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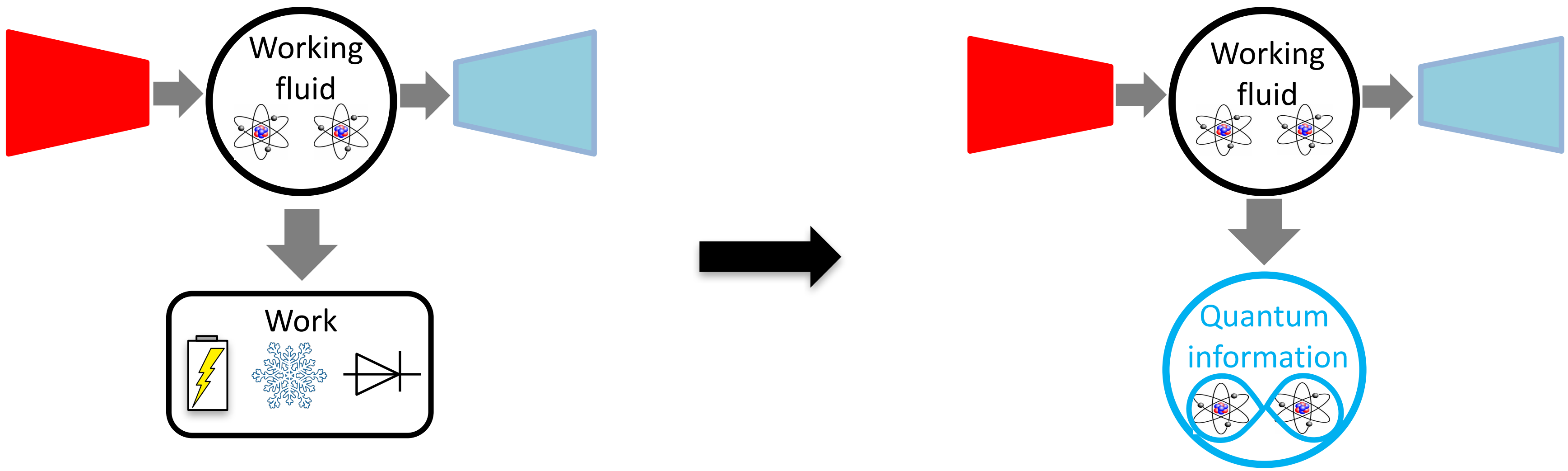
Autonomous nanoscale entanglement engines

Géraldine Haack

Junior group leader, Swiss Prima starting grant



The reset master equation: thermodynamics and open questions

BIRS workshop, Banff, 22.08.2019

“**Charge and Energy Transfer Processes: Open Problems in Open Quantum Systems**”

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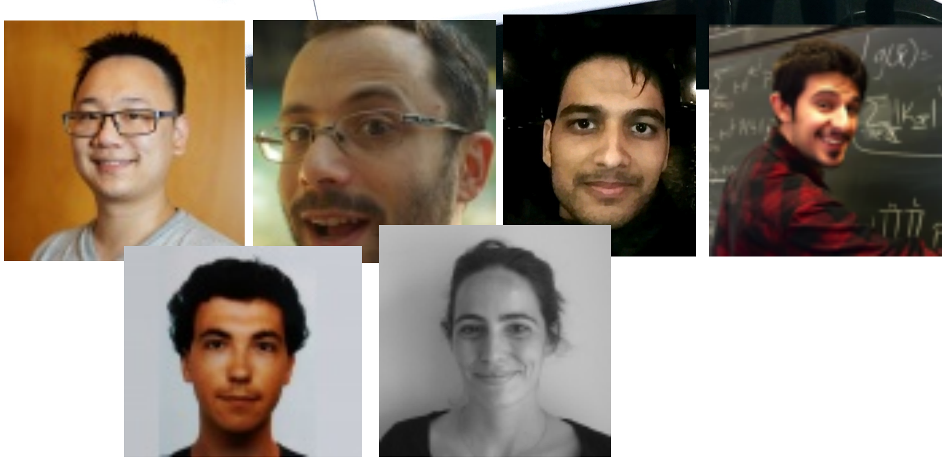
Quantum Correlations Group (Prof. Nicolas Brunner)

