NCAR
ASP Graduate Student Visitor
Host: Peter H. Lauritzen

Stony Brook University
Ph.D Advisor: Kevin A. Reed

Evaluating physics on a separate grid:
physics-dynamics coupling (pdc) with element based high-order Galerkin methods (e.g., CAM-SE)

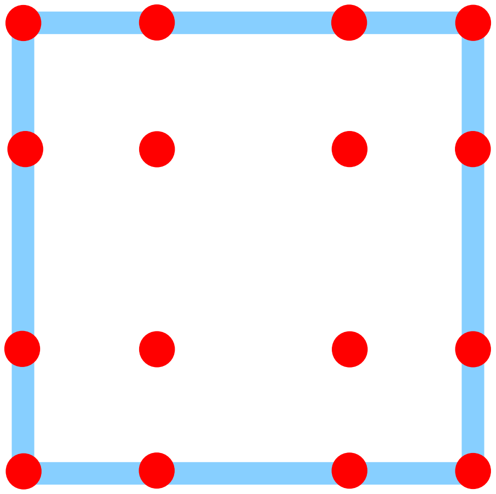
Adam R. Herrington, Ph.D.

BIRS PDC Workshop
October 2019

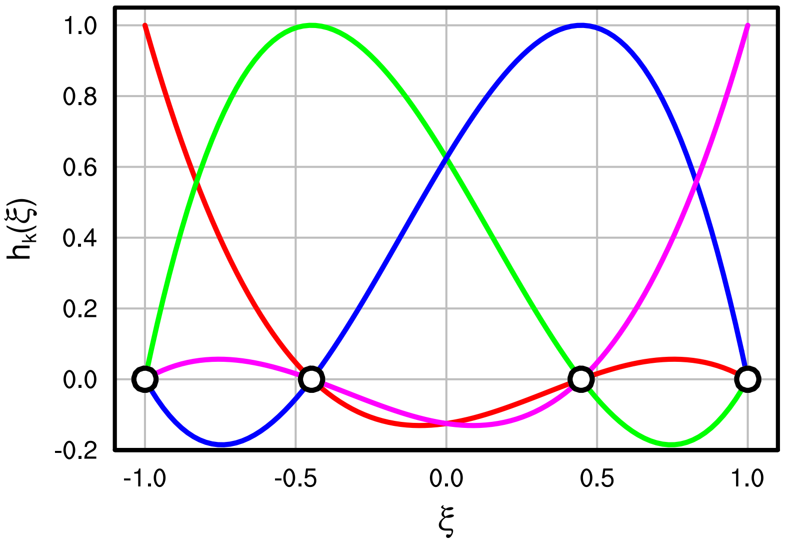
Conventional PDC paradigm

Evaluate physics on dynamics grid: the GLL grid

Galerkin method defines nodal point values



Degree three Lagrange basis set
(puts the ‘spectral’ in ‘spectral-element’)



Conventional PDC paradigm

Evaluate physics on dynamics grid: the GLL grid

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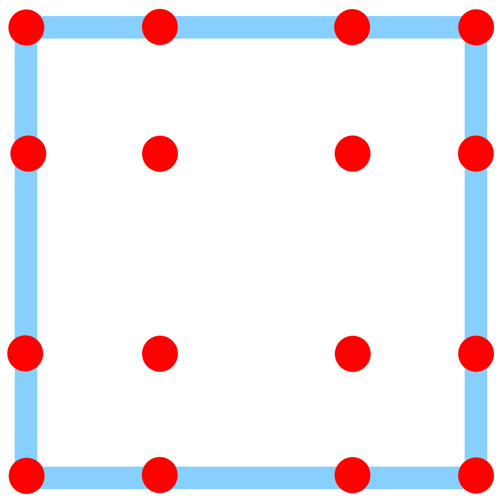
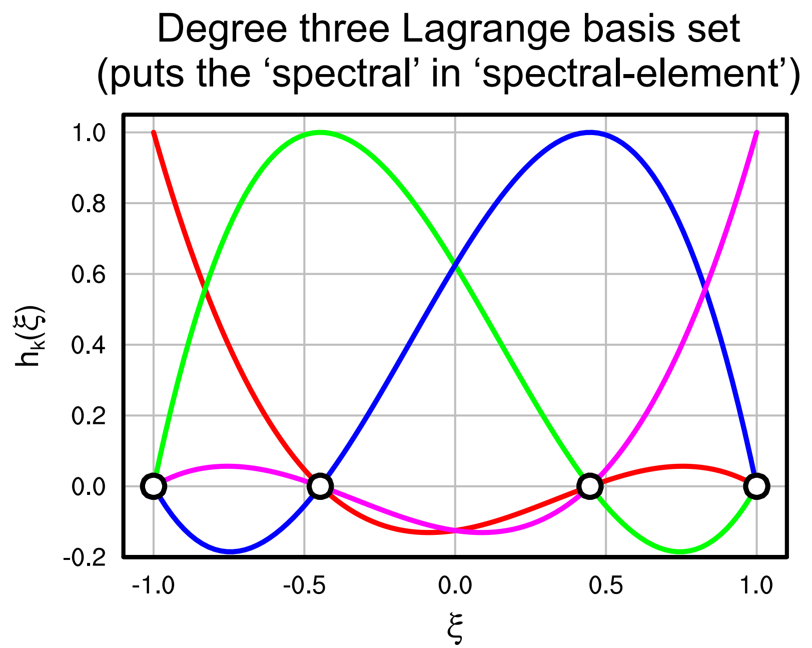


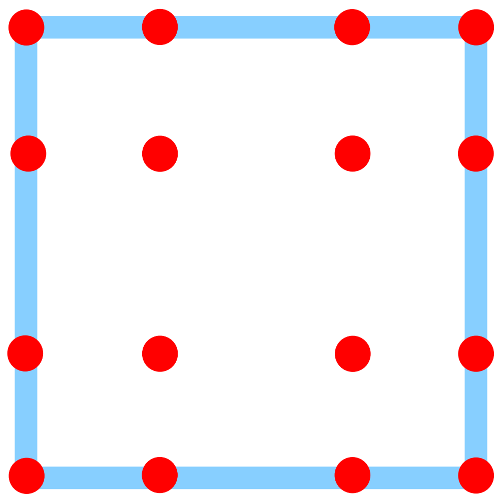


Issue('ish) 1: Is this a 'large-scale state,' as physics assumes?

Conventional PDC paradigm

Evaluate physics on dynamics grid: the GLL grid

Galerkin method defines nodal point values



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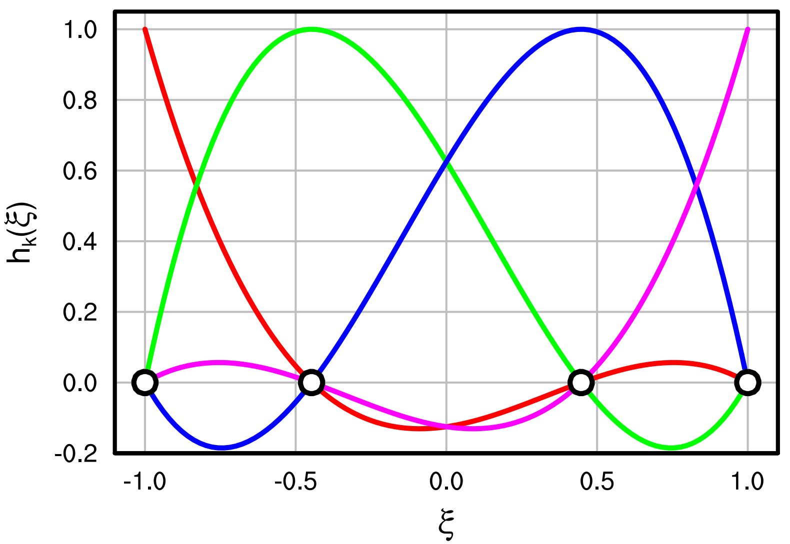


Issue('ish) 1: Is this a 'large-scale state,' as physics assumes?

