\vec{v}^2}{2} + \Pi_1\right) + \nabla \cdot \left(p' \, \vec{v} - \rho \vec{F}_{ke}\right) \approx \rho \, b \, w \; - \rho \varepsilon_k$$

Viscous Dissipation of KE

APE Budget

$$\rho \frac{D\Pi_2}{Dt} - \nabla \cdot (\rho \vec{F}_{ape}) = -\rho b w \; - \rho \varepsilon_p$$

Thermal Dissipation of APE

Heat Budget

$$\rho \frac{DB_r}{Dt} - \nabla \cdot \left(\rho \vec{F}_r\right) = \rho(\varepsilon_k + \varepsilon_p)$$

$$\frac{\varepsilon_p}{\varepsilon_k} \approx 0.2$$

HEAT CONSERVATION, OLD AND NEW

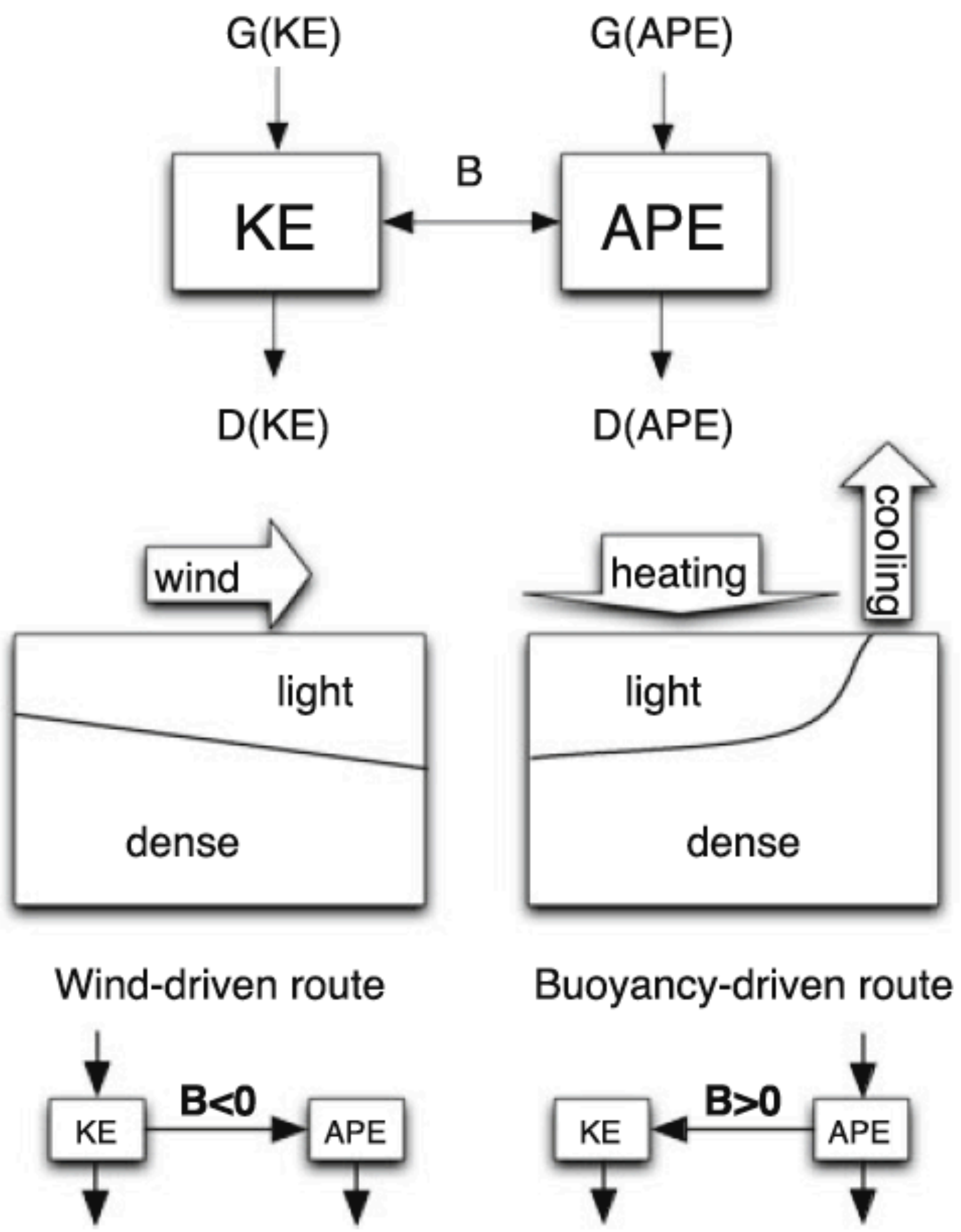
New definition of heat

$$\rho \frac{DB_r}{Dt} - \nabla \cdot (\rho F_r) = \rho(\varepsilon_p + \varepsilon_k)$$

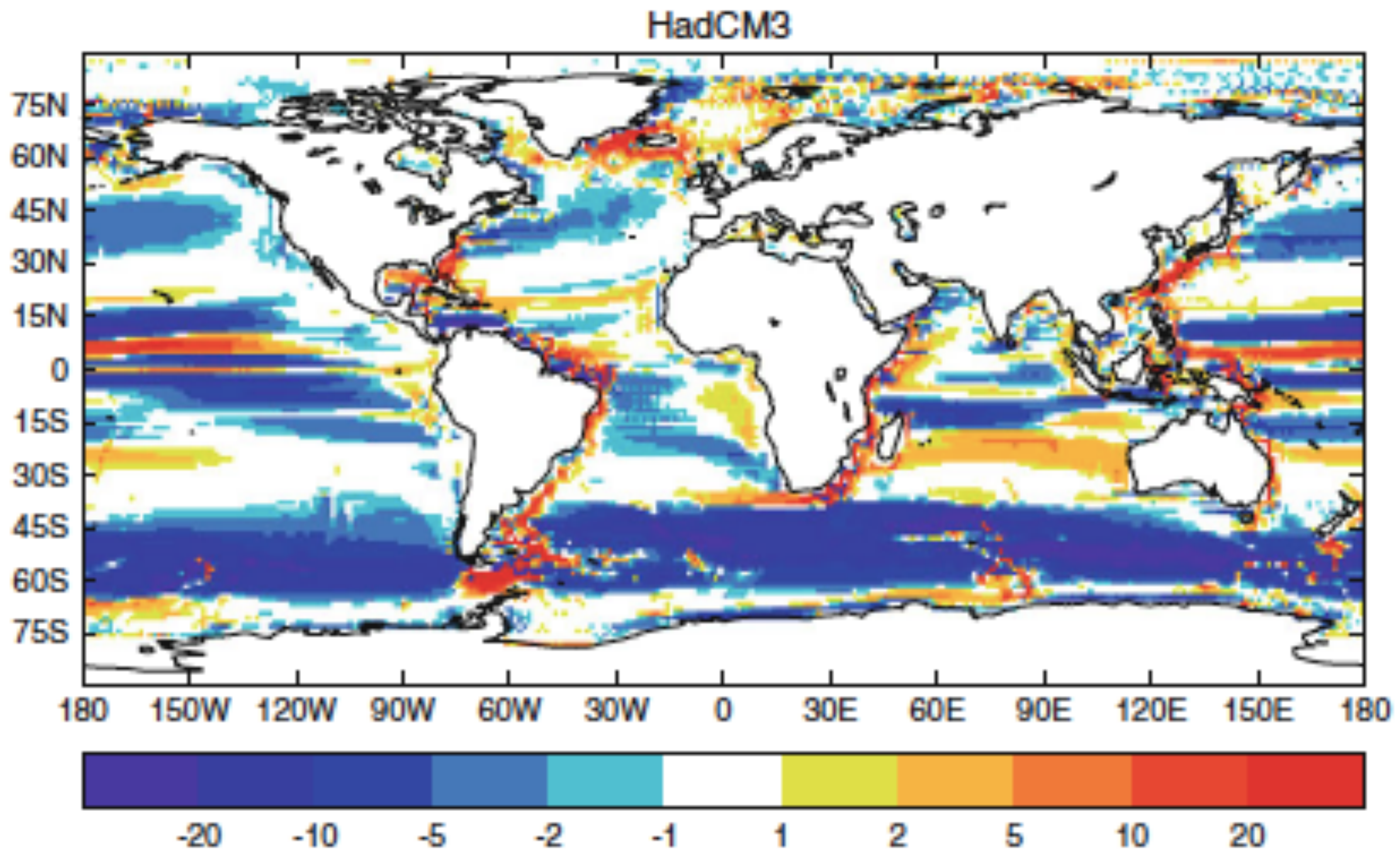
Existing heat definitions

$$\rho c_{p0} \frac{D\Theta}{Dt} - \nabla \cdot (\rho F_\Theta) = \rho\left(\varepsilon_p + \varepsilon_k\right) + \rho\dot{\Theta}_{irr}$$