Quantum Information Theory & Quantum Thermodynamics



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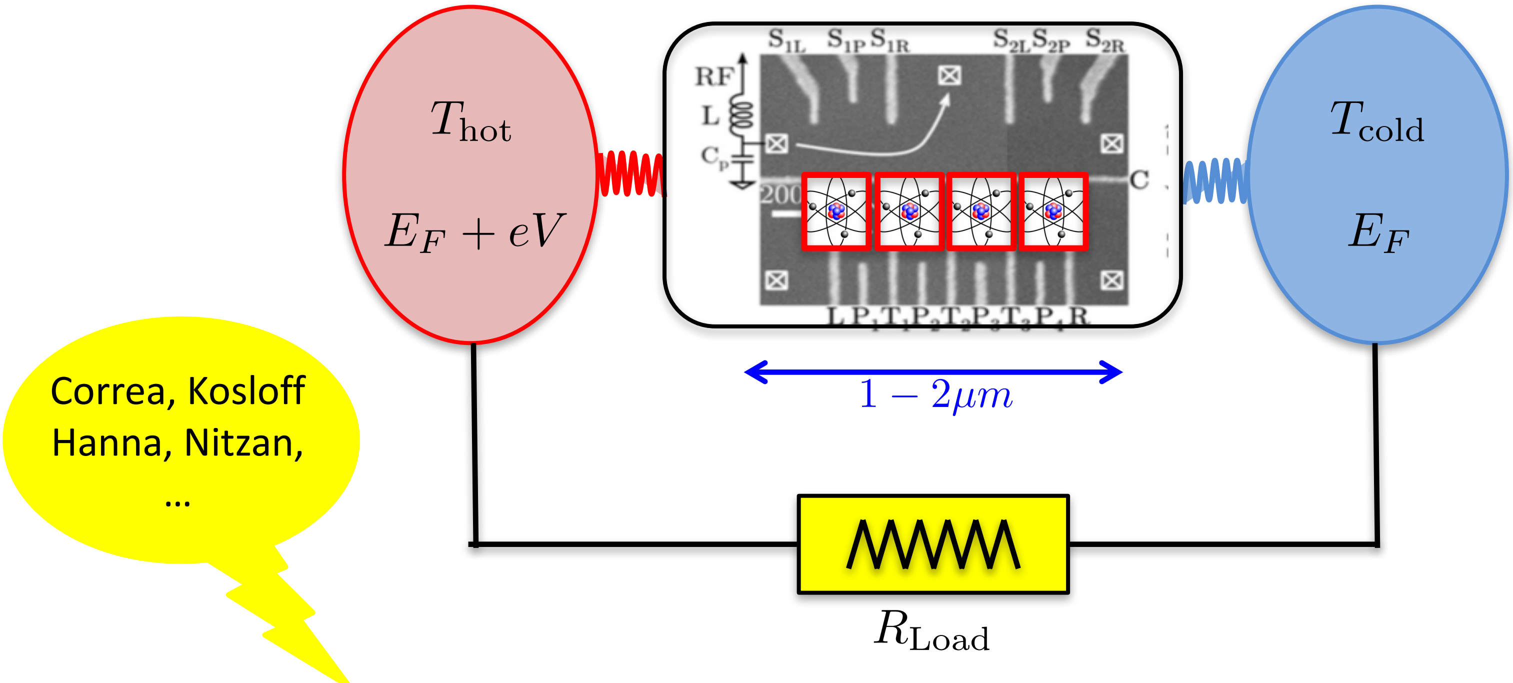
Jonatan B. Brask
(DTU, Denmark)

Ralph Silva (ETH Zürich), Marti Perarnau-Llobet (TU Munich), Patrick P. Potts (Lund Sweden)

Exploiting quantum properties of the working fluid



Typical outputs : Power (heat engine), cooling ratio (fridge)

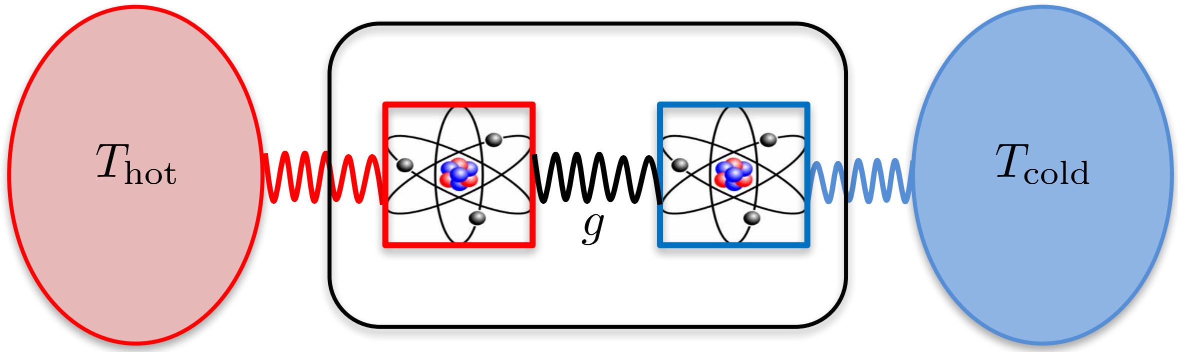
Take advantage of quantum properties to enhance efficiency

Haack, Giazotto, arXiv:1905.12672, Chiaracane et al., arXiv:1908.05139

Can we use thermal machines to generate quantum resources?

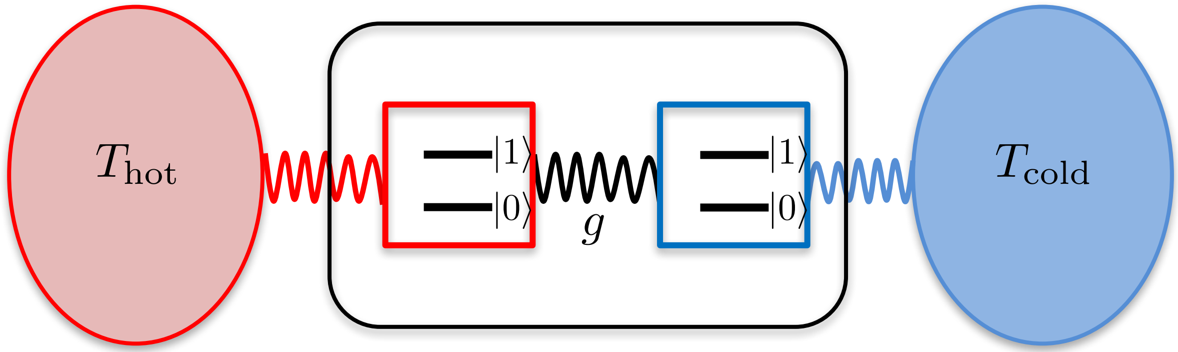
Entanglement? Multipartite entanglement?