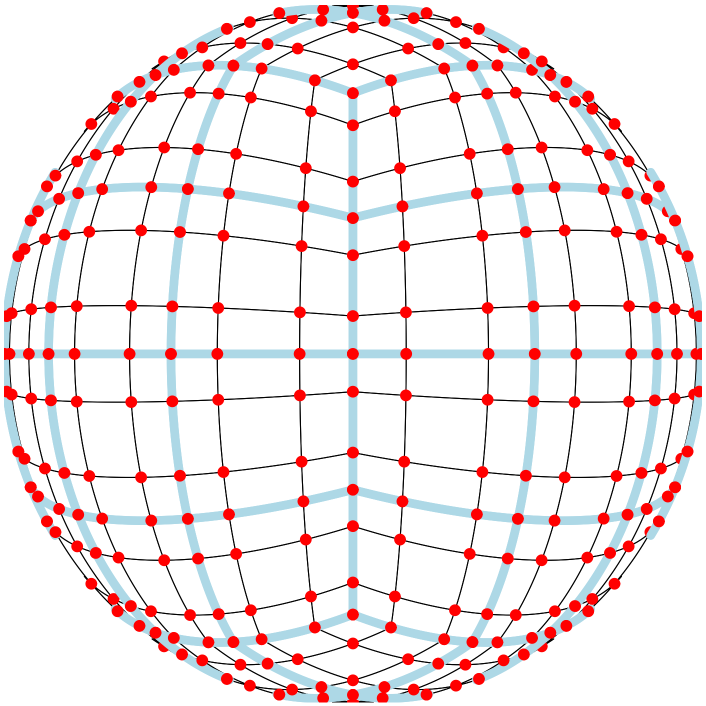
FV based remapping to coupler req's control volumes

Issue 2: No formal definition of a control volume.

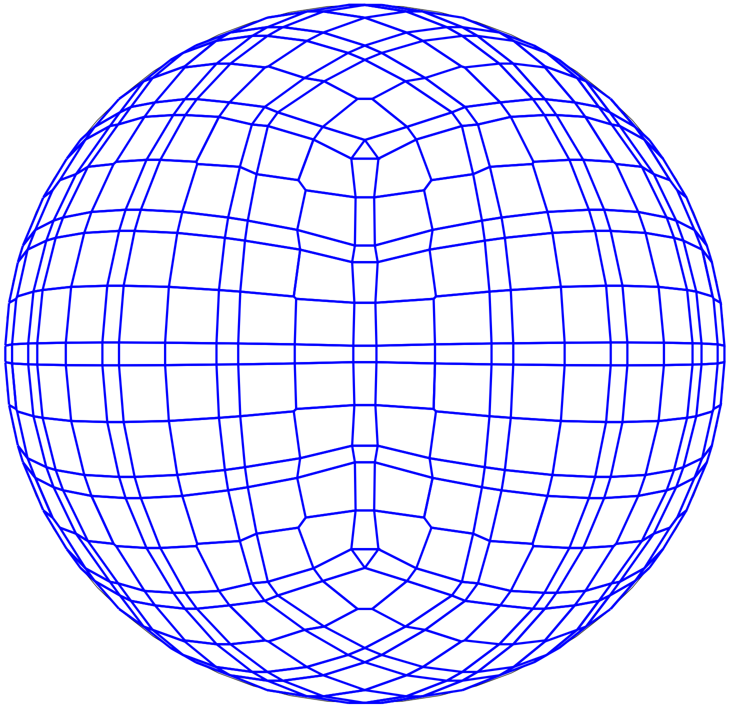
Control Volumes in CAM-SE

"…their spherical areas exactly match the Gaussian weight multiplied by the metric term (these weights are used for integrating the basis functions over the elements and can therefore, in this context, be interpreted as areas)." (Herrington et al. 2018, MWR)

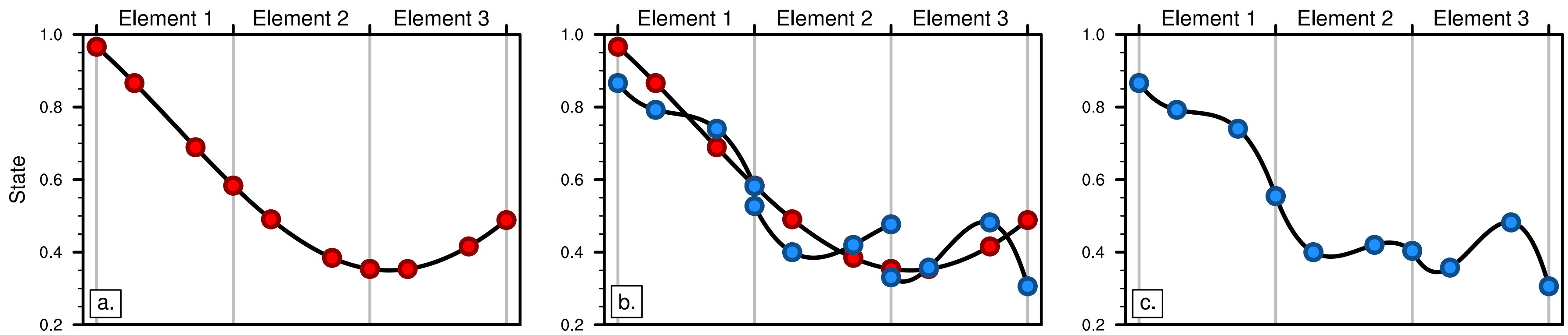
GLL grid



Phys/coupler grid

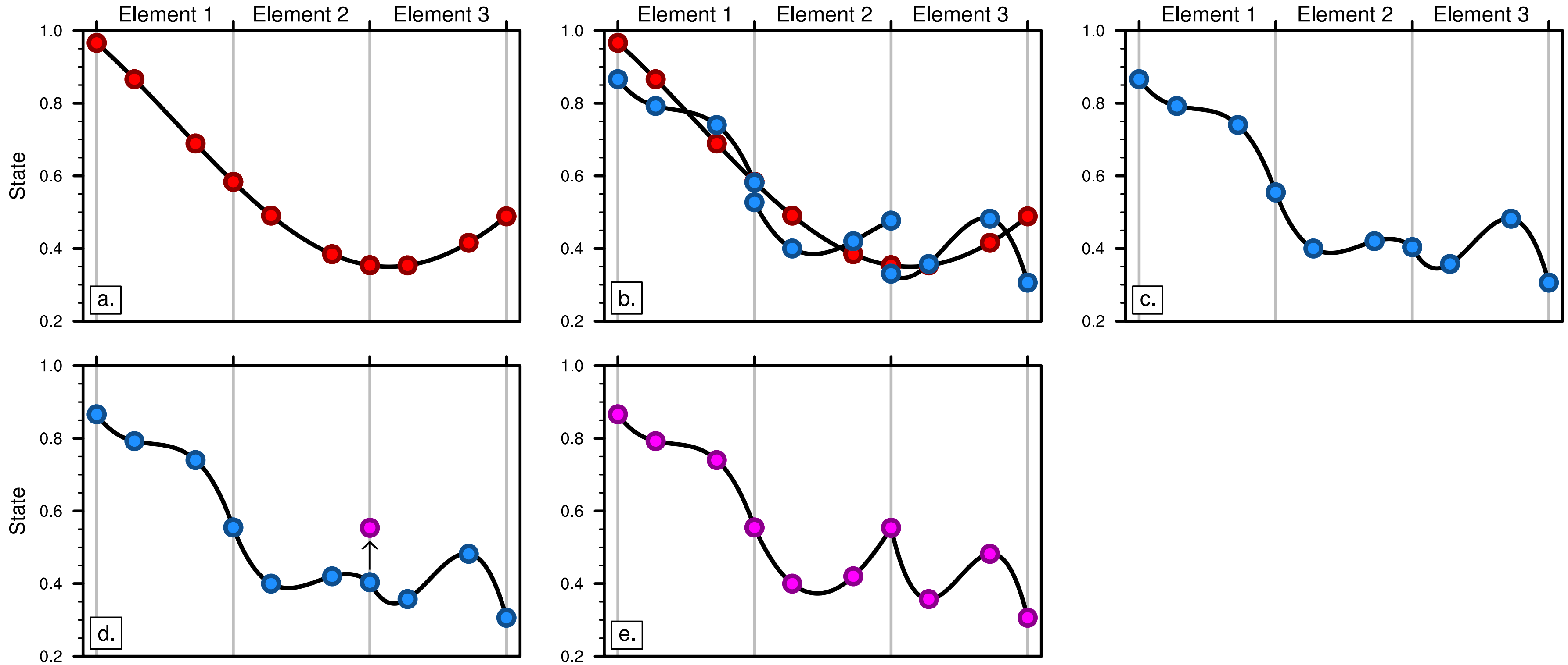


Grid Imprinting in CAM-SE



Issue 3: Physics can exacerbate grid imprinting.