SIMULATED CONVERSIONS BETWEEN APE AND KE: WIND-DRIVEN AND BUOYANCY-DRIVEN ROUTES



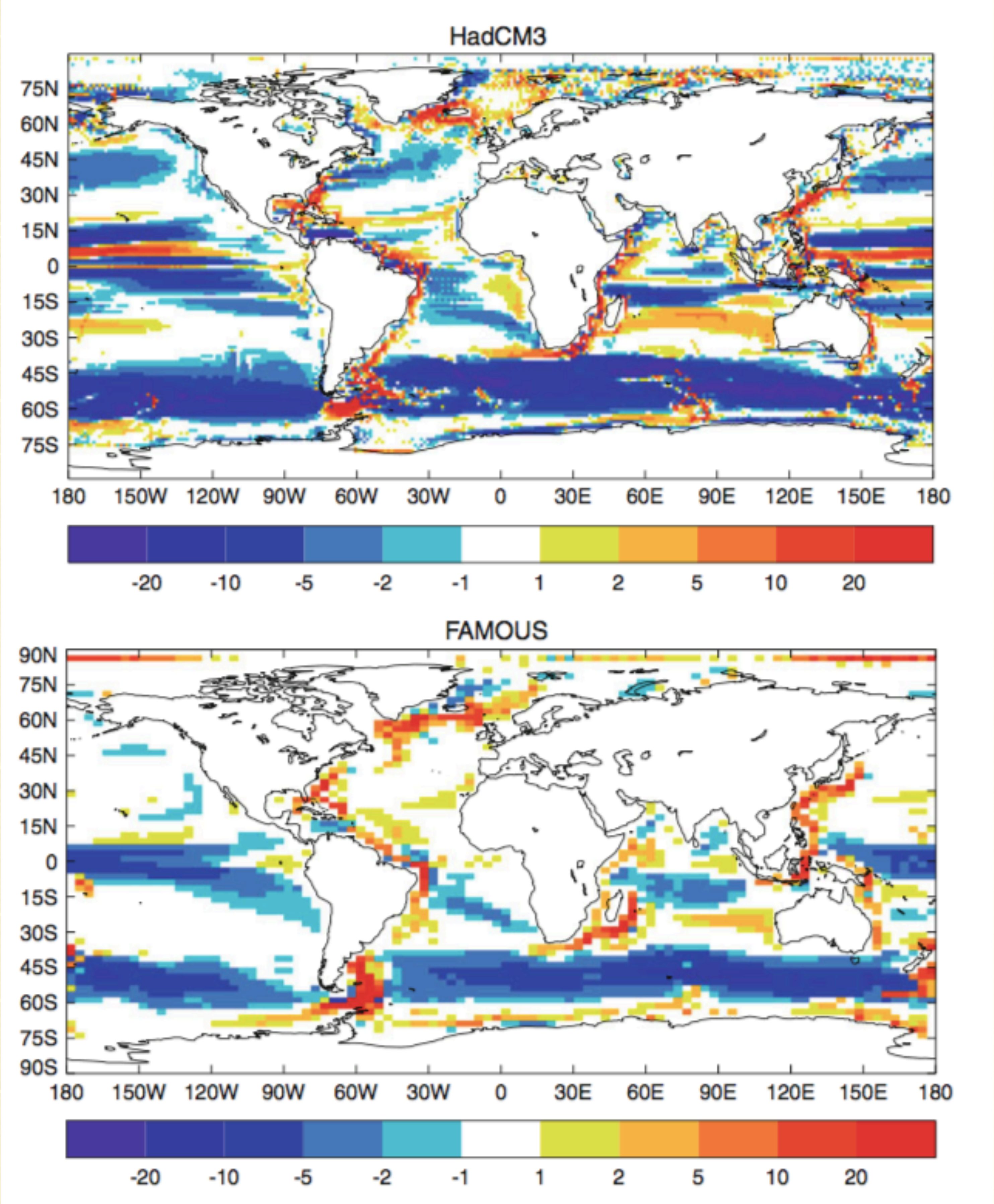
Gregory and Tailleux (2011)



Buoyancy-driven conversions = $O(20\ mW.m^{-2})$ taking place in about 1/10 of the total ocean area

Buoyancy-driven APE production =

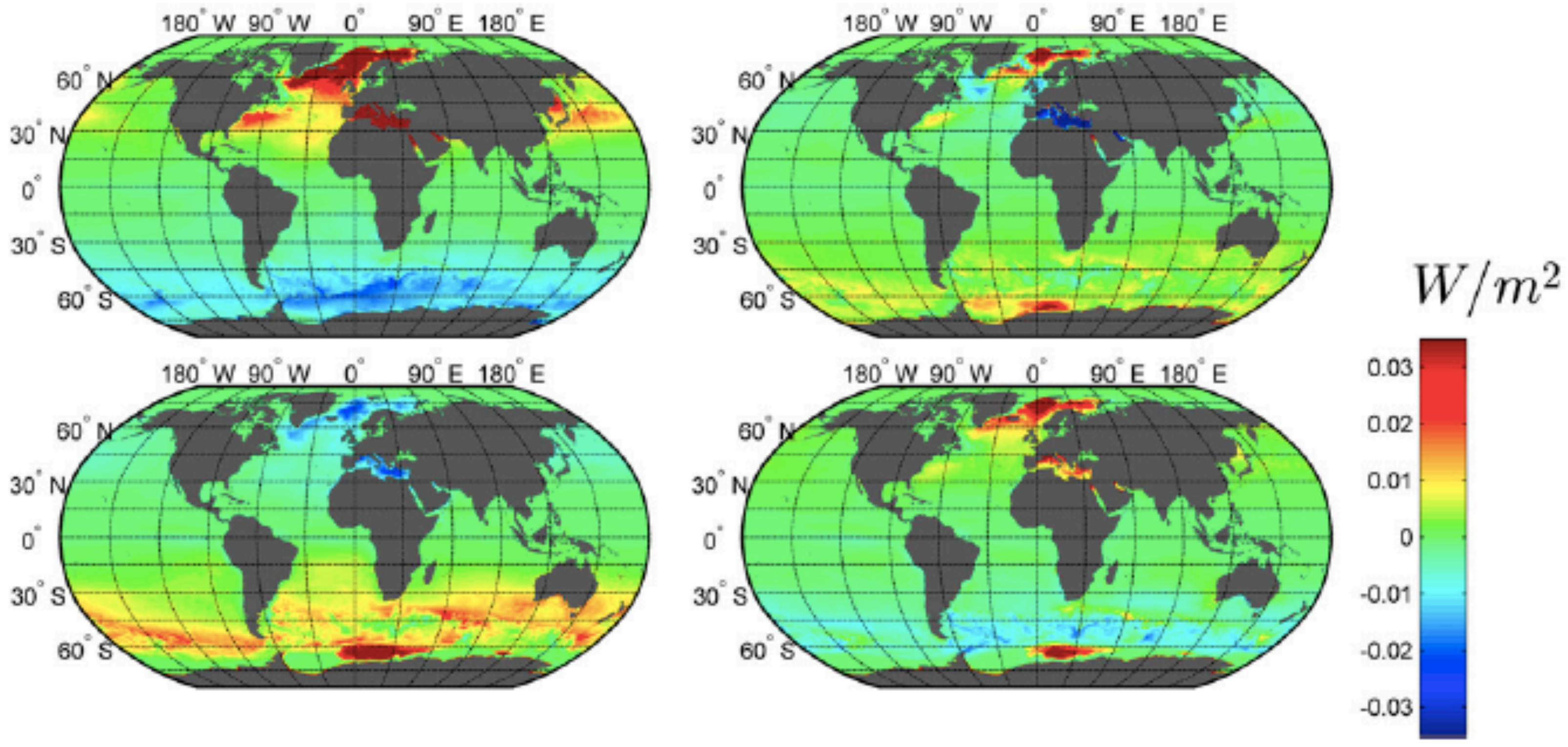
$20.10^{-3}\ W.m^{-2} \times 3.10^{14}\ m^2 \times \frac{1}{10} \approx \mathbf{0.6\ TW}$



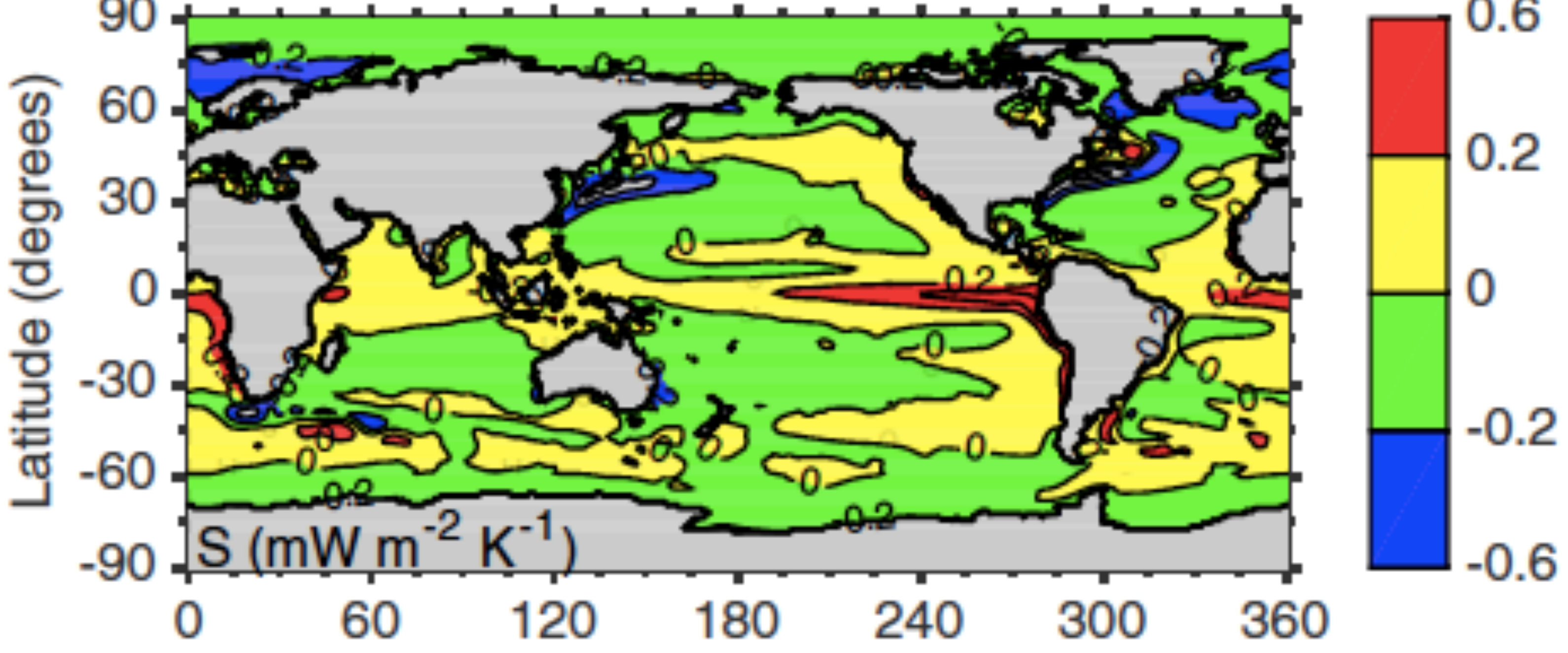
Gregory and Tailleux (2011)