Steady-state entanglement quantum engine



Brask, Haack, Brunner, Huber, NJP 17 (2015)

Steady-state entanglement quantum engine



Brask, Haack, Brunner, Huber, NJP 17 (2015)

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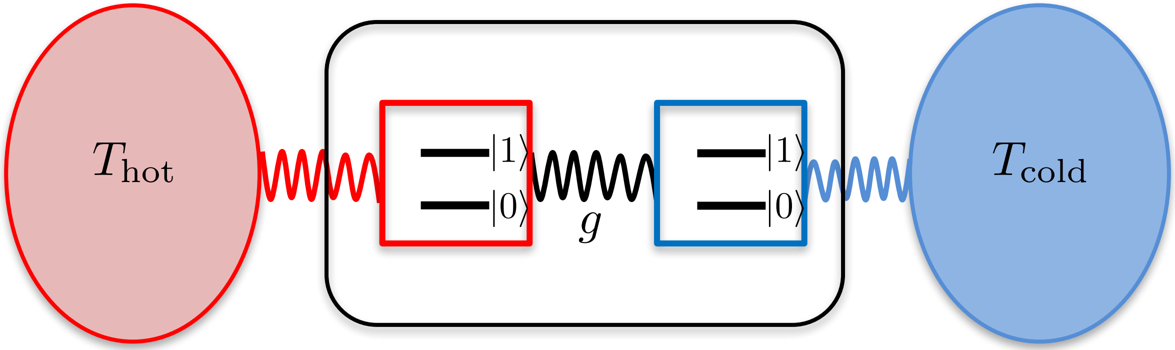
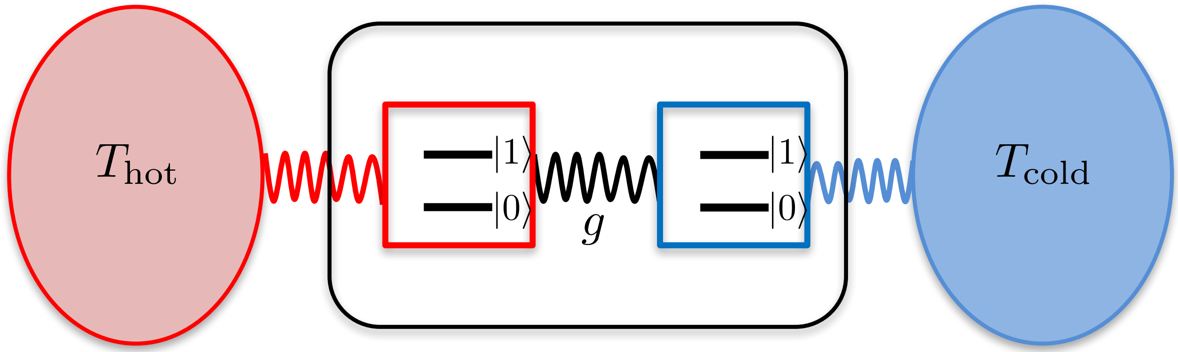


$$H_s = E\big(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2\big)$$

$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

Brask, Haack, Brunner, Huber, NJP 17 (2015)

Steady-state entanglement quantum engine



$$H_s = E\big(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2\big)$$

$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

- Time-independent interaction Hamiltonian, time-independent bath couplings

  ⟶ Thermodynamics: no work, only heat exchange

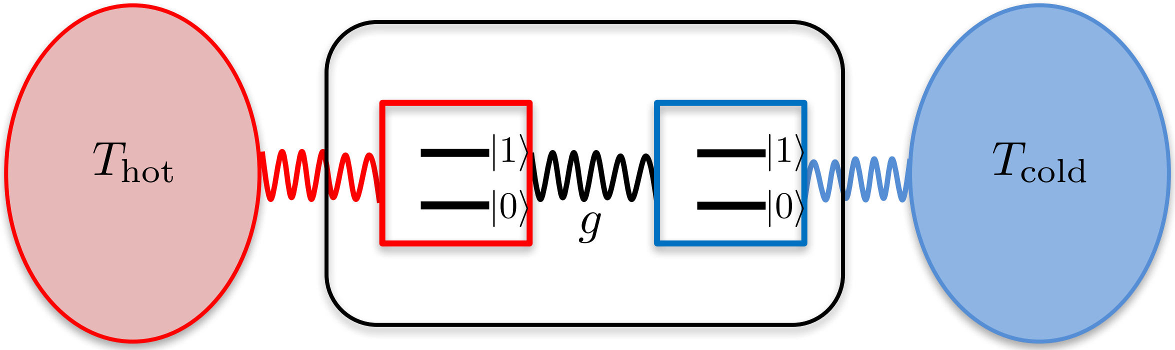
- Autonomous quantum thermal machine

- Ground state is a product state when $g < E$ (weak inter-qubit coupling)

- Solve master equation to obtain the steady-state solution

Brask, Haack, Brunner, Huber, NJP 17 (2015)