Grid Imprinting in CAM-SE



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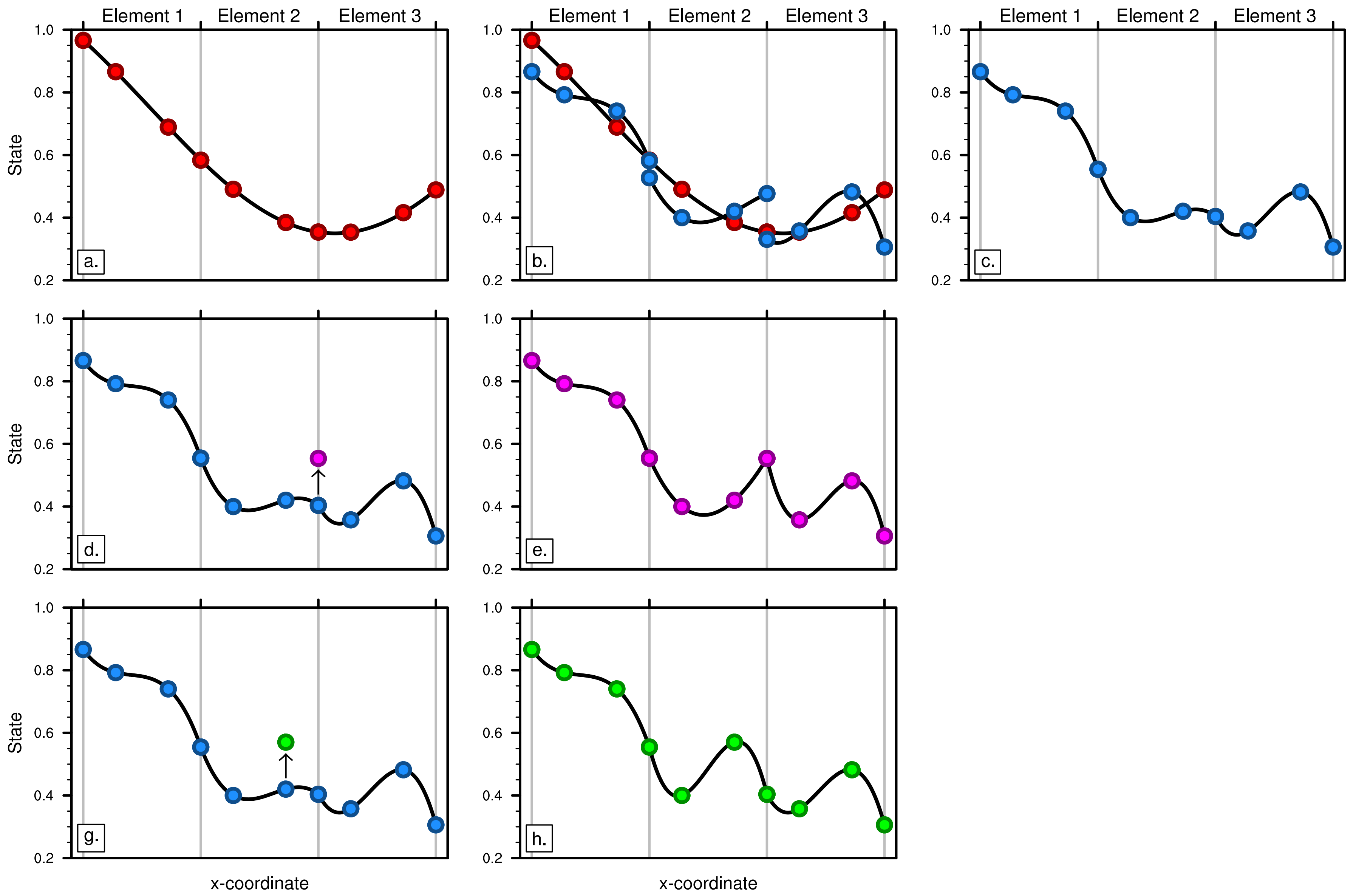
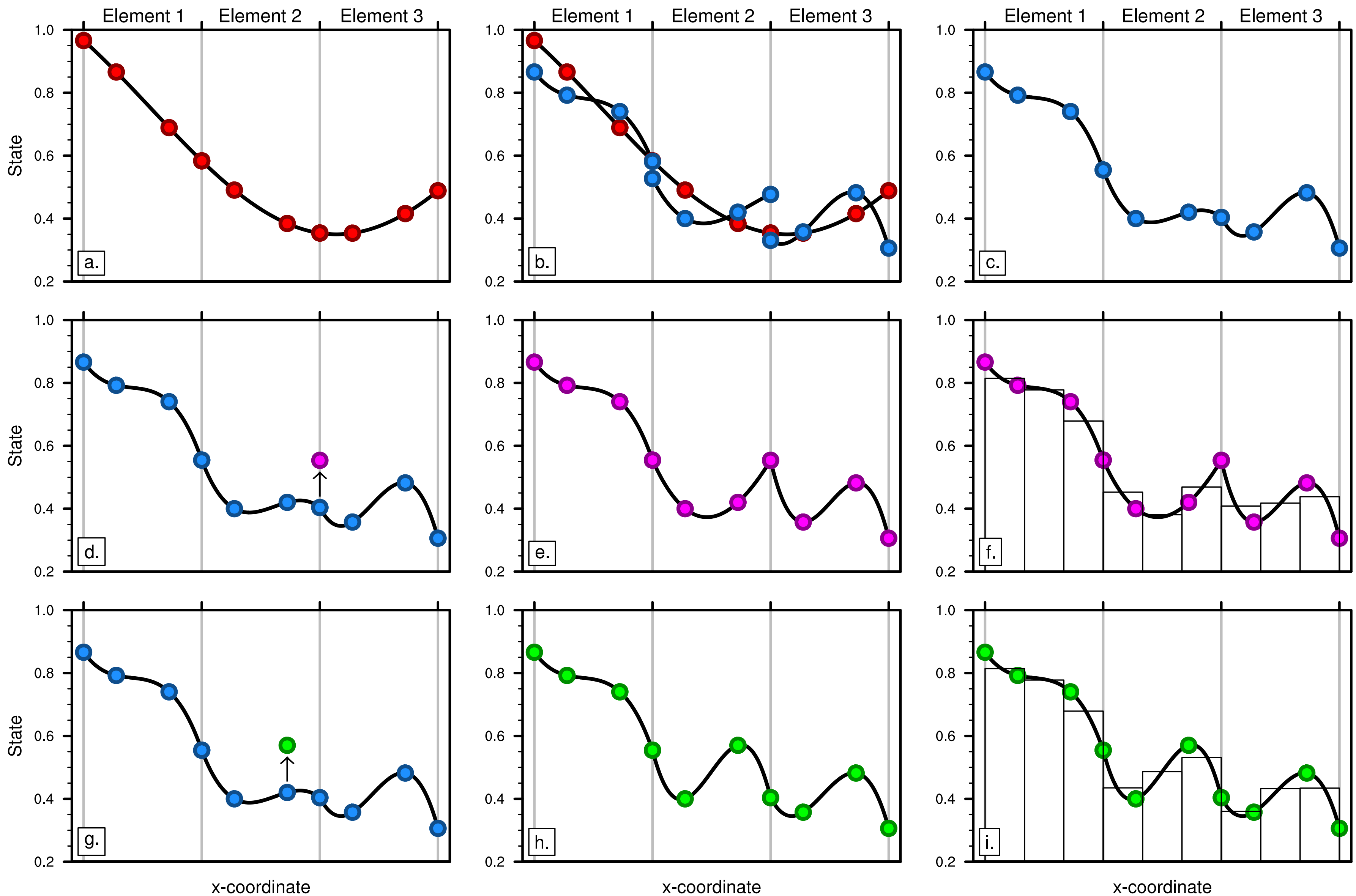
Grid Imprinting in CAM-SE