APE/KE conversion rate
Work against pressure gradient
mW/m^2

Seasonal APE production (Zemskova et al, 2015)

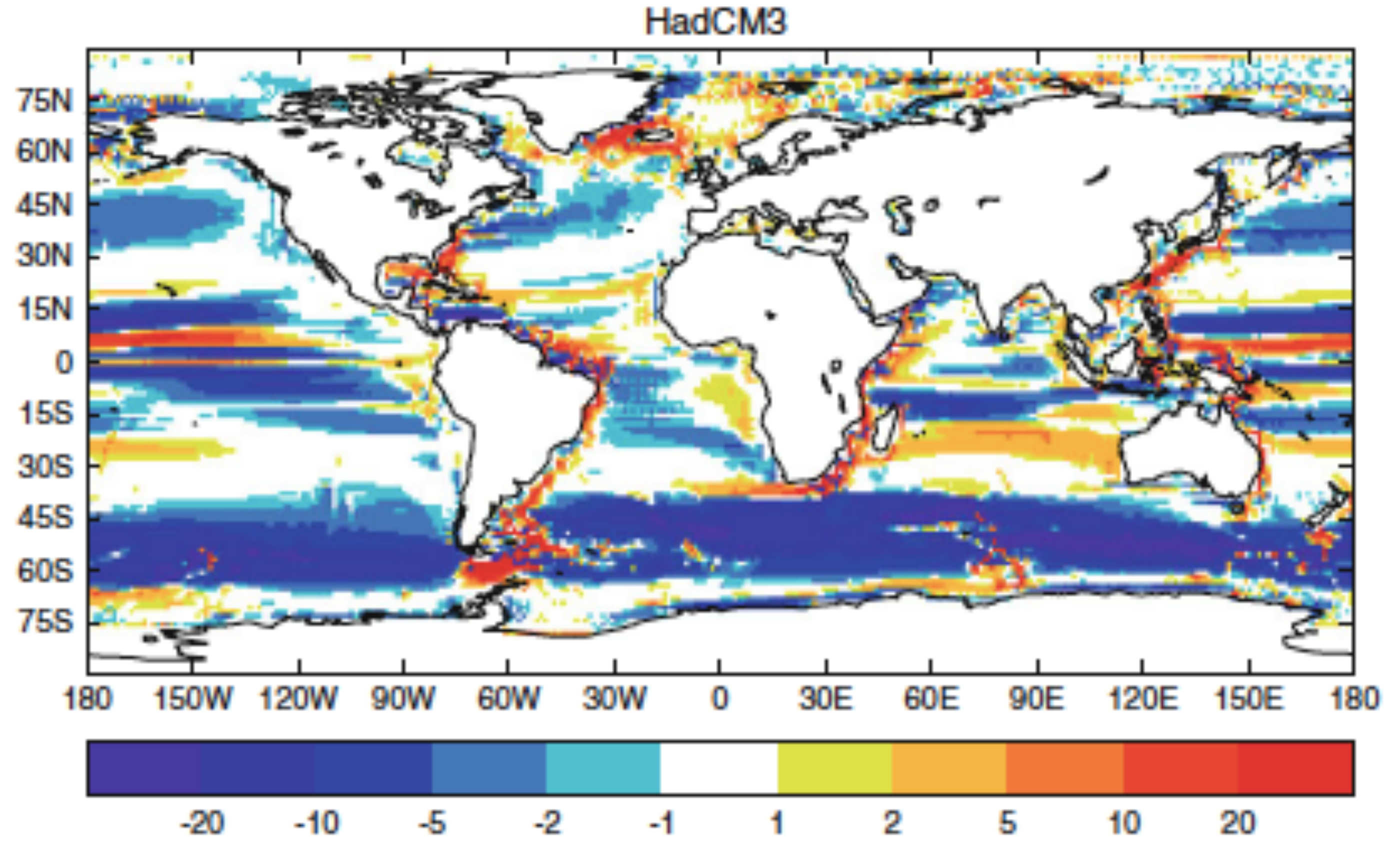


(b) Ocean Surface Entropy Flux



Bannon and Najjar (2018)

Conversions between APE and KE



Carnot Power = **O(100 TW)**

(Bannon and Najjar, 2018)

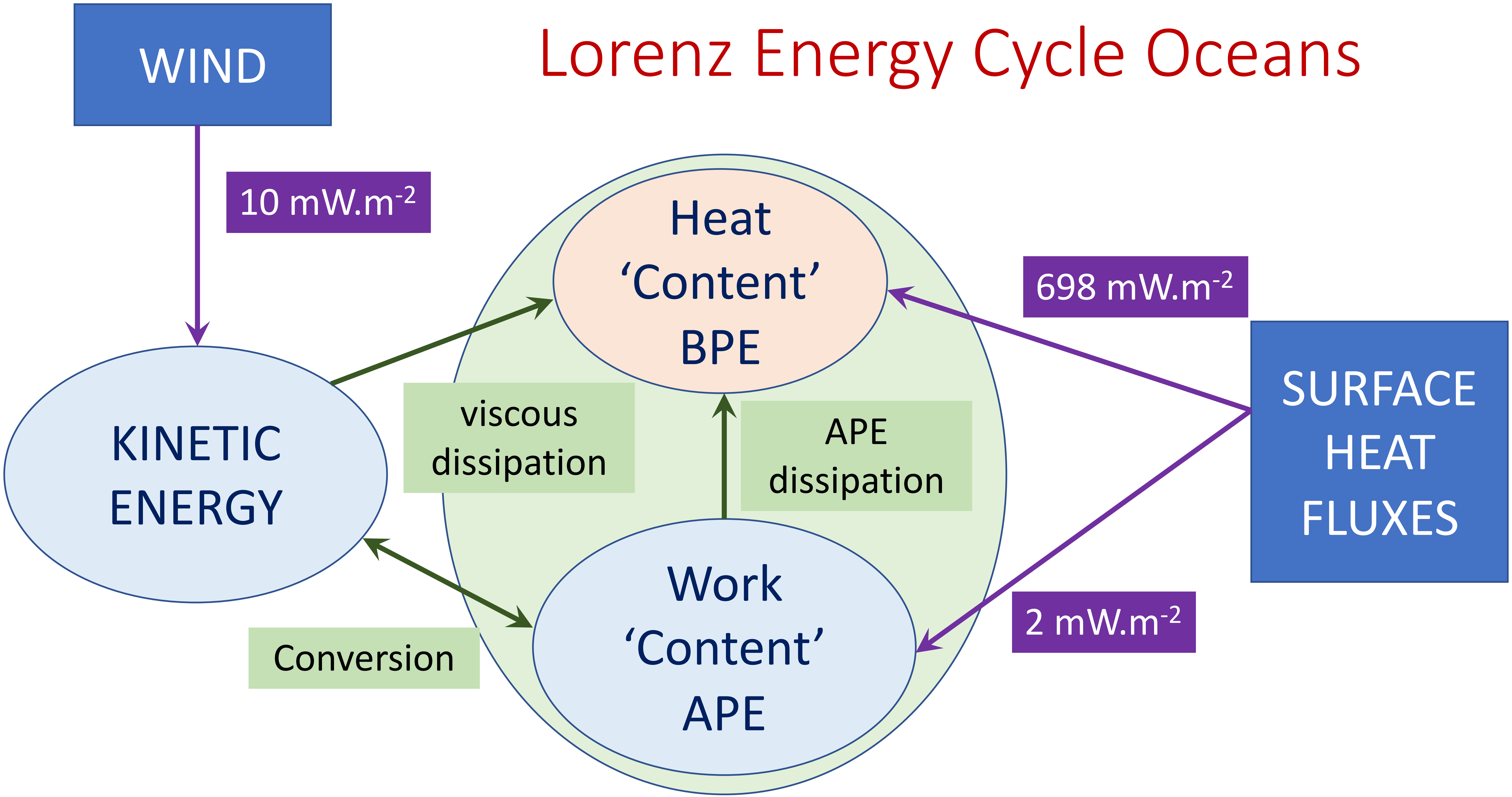
APE production = **O(0.5 TW)**

(Zemskova et al., 2015)