Dynamics



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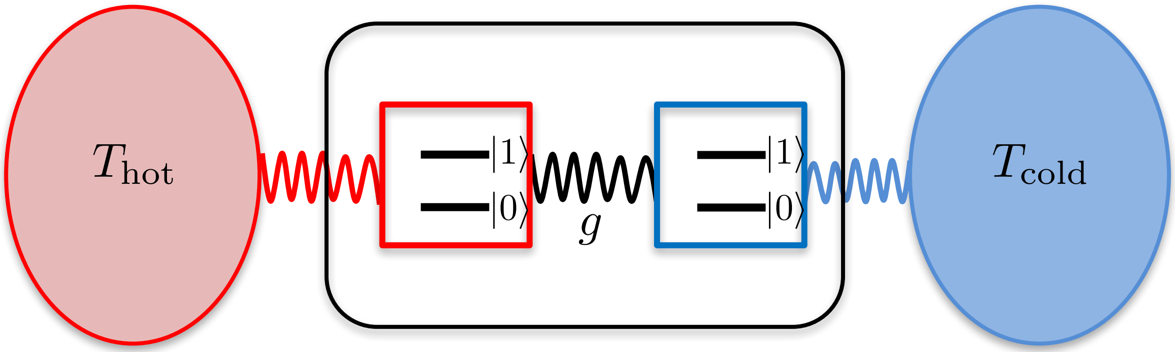
- Probabilistic reset: $\rho(t+dt) = -i[H_s, \rho(t)]\,dt + \gamma dt\,\tau + (1-\gamma dt)\rho(t)$

Thermal state $\tau = r|0\rangle\langle 0| + (1-r)|1\rangle\langle 1|$

Ground state population $r = \dfrac{1}{1+e^{-E/(k_B T)}}$

Dynamics



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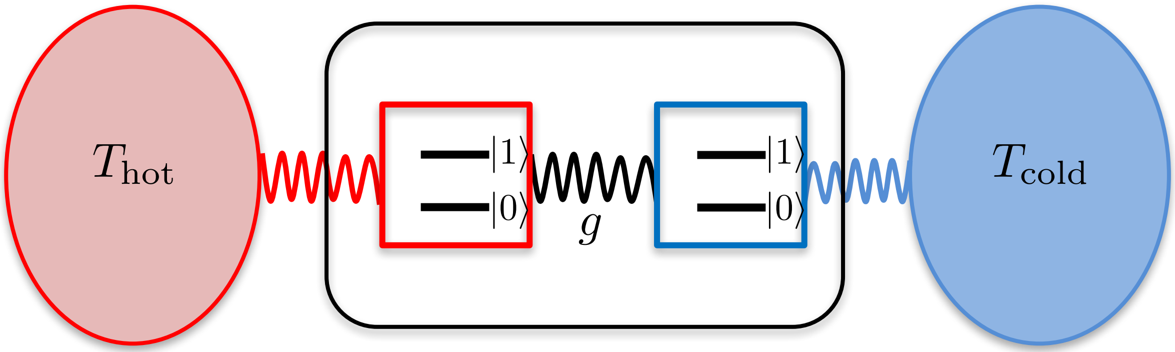
Brask, Haack, Brunner, Huber, NJP 17 (2015)

Dynamics



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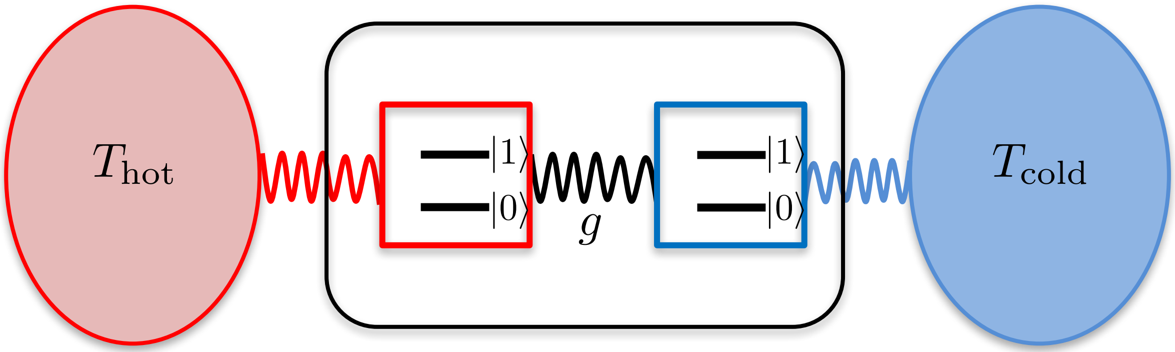
- For two qubits: $\dot{\rho}(t) = -i[H_s + H_{int}, \rho(t)] + \gamma_h\,(\tau_h \otimes \mathrm{Tr}_h \rho(t) - \rho(t)) + \gamma_c\,(\mathrm{Tr}_c \rho(t) \otimes \tau_c - \rho(t))$

Dynamics



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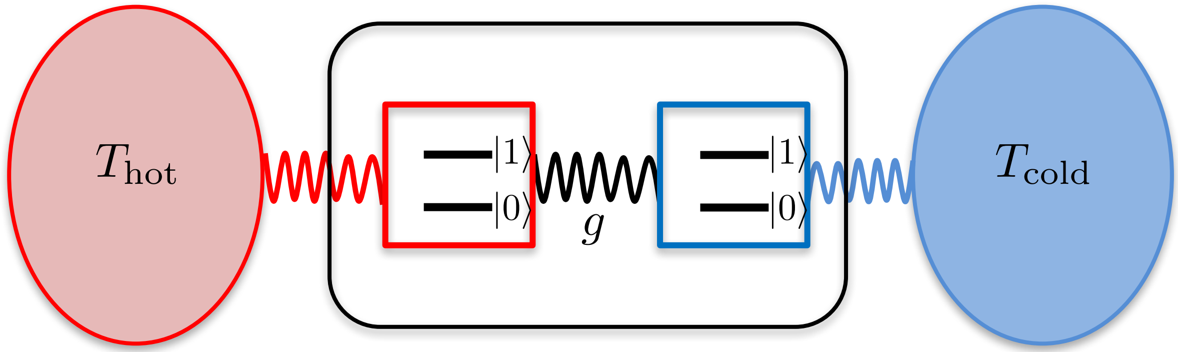
⇨ talk by C. Koch on optimal control for reset