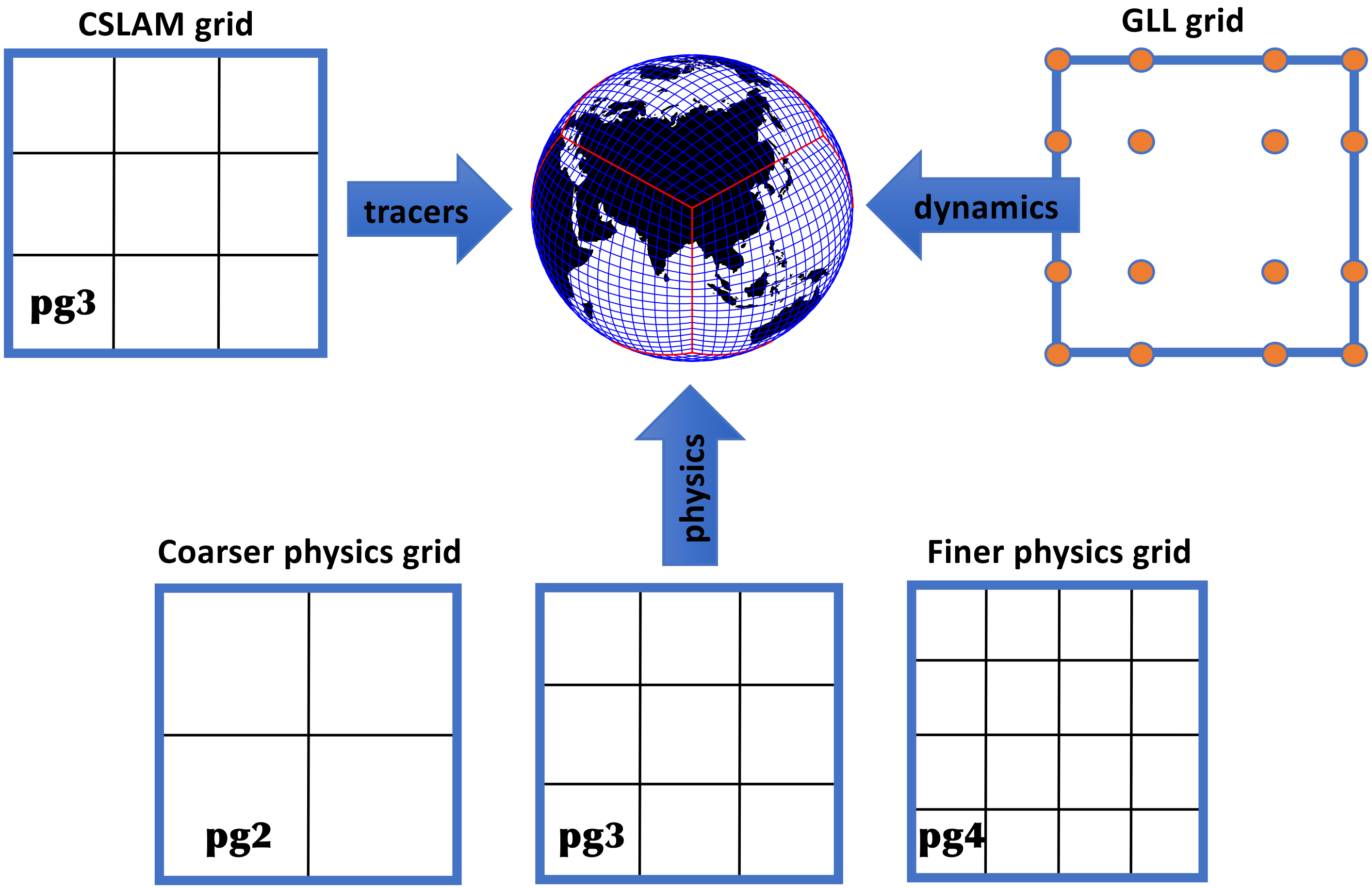
Grid Imprinting in CAM-SE



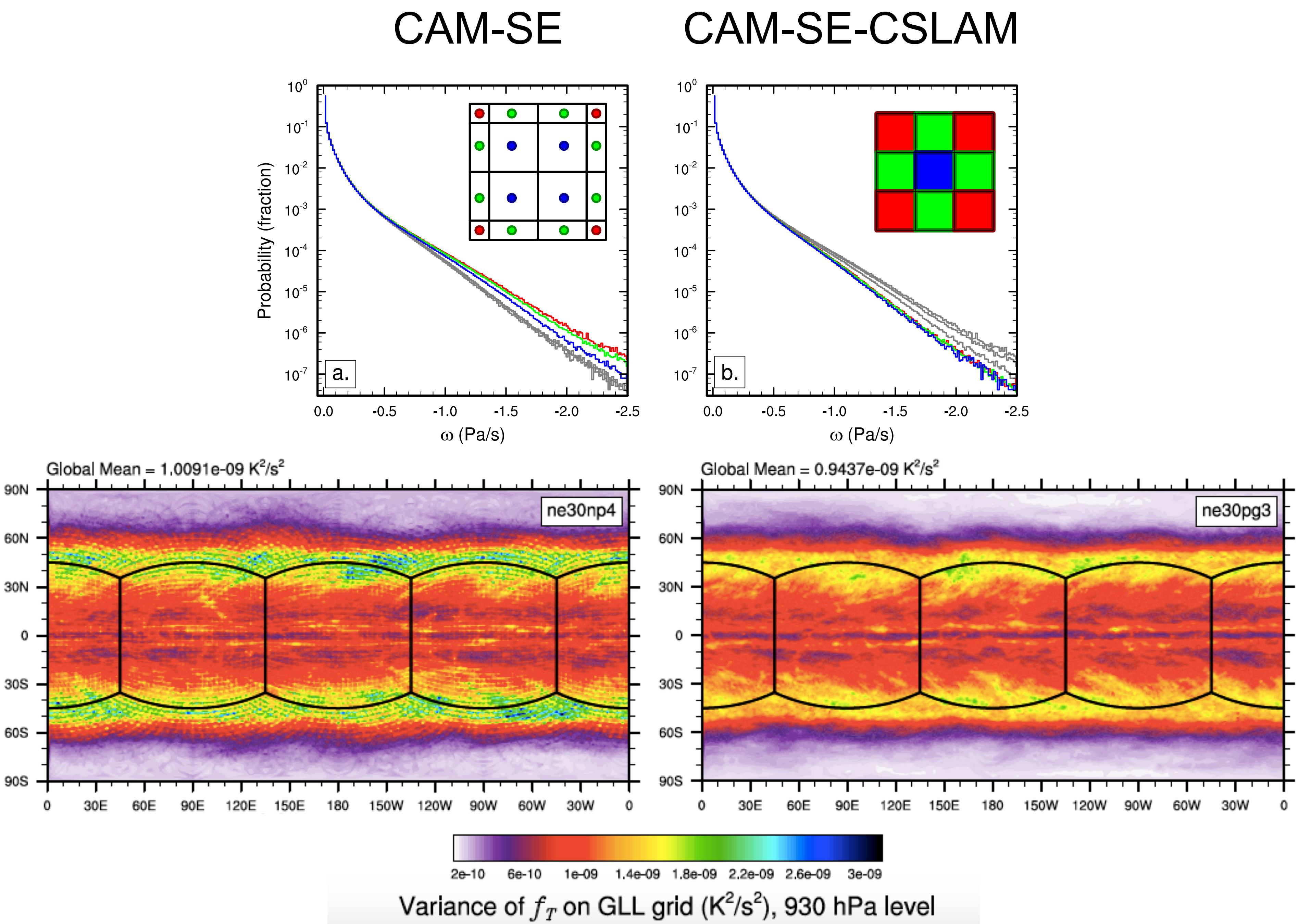
Herrington et al. 2018, MWR

CAM-SE-CSLAM

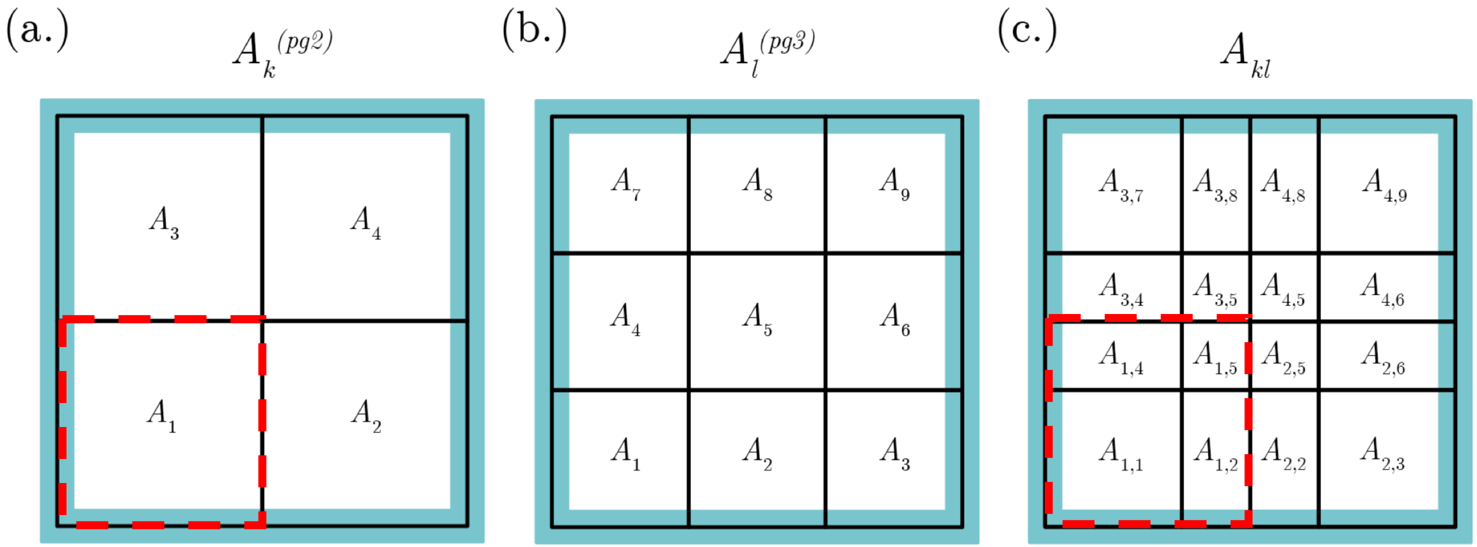
Conservative **S**emi-**La**grangian **M**ulti-tracer



Lauritzen et al. 2017; Herrington et al. 2018; 2019


CAM-SE
CAM-SE-CSLAM
Probability (fraction)
a.
b.
ω (Pa/s)
ω (Pa/s)
Global Mean = 1.0091e-09 K²/s²
Global Mean = 0.9437e-09 K²/s²
ne30np4
ne30pg3
2e-10