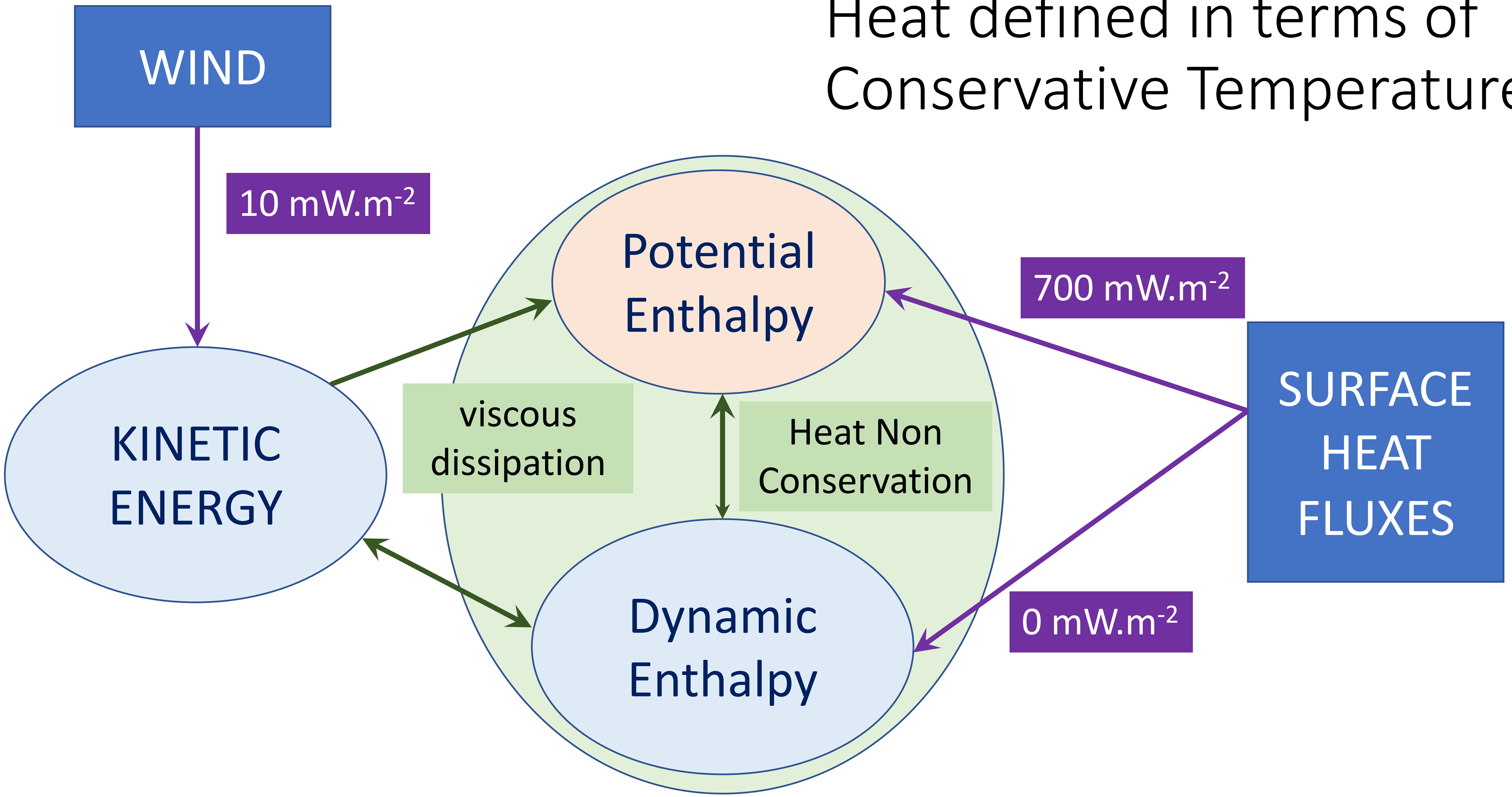
Indirect estimate of APE production

(Gregory and Tailleux, 2011) = **0.6 TW**

Lorenz Energy Cycle Oceans



Heat defined in terms of Conservative Temperature



BOUSSINESQ ENERGETICS

- Because Boussinesq decouples dynamics from thermodynamics, we did not know until not long ago:
  - Whether seawater Boussinesq equations had a well defined conservation principle
  - Whether standard thermodynamic properties (internal energy, enthalpy, entropy) could still be defined for it
  - Whether its energetics is traceable to that of the full compressible Navier-Stokes equations

CLOSING ENERGETICS OF BOUSSINESQ MODELS IS EASY

$$\frac{Dv}{Dt} + fk \times v + \frac{1}{\rho_0}\nabla p' = F(v)$$

$$\frac{1}{\rho_0}\frac{\partial p'}{\partial z} = b_{gcm} = buoyancy$$

$$\frac{D\theta}{Dt} = \nabla \cdot F_\theta + \dot{\theta}_{irr}$$

$$\frac{DS}{Dt} = \nabla \cdot F_S$$

$$b_{gcm} = -\frac{g}{\rho_0}\left(\rho_{gcm} - \rho_0\right)$$

$$p' = p + \rho_0 g z$$

$$\nabla \cdot v + \frac{\partial w}{\partial z} = 0$$

$$\rho_0 \frac{D}{Dt}\frac{v^2}{2} + \nabla \cdot (p' v_{3d}) = \rho_0 b_{gcm}(S, \theta, z)\, w + \rho_0 v \cdot F(v)$$

CLOSING ENERGETICS OF BOUSSINESQ MODELS IS EASY

Write: $$b(S,\theta,z)\frac{Dz}{Dt} = \frac{D}{Dt}\int_0^z b(S,\theta,z')dz' - \int_0^z b_S dz'\frac{DS}{Dt} - \int_0^z b_\theta dz'\frac{D\theta}{Dt}$$

Set $\dot{\theta}_{irr}$ so that the term $\dot{E}_{noncons} = 0$ in Boussinesq energy conservation equation:

$$\rho_0\frac{D}{Dt}\left(\frac{v^2}{2} + gz - \int_0^z b(S,\theta,z')dz'\right) + \nabla\cdot(p'v') = \nabla\cdot F_e + \dot{E}_{noncons}$$

To what extent does it matter that Boussinesq total energy differs from true total energy?

BOUSSINESQ THERMODYNAMICS

T,S are the observable quantities. Boussinesq approximation should not affect these.

Gibbs function: g(S,T,p) encodes all possible information about thermodynamics of seawater

$$g_b(S,T,z) = g(S,T,p_0(z))$$

$$dg_b = -\eta_b dT + \mu_b dS - \upsilon_b \rho_0 g\, dz$$

$$d(g_b + gz) = -\eta_b dT + \mu_b dS + g(1 - \rho_0 \upsilon_b) dz$$

$$d(g_b + gz + T\eta_b) = T d\eta_b + \mu_b dS - b\, dz$$

$$b\frac{Dz}{Dt} = -\frac{D}{Dt}(h_b + gz) + T\frac{D\eta_b}{Dt} + \mu_b\frac{DS}{Dt}$$

$$\rho_0 \frac{D}{Dt}\left(\frac{v^2}{2} + h_b + gz\right) + \nabla\cdot(p' v_{3d}) = \rho_0\left(T\frac{D\eta_b}{Dt} + \mu_b\frac{DS}{Dt}\right) + \rho_0 v\cdot F(v)$$

CLOSING BOUSSINESQ ENERGY BUDGET

Inviscid, adiabatic, isohaline processes:

$$\rho_0 \frac{D}{Dt}\left(\frac{v^2}{2} + h_b + gz\right) + \nabla \cdot (p' v_{3d}) = 0$$

Boussinesq

$$\rho \frac{D}{Dt}\left(\frac{v_{3d}^2}{2} + AAE + h_b + gz\right) + \nabla \cdot (p' v_{3d}) = 0$$

Fully compressible Navier-Stokes equations

Boussinesq neglects $\frac{w^2}{2} + \frac{p'^2}{2\rho c_s^2} + h(\eta, S, p_r(z)) - h(\eta, S, p_0(z))$

$$-\frac{g}{\rho_0}(\rho_{gcm} - \rho_0) = g(\rho_0 v_b - 1)$$

Requires $b_{gcm} = b$

$$\rho_{gcm} = \rho_b - \frac{(\rho_b - \rho_0)^2}{\rho_b} \neq 0$$

CLOSING ENERGY BUDGET OF BOUSSINESQ MODELS

$$b_{gcm} = b_{true} - \frac{g(\rho_b - \rho_0)^2}{\rho_b \rho_0}$$

$$b_{gcm}\frac{Dz}{Dt} = b_{true}\frac{Dz}{Dt} - \delta b\frac{Dz}{Dt} = -\frac{D}{Dt}(h_b + gz) + (T - \delta T)\frac{D\eta_b}{Dt} + (\mu_b - \delta\mu_b)\frac{DS}{Dt} - \int_0^z \delta b(\eta_b, S, z')$$

THERMODYNAMICS OF "MEAN" VARIABLES

Problem: Mean (averaged) variables are assumed to satisfy the same thermodynamic relations as the non-averaged variables. Example for Boussinesq Ocean Models: $\rho_m = \rho(S_m, \theta_m, p_0(z))$

For hydrostatic models: $\frac{\partial p_m}{\partial z} = -\rho_m g$

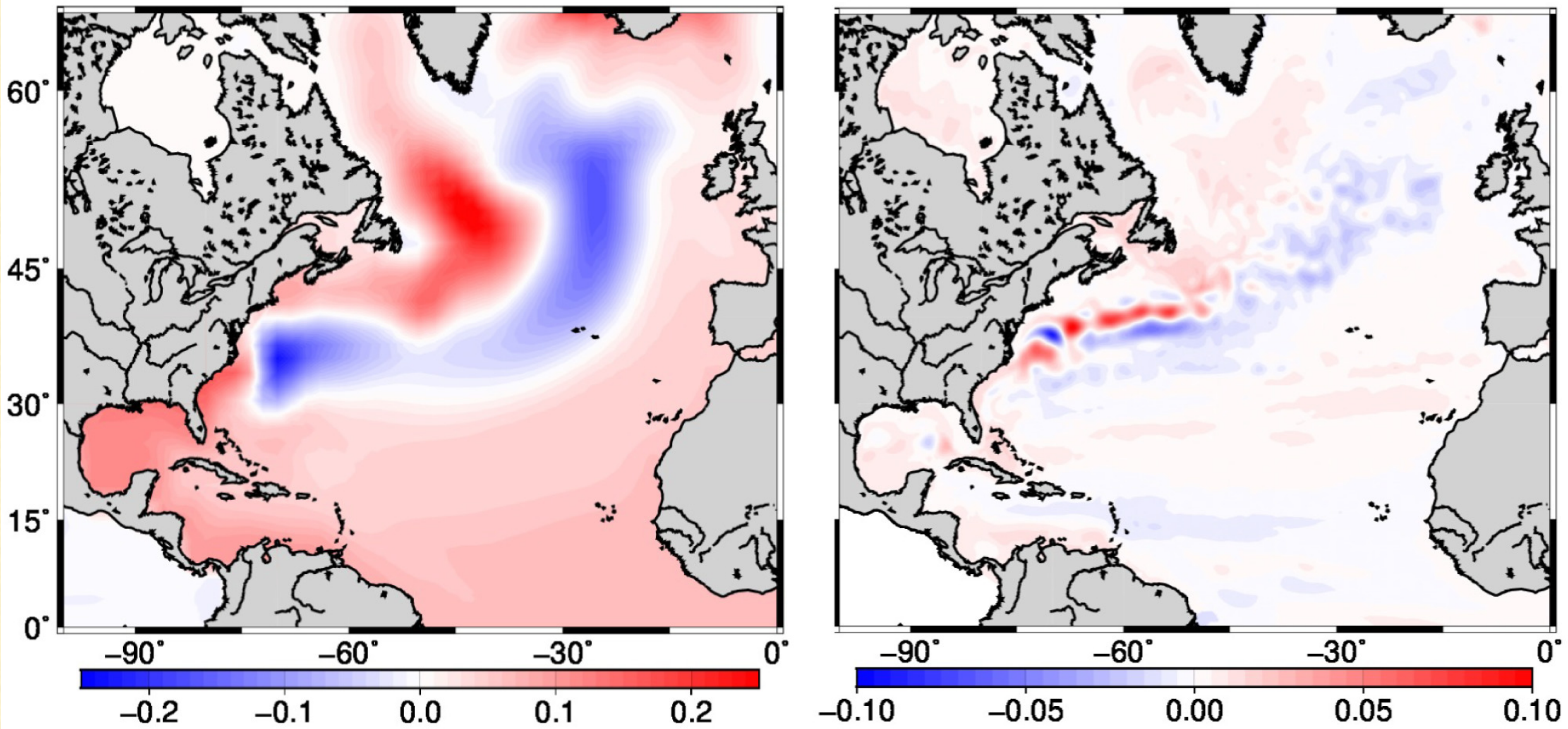
However, for nonlinear functions of their arguments:

$$\overline{\rho} = \rho\left(\overline{S}, \overline{\theta}, p_0(z)\right) + \rho_{SS}\,\frac{\overline{S'^2}}{2} + \rho_{S\theta}\,\overline{S'\theta'} + \rho_{\theta\theta}\,\frac{\overline{\theta'^2}}{2} + \cdots = \rho\left(\overline{S}, \overline{\theta}, p_0\right) + \delta\rho_{eddy}$$

$$\frac{\partial \overline{p}}{\partial z} = -(\overline{\rho} + \delta\rho_{eddy})g$$

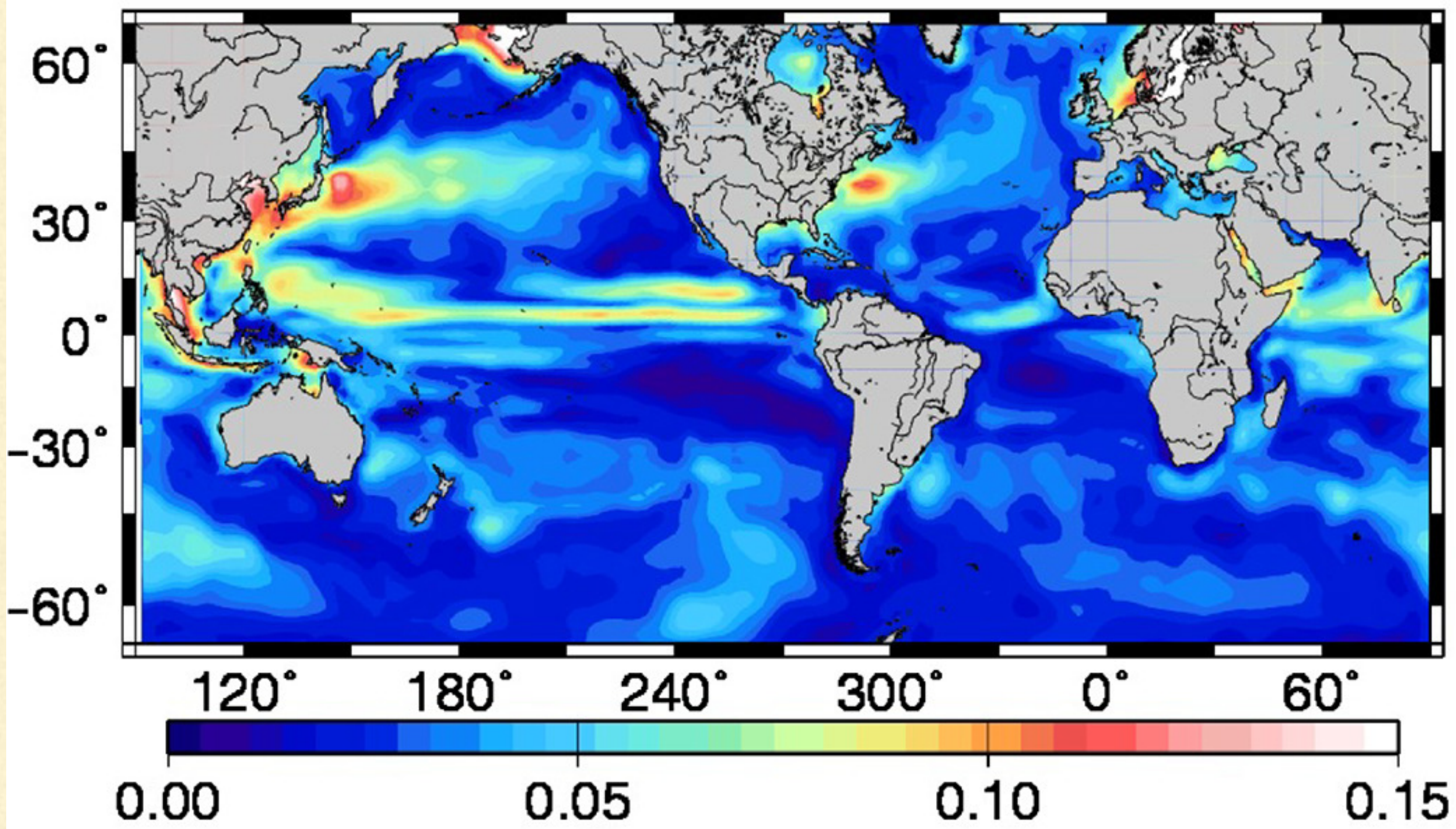
BRANKART (2013), BRANKART ET AL. (2015), ZANNA ET AL. 2018