Steady-state entanglement quantum engine



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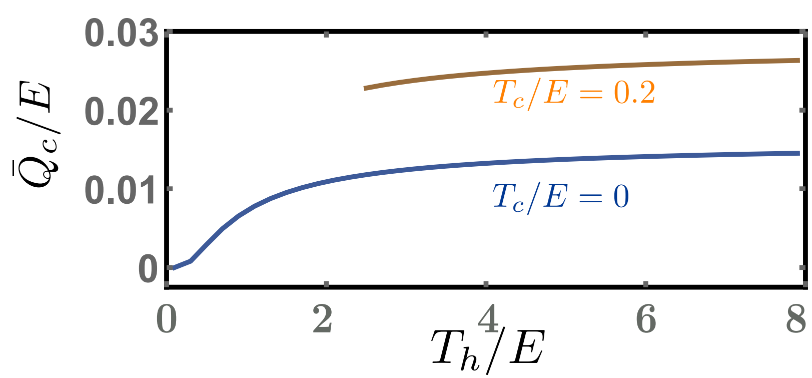
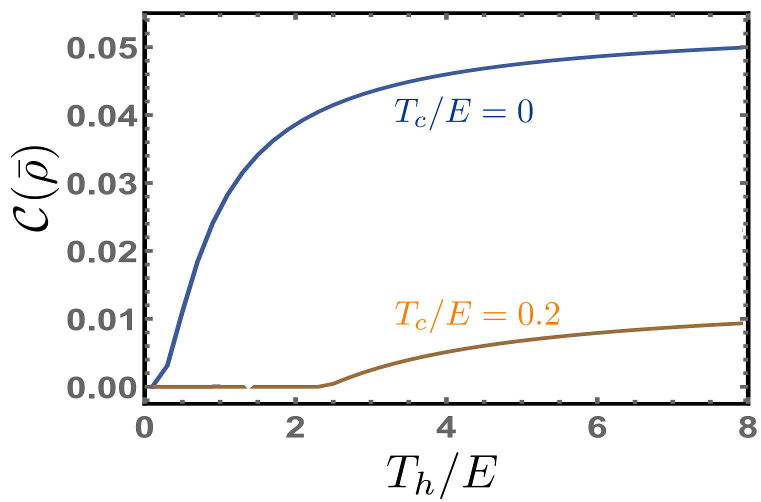
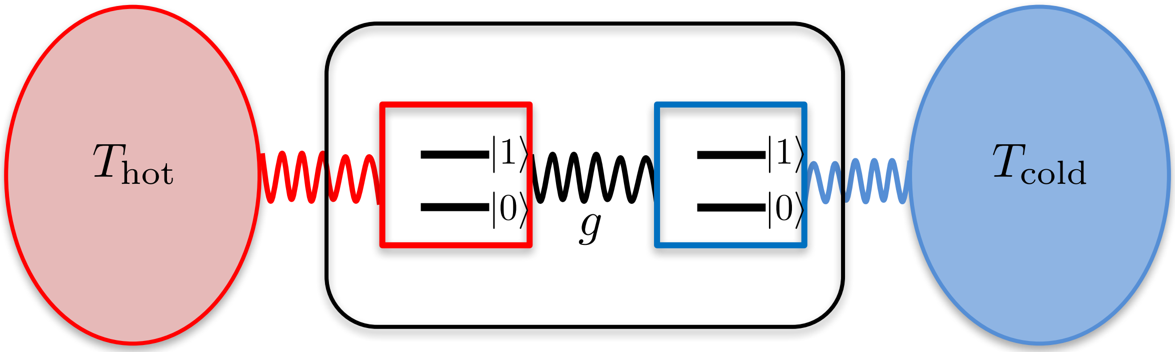




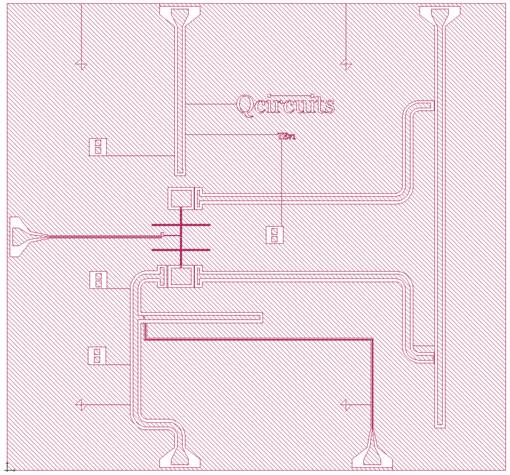
Steady-state heat flow sustains the generation of entanglement

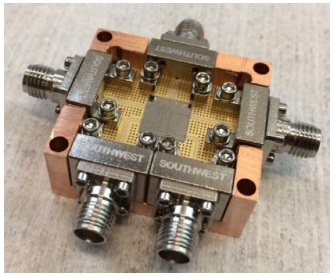
Brask, Haack, Brunner, Huber, NJP 17 (2015)

Steady-state entanglement quantum engine



Circuit QED platform
Huard group (ENS Lyon, France)