6e-10
1e-09
1.4e-09
1.8e-09
2.2e-09
2.6e-09
3e-09
Variance of f_T on GLL grid (K²/s²), 930 hPa level

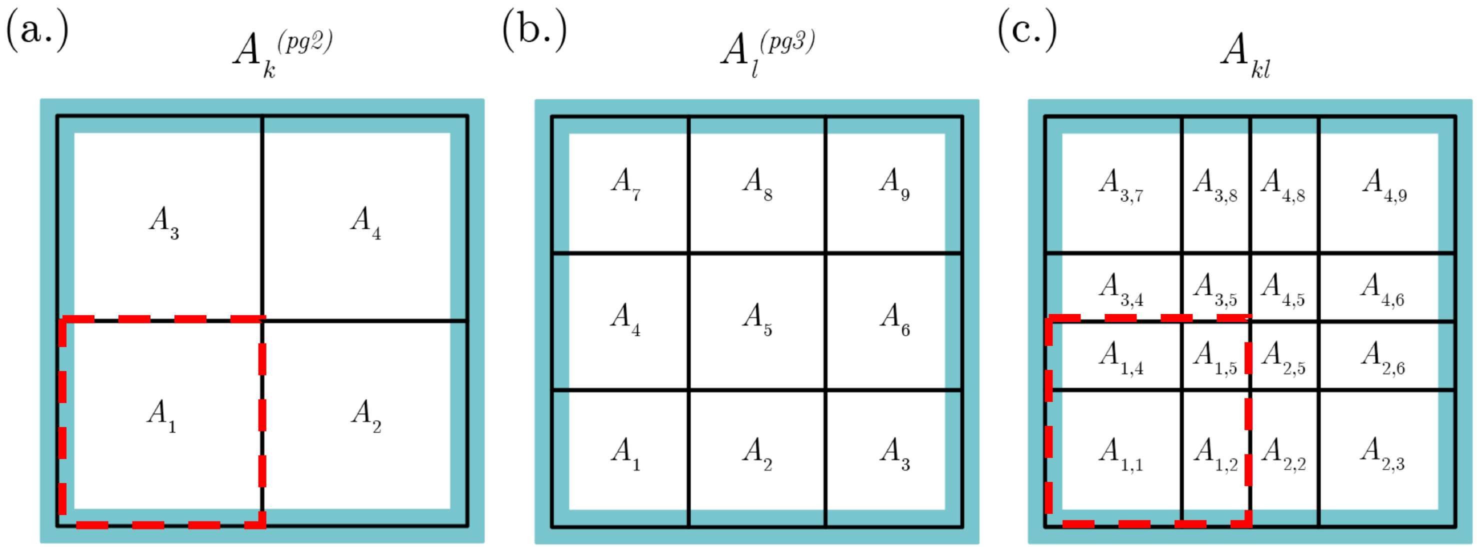
The maps are not reversible



CSLAM -> pg2

$$\overline{m}_k^{(pg2)} = \frac{\overline{m\Delta p}_k^{(pg2)} = \frac{1}{\Delta A_k^{(pg2)}} \sum_{\ell=1}^{nc^2} \langle m\delta p\rangle_{k\ell}\,,}{\overline{\Delta p}_k^{(pg2)} = \frac{1}{\Delta A_k^{(pg2)}} \sum_{\ell=1}^{nc^2} \langle \delta p\rangle_{k\ell}\,,}$$