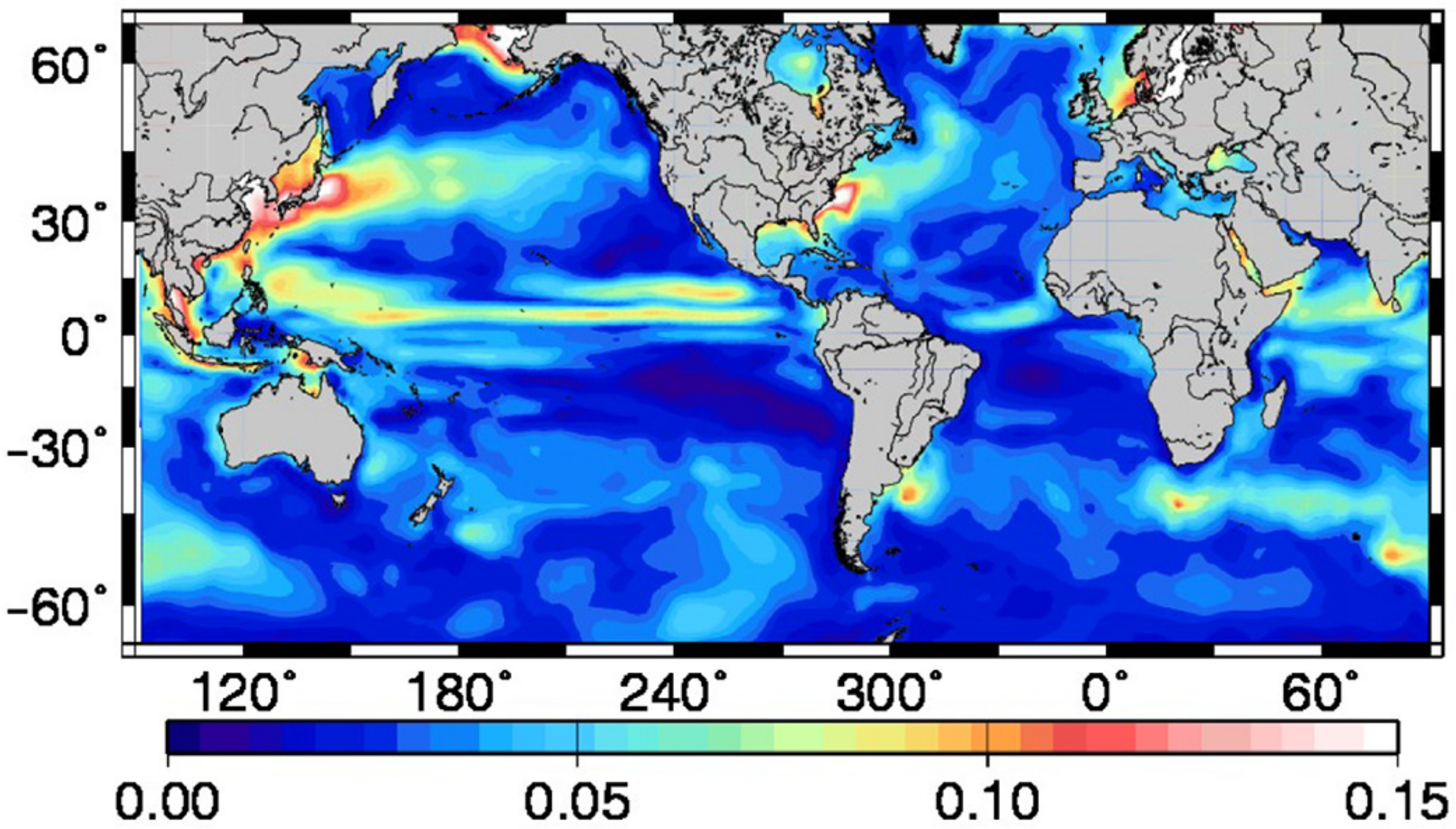
ORCA2 (2deg) **NATL25 (1/4 deg)**



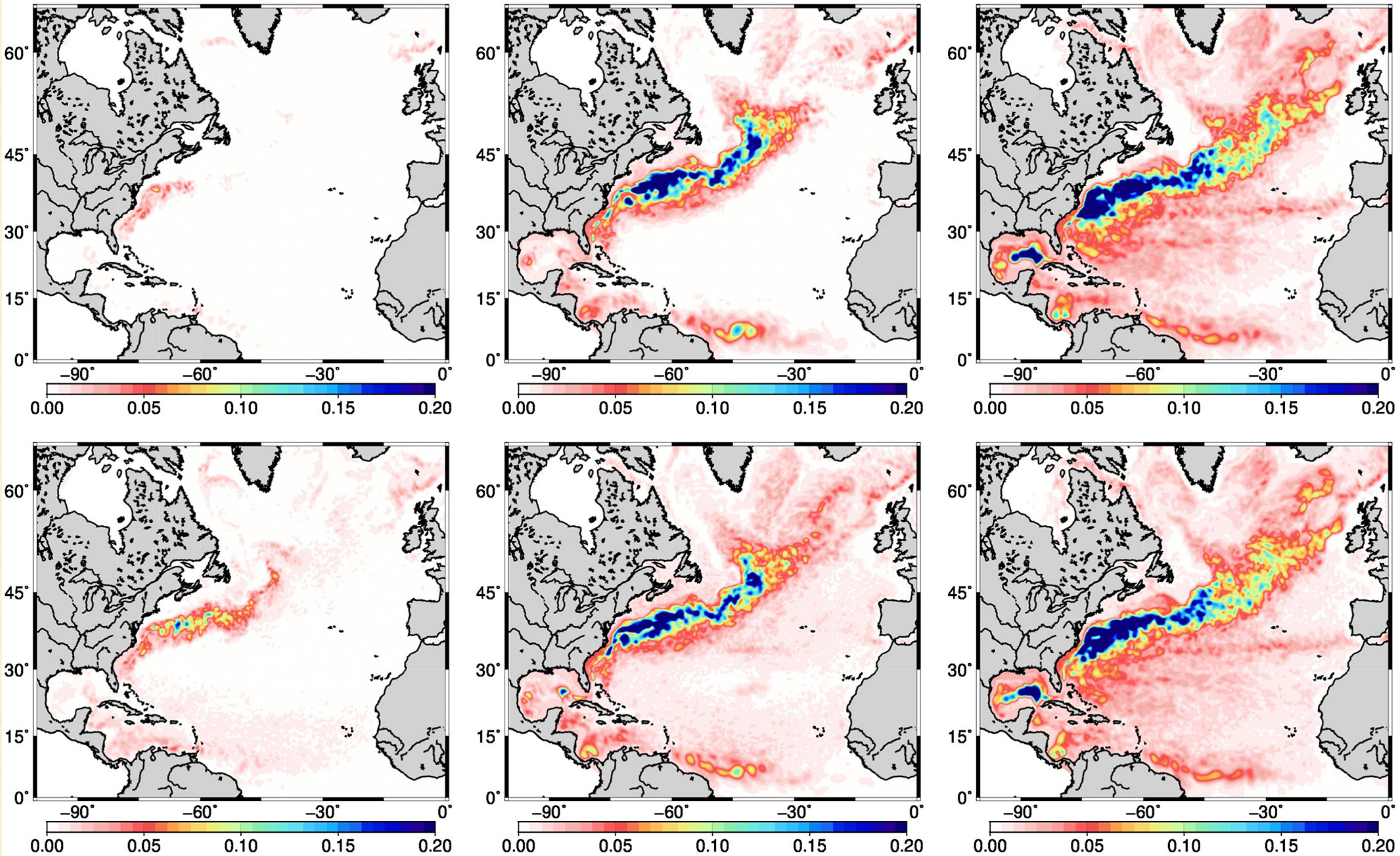
Effects of subgridscale T/S variability on mean density. Impacts on Sea Surface Height (SSH)

BRANKART 2013, BRANKART ET AL. (2015), ZANNA ET AL., 2018





BRANKART 2013, BRANKART ET AL., (2015), ZANNA ET AL. 2018



IMPORTANCE OF AVERAGING OPERATOR CORRECTLY SIMULATING T/S CRUCIAL

$$\vec{v}_{gcm} \cdot \nabla C_{gcm} = \nabla(K\, \nabla \mathrm{C}_{\mathrm{gcm}})$$

Lagrangian averaging

$$\overline{v}^L \cdot \nabla \overline{c}^L = \overline{\dot{c}}^L$$

$$\frac{D\overline{v}^L}{Dt} + 2\Omega \times \overline{v}^L + \frac{1}{\rho}\nabla p = D_v + F_{eddy}$$

Eulerian averaging

$$\left(\overline{v} + v_{eddy}\right) \cdot \nabla \overline{C} = \nabla \cdot \left(K \nabla \overline{C}\right)$$

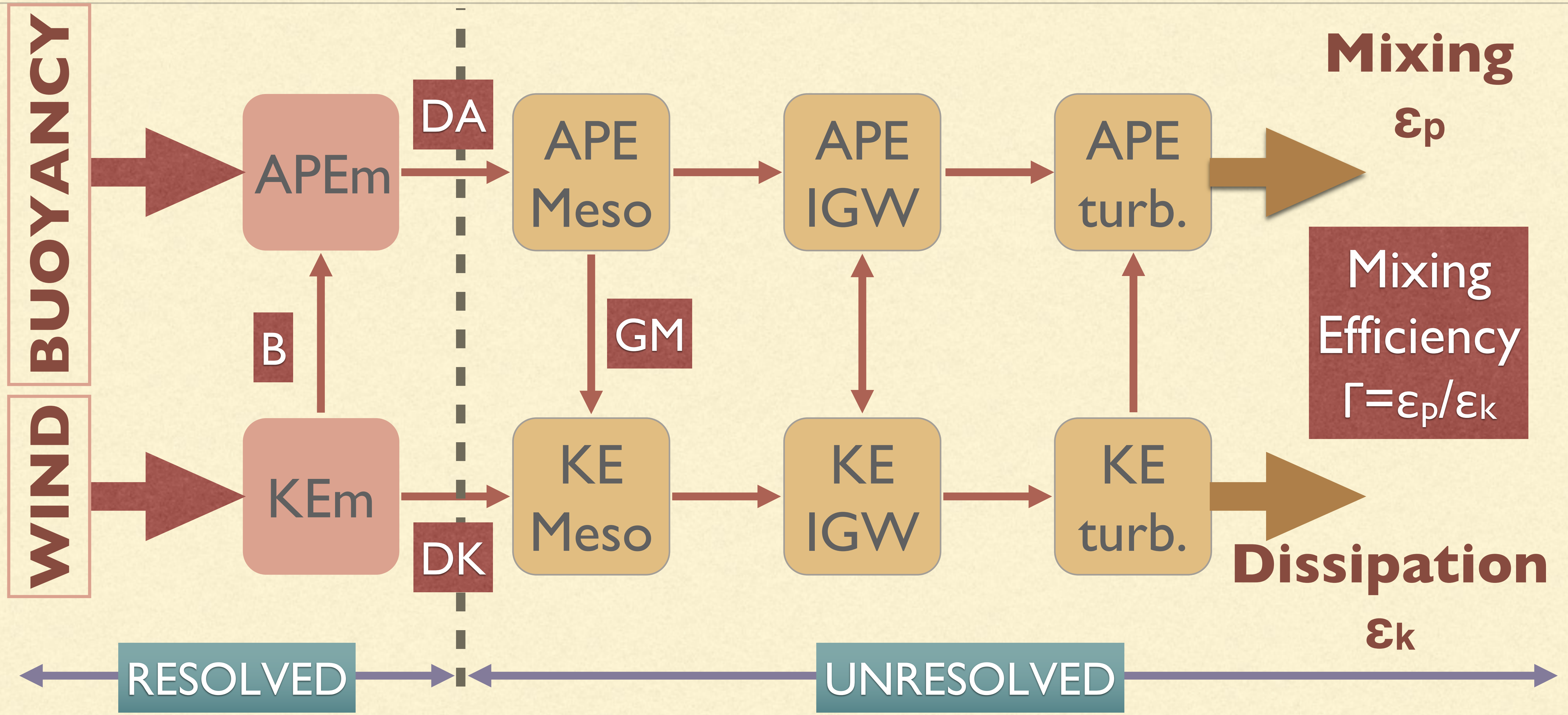
$$\nabla \cdot \overline{v'C'} = v_{eddy} \cdot \nabla C \; - \nabla \cdot (K \nabla \overline{C})$$

$$\frac{D\overline{v}}{Dt} + 2\Omega \times \overline{v} + \frac{1}{\rho}\nabla p = D_v$$

LINKING MIXING TO SOURCES OF STIRRING ENERGETICALLY CONSISTENT MODELS (EDEN,..)