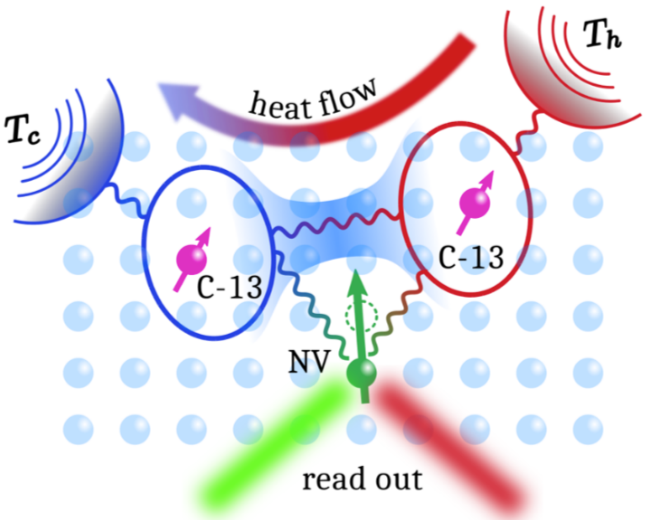




Thermal baths : Spectral density of current noise

$$\propto E\,\mathrm{Re}[Y(E)]\frac{1}{1-e^{-E/(k_B T)}}$$

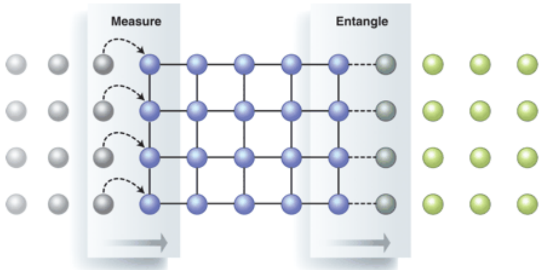
NV centres
Houck and Berg-Sorensen (DTU, Denmark)



(Proposal)

Multipartite entanglement?

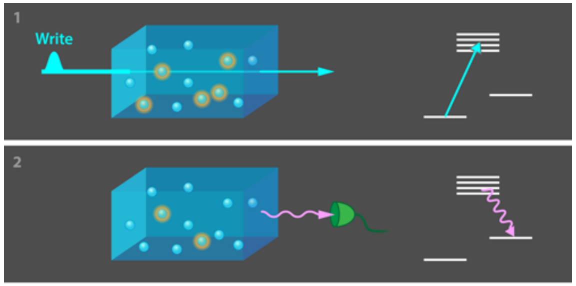
Cluster states



O'Brien, Science 318 (2007)

Quantum Computing

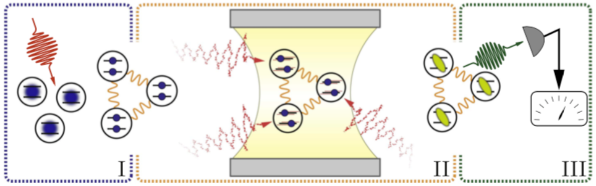
Dicke states / W-states



Nunn, Physics 10 (2017)

Quantum memories

GHZ states



Haase *et al.*, NJP 20 (2018)

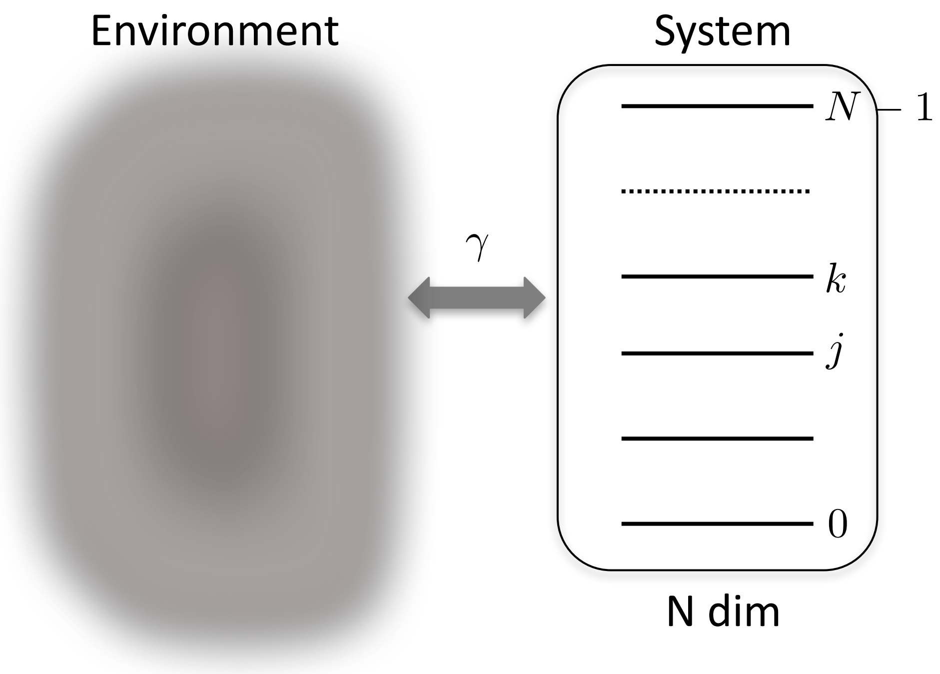
Quantum metrology

Which quantum states can be generated via an autonomous thermal machine?

Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)
Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

The reset evolution equation

Open questions on open quantum systems



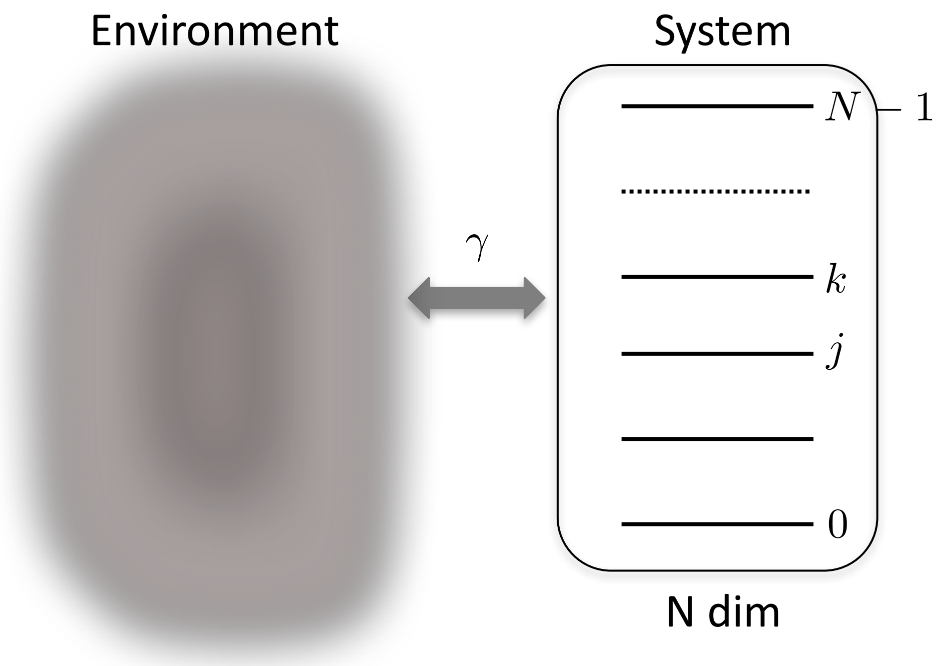
$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\left(\tau - \rho(t)\right)$$

Reset state not specified

$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\left(\sigma - \rho(t)\right)$$

The reset evolution equation

Open questions on open quantum systems



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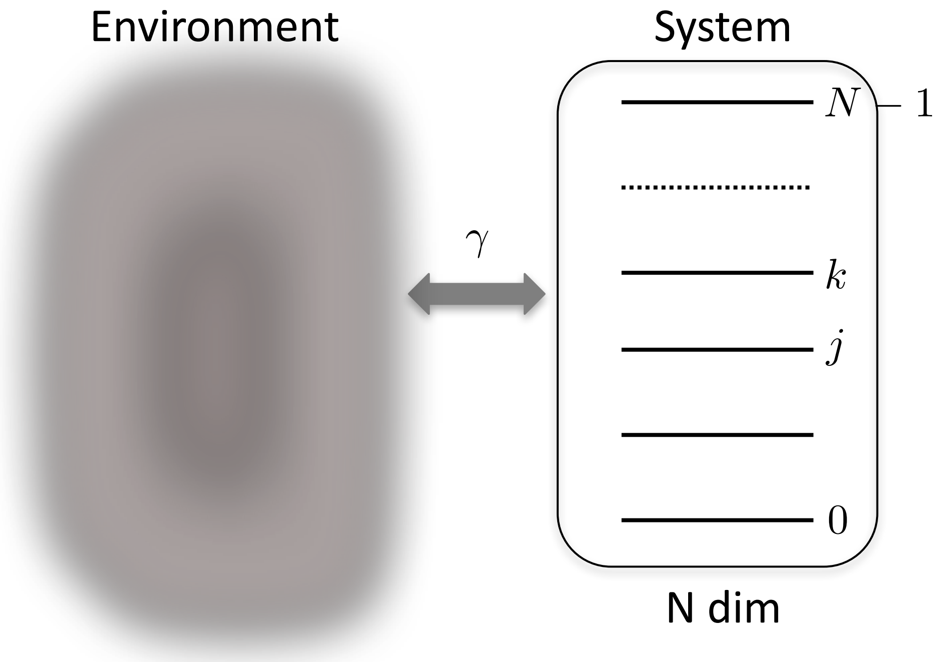
- Linearity ?
- Equivalent to a Lindblad-type ME ?
- Relevant for experiments ?
- Are the laws of thermodynamics valid ? For any reset state ?

The reset evolution equation

Open questions on open quantum systems



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Reset state not specified

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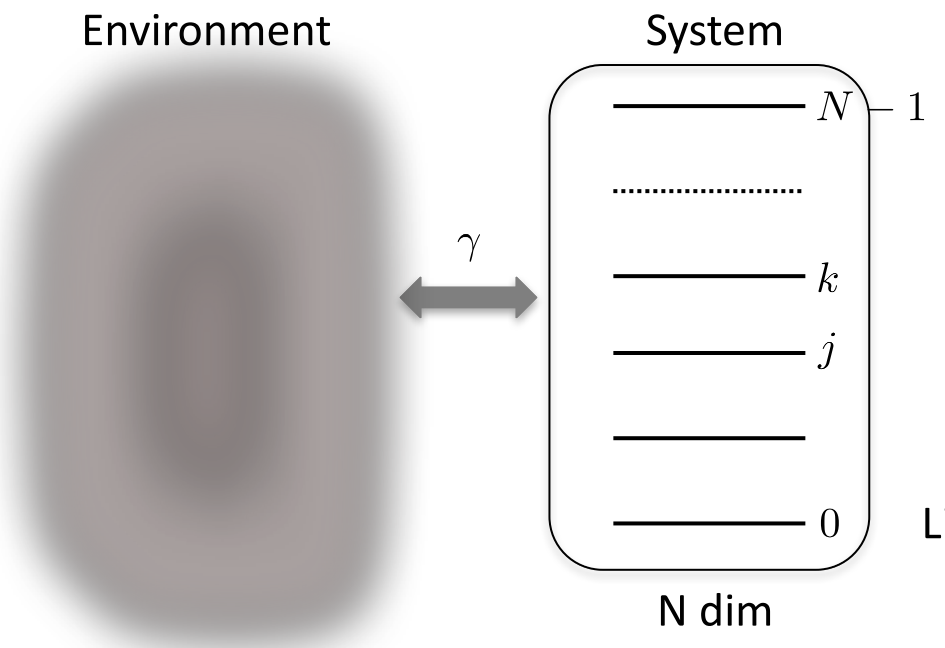
- Linearity ? $\dot{\rho}(t) = \gamma\,(\sigma\,\mathrm{Tr}[\rho(t)] - \rho(t))$ $\longrightarrow$ $\frac{d}{dt}(a\rho_1 + b\rho_2) = \ldots = a\frac{d\rho_1}{dt} + b\frac{d\rho_2}{dt}$
- Equivalent to a Lindblad-type ME ?
- Relevant for experiments ?
- Are the laws of thermodynamics valid ? For any reset state ?