The maps are not reversible



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pg2 -> CSLAM

p-level thickness mapped from pg2 -> pg3

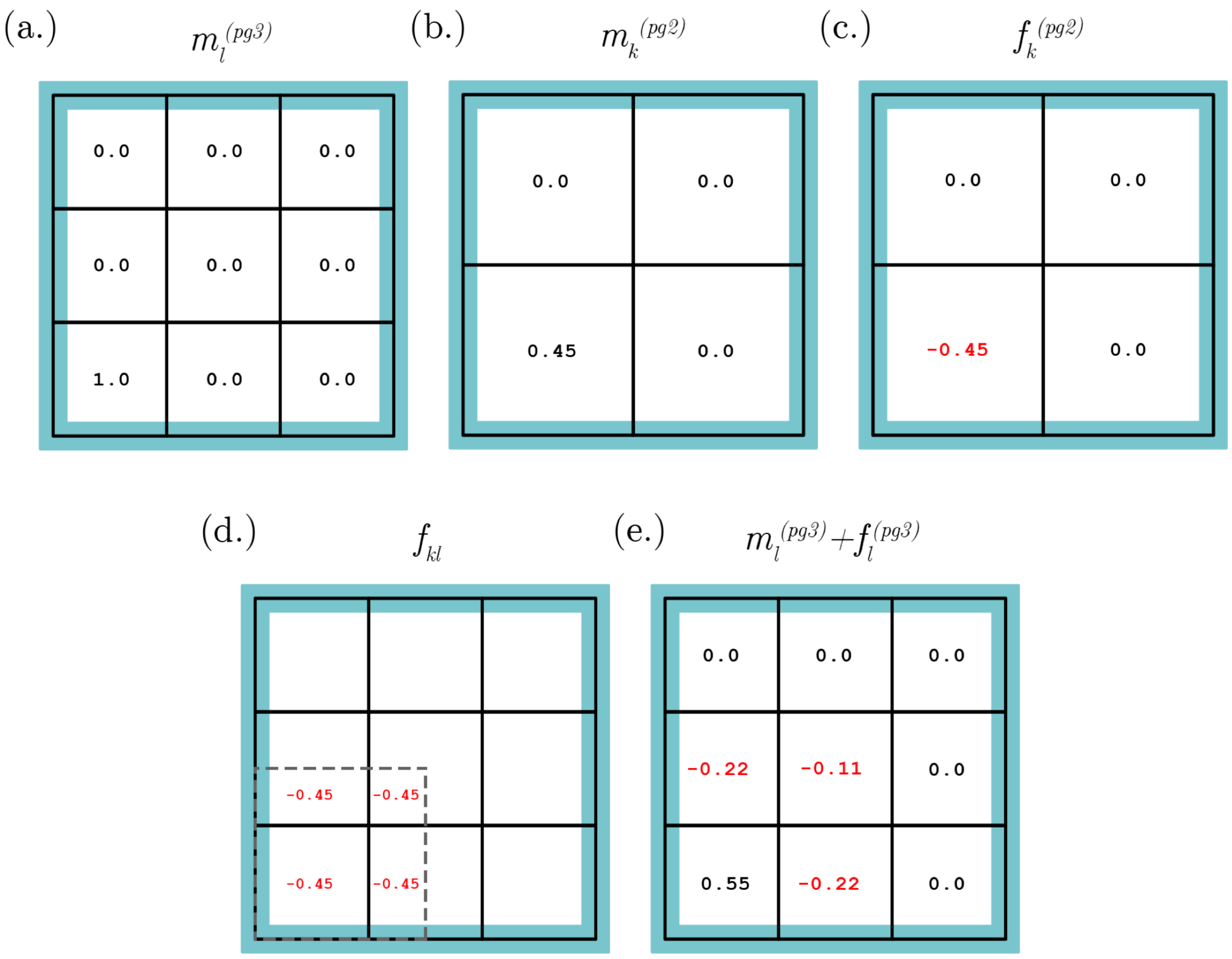
$$\overline{m}_k^{(pg3)} = \frac{\widetilde{\overline{f\Delta p_k}}^{(pg3)}}{\overline{\Delta p_k}^{(pg3)}},$$

$$\widetilde{\overline{\Delta p_k}}^{(pg3)} \neq \overline{\Delta p_k}^{(pg3)},$$

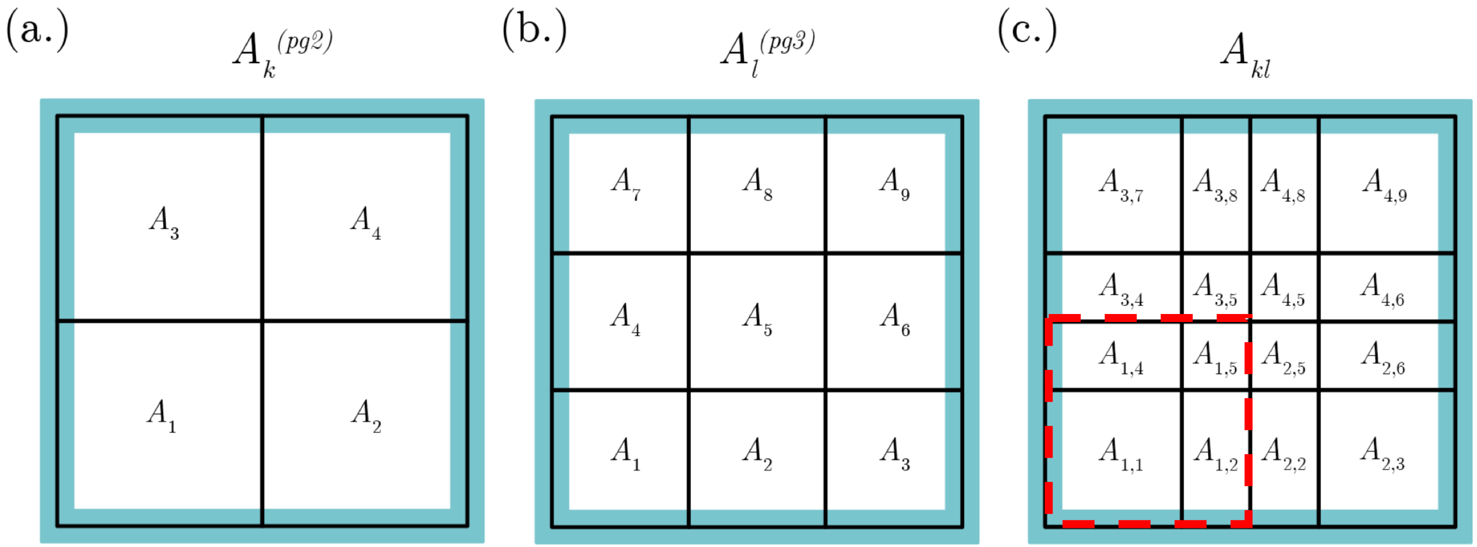
$$\overline{m}_k^{(pg3)} = \frac{\widetilde{\overline{f\Delta p_k}}^{(pg3)}}{\widetilde{\overline{\Delta p_k}}^{(pg3)}},$$

Mapping tracer tend from pg2 to CSLAM

The ‘Negativity Problem’