- Usually based on the so-called Osborn model linking diapycnal mixing to viscous dissipation: $K_v = \Gamma\, \varepsilon_k / N^2$
- Motivates using a turbulent kinetic energy (TKE) equation to predict ε_k, but requires knowledge of the mixing efficiency parameter Γ (often assumed constant = 0.2)
- Alternative would be to use alternative form $K_v = \varepsilon_p / N^2$
- Would suggest to construct a turbulent potential energy (TPE) dissipation to predict ε_p
- Atmosphericists currently developing turbulent schemes based on two turbulent energies equation for KE and APE

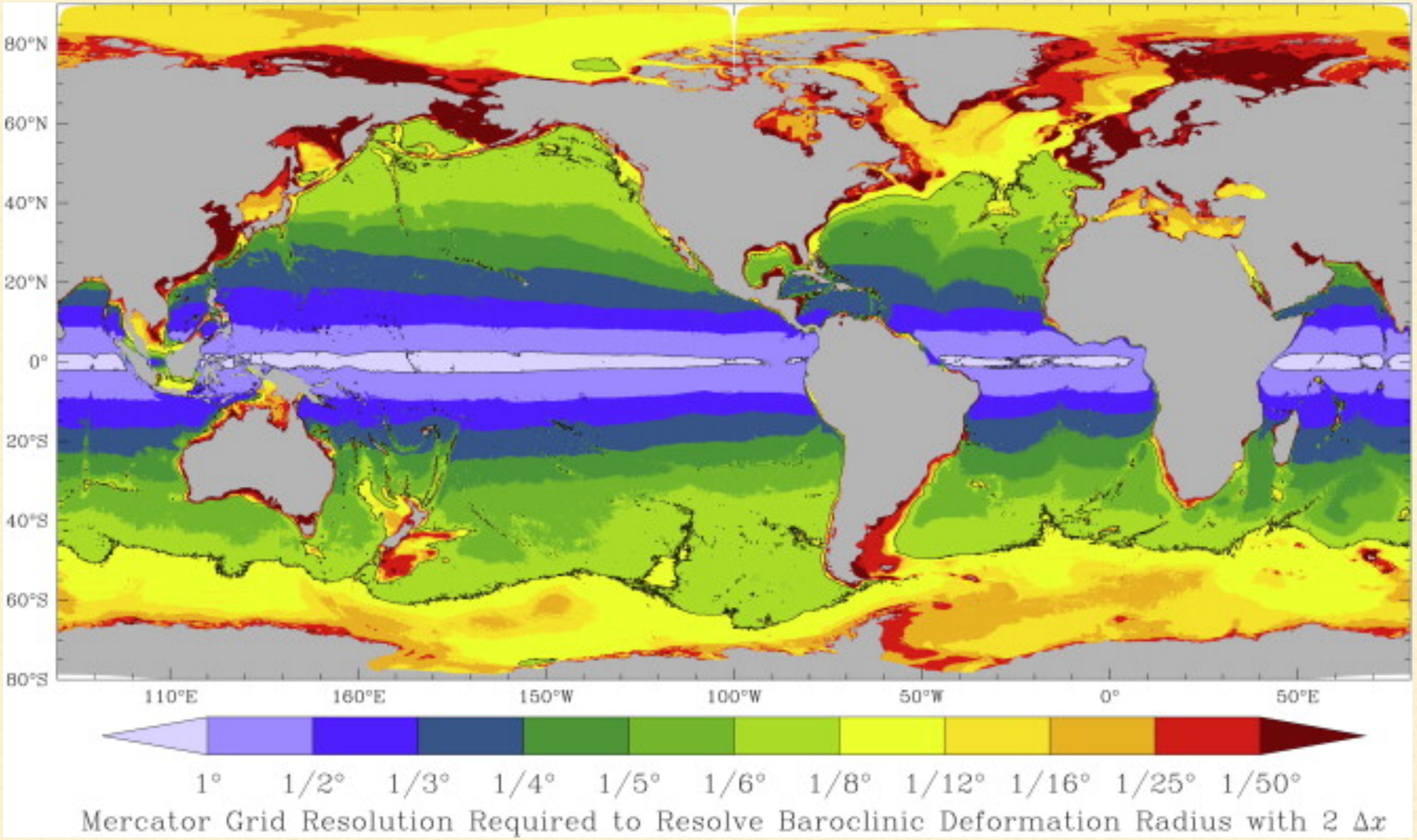
RESOLVED AND UNRESOLVED ENERGY PATHWAYS - COARSE RESOLUTION MODELS



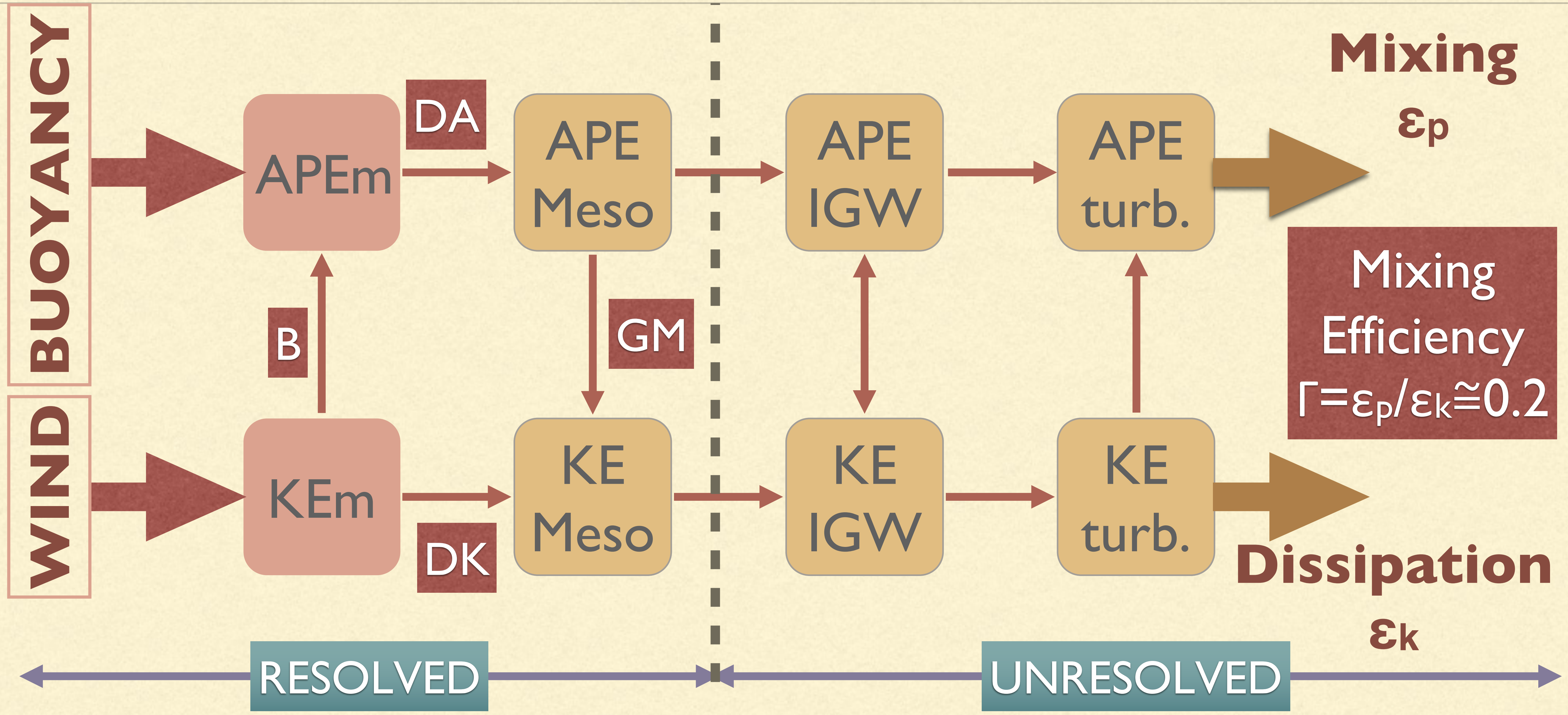
$$DA + DK = \varepsilon_p + \varepsilon_k = (1+\Gamma)\varepsilon_k$$

RESOLUTION NEEDED TO RESOLVE BAROCLINIC ROSSBY RADIUS OF DEFORMATION

Hallberg (2013)

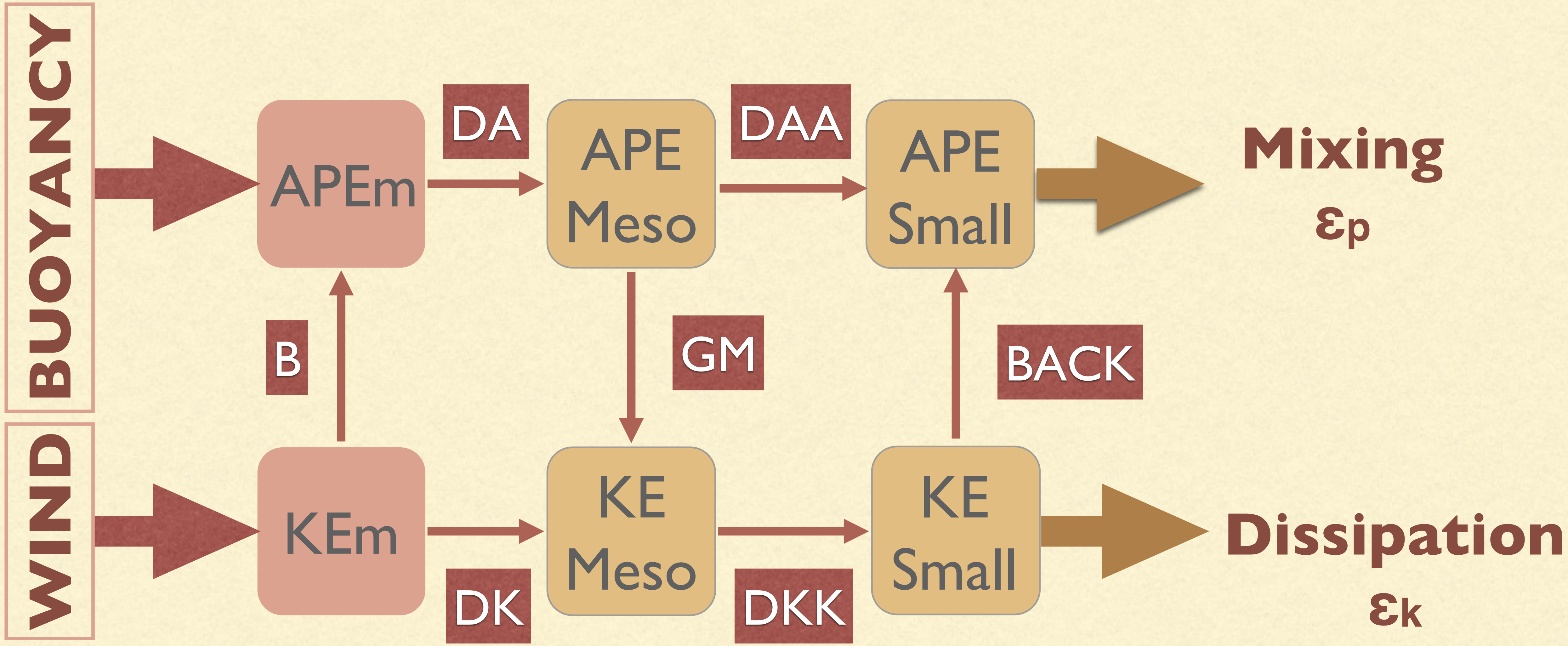


RESOLVED AND UNRESOLVED ENERGY PATHWAYS - HIGH RESOLUTION MODELS



DA = BUO + B   DAA = DA - GM   EP = DAA+BACK

DK = WIND - B   DKK = DK + GM   EK = DKK - BACK



EP = DA - GM + BACK

EK = DK + GM - BACK

DA = EP + GM - BACK

DK = EK - GM + BACK

MIXING PARAMETERISATIONS

$$\frac{DS}{Dt} = -\mathbf{\Psi}_{gm} \times \nabla S + \nabla \cdot (\mathbf{K}\nabla S)$$

$$\frac{D\theta}{Dt} = -\mathbf{\Psi}_{gm} \times \nabla \theta + \nabla \cdot (\mathbf{K}\nabla \theta)$$

$$\mathbf{K} = K_i(\mathbf{I} - \mathbf{d}\mathbf{d}^T) + K_d\mathbf{d}\mathbf{d}^T \qquad \mathbf{d} = \frac{\nabla\gamma}{|\nabla\gamma|}$$

Isopycnal Mixing (K_i)

Diapycnal Mixing (K_d)

Diapycnal Direction ($\mathbf{d}$)

EP DUE TO ROTATED DIFFUSION IN NON-EDDY RESOLVING MODELS. HAS IT THE RIGHT FORM?