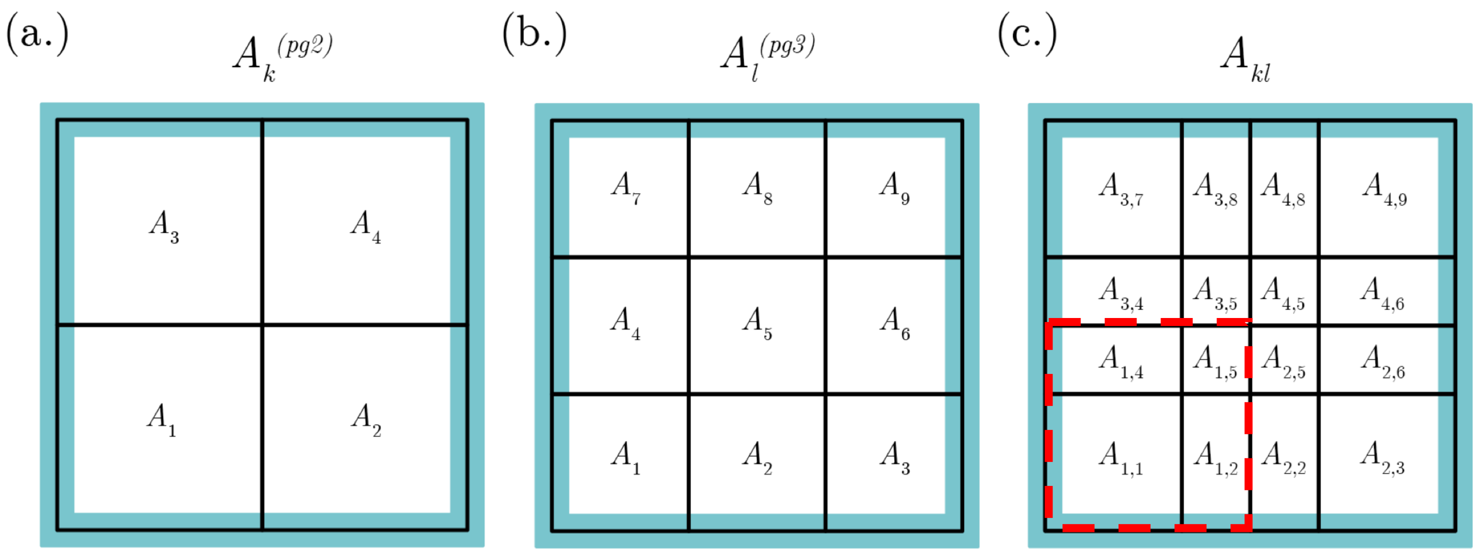
Mapping tracer tend from pg2 to CSLAM



$\Delta m_{k\ell}^{(excess)} = \overline{m}_{k\ell} - \overline{m}_{k}^{(min)}$, excess mixing ratio such that no local minima is produced

Amount of mass that can be removed on overlap grid per $A_k^{(pg2)}$: $\sum_{\ell} \Delta m_{k\ell}^{(excess)} \overline{\Delta p}_{k\ell} \delta A_{k\ell}$.

Mapping tracer tend from pg2 to CSLAM



$\Delta m_{k\ell}^{(excess)} = \overline{m}_{k\ell} - \overline{m}_k^{(min)}$, excess mixing ratio such that no local minima is produced

Amount of mass that can be removed on overlap grid per $A_k^{(pg2)}$: $\sum_\ell \Delta m_{k\ell}^{(excess)} \overline{\Delta p}_{k\ell} \delta A_{k\ell}$.

To ensure the mass removed by physics does not exceed this amount, solve for γ_k:

$$\Delta A_k^{(pg2)} \overline{\Delta p}_k^{(pg2)} \overline{f}^{(pg2)} = \gamma_k \sum_\ell \Delta m_{k\ell}^{(excess)} \overline{\Delta p}_{k\ell} \delta A_{k\ell},$$

The physics mass increment on overlap grid: $\gamma_k \Delta m_{k\ell}^{(excess)} \overline{\Delta p}_{k\ell} \delta A_{k\ell}$,

Mapping tracer tend from pg2 to CSLAM

In an aqua-planet simulation, mass leaks of water vapor improve from 10^{-7} to 10^{-16} Pa per time-step (i.e., within machine-precision)

errors computed after Lauritzen and Williamson (2019)

$\Delta m_{k\ell}^{(excess)} = \overline{m}_{k\ell} - \overline{m}_k^{(min)}$, excess mixing ratio such that no local minima is produced

Amount of mass that can be removed on overlap grid per $A_k^{(pg2)}$: $\sum_\ell \Delta m_{k\ell}^{(excess)} \overline{\Delta p}_{k\ell} \delta A_{k\ell}$.

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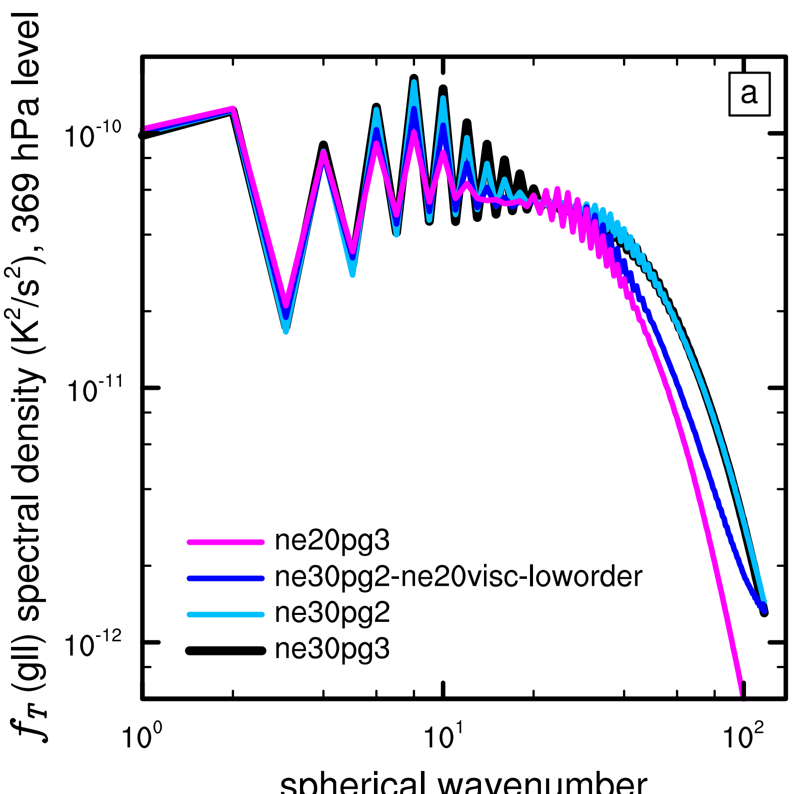
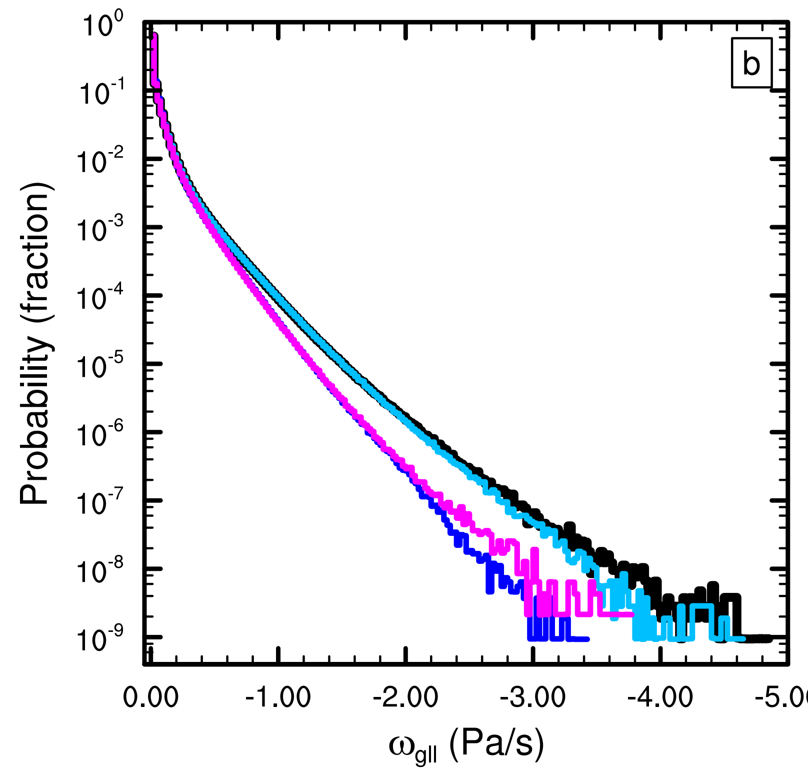
The physics mass increment on overlap grid: $\gamma_k \Delta m_{k\ell}^{(excess)} \overline{\Delta p}_{k\ell} \delta A_{k\ell}$,

Herrington et al, 2019, JAMES

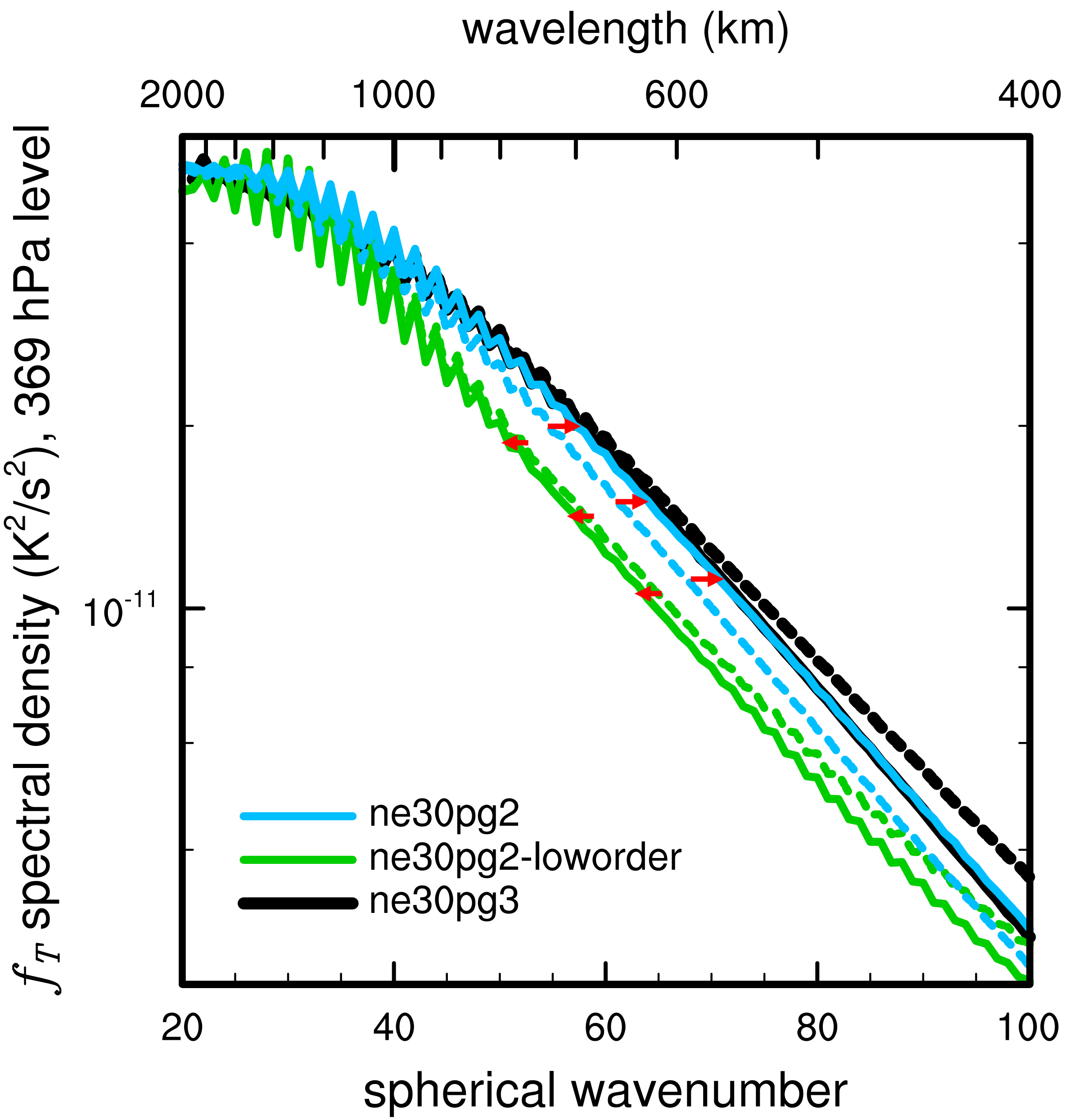
ne20pg3 v. ne30pg2 v. ne30pg3

Grid name	Δx_{dyn}	Δt_{dyn}	Δx_{phys}	Δt_{phys}
ne20pg3	166.8km	300s	166.8km	1800s
ne30pg2	111.2km	300s	166.8km	1800s
ne30pg3	111.2km	300s	111.2km	1800s





Mapping phys tend from pgX to GLL grid

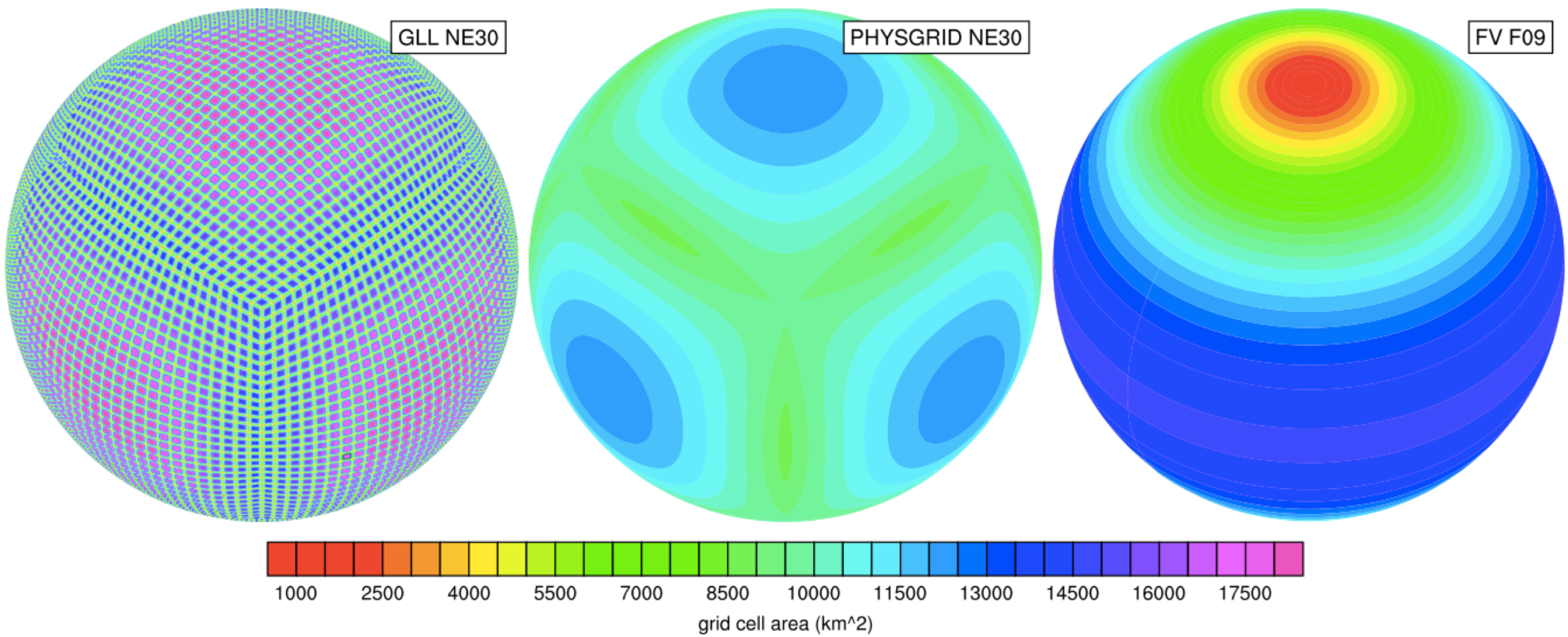


Dotted Lines
– Phys tend on pgX grid

Solid Lines
– Phys tend on GLL grid

Conclusions

- **Problems with PDC method of evaluating physics on GLL grid**
  - Physics requires ‘large-scale state’ and nodal point values are not representative of the state in it’s vicinity.
  - Evaluating physics at element boundaries exacerbates grids imprinting.
- **FV based remapping to coupler requires control volumes but there is no formal definition of a control volume**
  - CAM-SE control volumes are not volume mean state.
- **A separate finite-volume physics grid is a solution to these problems**
  - Mapping between grids still preserves important design aspects (mass conservation, consistency, preserves shape and linear correlations)
  - Through careful consideration, a lower resolution physics grid can preserve tracer mass, and will not alias onto the resolved scales of motion.



GLL NE30
PHYSGRID NE30
FV F09
1000
2500
4000
5500
7000
8500
10000
11500
13000
14500
16000
17500
grid cell area (km^2)

Forcing Spectra