Local APE theory for SeaWater Boussinesq models (Tailleux, 2013)

$$e_a = \frac{g}{\rho_0} \int_{z_r(S,\theta,t)}^{z} \left[\rho(S,\theta,p_0(z')) - \rho_r(z',t)\right] \mathrm{d}z'$$

$$\rho(S,\theta,p_0(z_r)) = \rho_r(z_r,t)$$

$$\rho_0 \frac{De_a}{Dt} = (\rho - \rho_r) g w + g(z - z_r)\left(\overline{\rho}_S \frac{DS}{Dt} + \overline{\rho}_\theta \frac{D\theta}{Dt}\right) - g \int_{z_r}^{z} \frac{\partial \rho_r}{\partial t} \mathrm{d}z'$$

$$\overline{\rho}_i = \frac{1}{z - z_r} \int_{z_r}^{z} \rho_i(S,\theta,p_0(z'))\mathrm{d}z'$$

CONSTRAINTS ON DIABATIC MIXING PARAMETERISATION

$$\varepsilon_p = \frac{g}{\rho_0}\left[\mathbf{K}\left(\bar{\rho}_S \nabla S + \bar{\rho}_\theta \nabla \theta\right)\right] \cdot \nabla(z - z_r) + (z - z_r)\text{NonLinear}$$

$$K_v N^2 = K_i \mathbf{M}_i \cdot \mathbf{h}_i + K_d \mathbf{M}_d \cdot \mathbf{h}_d + (z - z_r)\text{NonLinear}$$

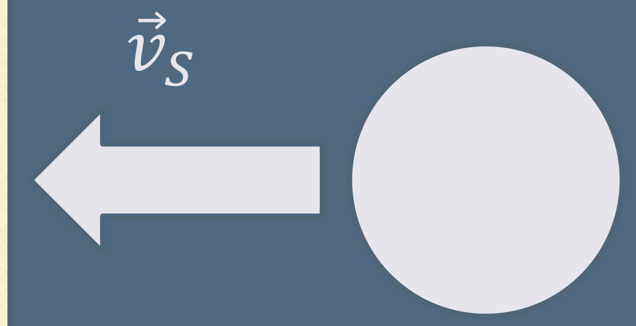
$$\mathbf{M} = \frac{g}{\rho_0}\left(\bar{\rho}_S \nabla S + \bar{\rho}_\theta \nabla \theta\right) \qquad \mathbf{h} = \nabla(z - z_r) = \mathbf{k} - \nabla z_r$$

Ki, Kd, and mixing direction needs to be such that above quantity is of same order of magnitude as what is measured observationally

PARTIAL THERMODYNAMIC PROPERTIES
"WHAT IS THE LATENT HEAT FLUX?"

The idea is that quantities such as enthalpy or internal energy can be defined separately for the freshwater and dissolved salt components in seawater, or for water vapour and dry air in the atmosphere, as if they were not part of a mixture

PARTIAL THERMODYNAMIC PROPERTIES



Dissolved salt

Velocity: $\vec{v}_S$

Density: $\rho_S = \rho S$

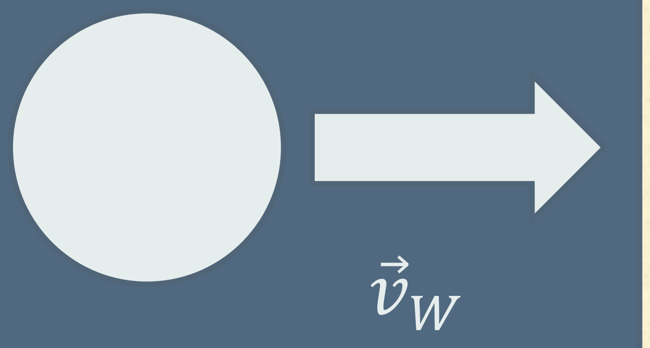
Enthalpy: $h_S = h + (1 - S)\dfrac{\partial h}{\partial s}$

Freshwater

Velocity: $\vec{v}_W$

Density: $\rho_W = \rho(1 - S)$

Enthalpy: $h_W = h - S\dfrac{\partial h}{\partial s}$



Mixture: Seawater

Velocity: $\vec{v} = S\vec{v}_S + (1 - S)\vec{v}_W$

Density: ρ

Enthalpy: $h = S\, h_S + (1 - S)h_W$

Salt Flux: $\vec{J}_S = S(1 - S)(\vec{v}_S - \vec{v}_W)$

Dissolved salt and freshwater travel with their own velocity and individual thermodynamic properties

PARTIAL THERMODYNAMIC PROPERTIES IMPORTANT FOR:

Defining saturation water vapour pressure over seawater:

$$g_W(S, T, p_d + e_S) = g_V(T, q, p_d + e_S)$$

Defining latent heat of evaporation over seawater

$$L = h_v - h_W$$

Defining boundary conditions for seawater (similar idea for moist air):

$$\vec{v}_W \cdot \vec{n}\, dS = \text{evaporation} - \text{precipitation}$$

$$\vec{v}_S \cdot \vec{n}\, dS = 0$$

Saturation water vapour pressure and latent heat depends on salinity over the ocean!!

PARTIAL THERMODYNAMIC PROPERTIES IMPORTANT FOR:

Defining fluxes of properties

Entropy flux: $\rho\, \eta\, \vec{v} = \rho \left[S\, \eta_S\, \vec{v}_S + (1 - S)\eta_W\, \vec{v}_W - (\eta_S - \eta_W)\vec{J}_S \right]$

Enthalpy flux: $\rho\, h\, \vec{v} = \rho \left[S\, h_S\, \vec{v}_S + (1 - S)h_W\, \vec{v}_W - (h_S - h_W)\vec{J}_S \right]$

Salt Flux: $\rho \vec{J}_S = \rho_S\, (\vec{v} - \vec{v}_S) = \rho\, S(1 - S)(\vec{v}_S - \vec{v}_W)$

BPE (Heat) can occasionally become APE (Work)

Stable APE = 0
PE = Heat only

WARM, SALTY
LIGHT

COLD, FRESH
HEAVY

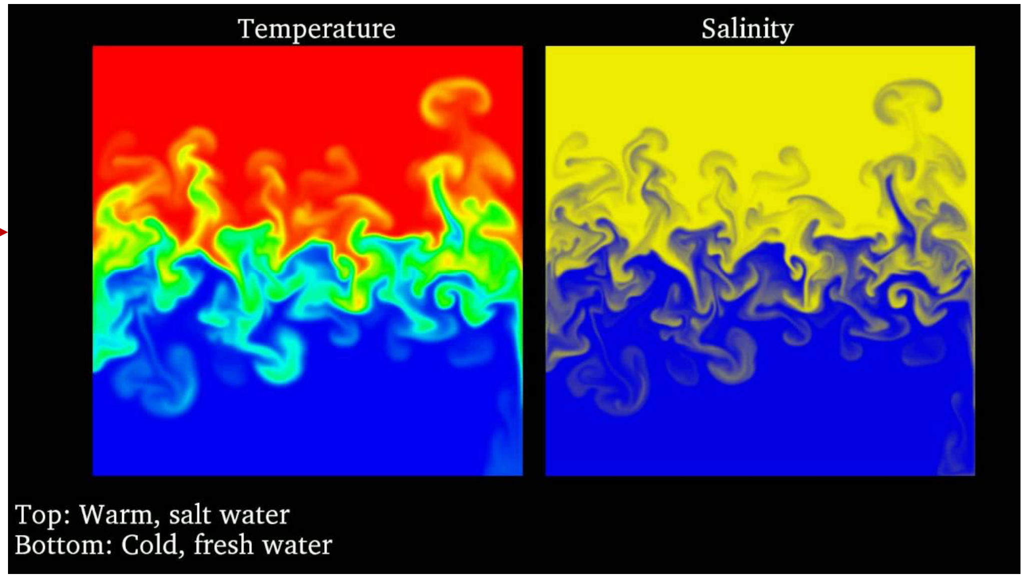
Mixing rate larger for Temperature than for Salt

Unstable APE > 0
PE = APE + BPE

MILD, SALTY
HEAVY

MILD, FRESH
LIGHT

Double Diffusive Instability



Equal mixing rates for Temperature and Salt

MILD, FRESH
LIGHT

MILD, SALTY
HEAVY

Stable APE = 0
PE = BPE only

SUMMARY

- Boussinesq equations can be made conservative but
  - Conserved Boussinesq energy may differ from true energy
  - Energy conservation requires that resolved + unresolved energy be conservative
  - Need to clarify link between resolved and unresolved energy reservoirs
  - Requires clarification of underlying averaging operator
- Partial thermodynamic properties key to understand coupling with atmosphere