Equivalent to a Lindblad master equation ?



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Reset state not specified

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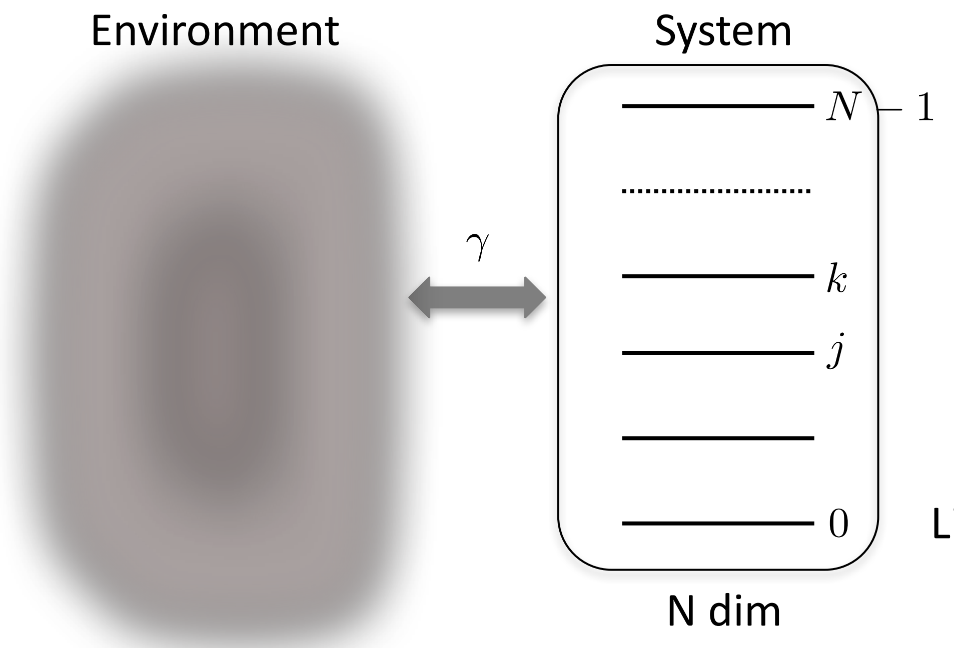
?

Lindblad dissipators: $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$

Equivalent to a Lindblad master equation ?



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?

Lindblad dissipators: $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$

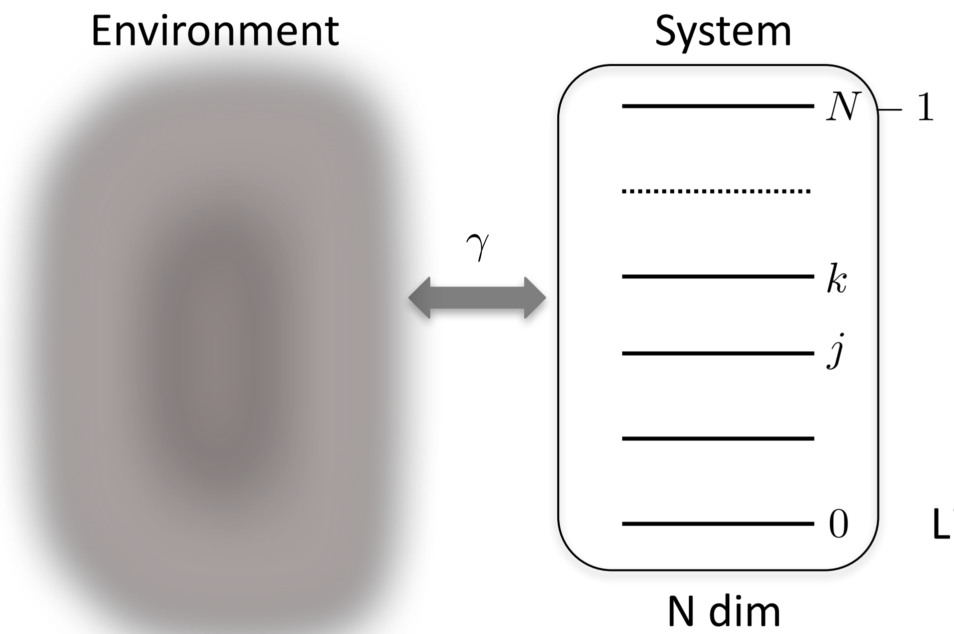
- Rate equations for the populations for Lindblad dissipators

$$\dot{p}_0 = \sum_{j>0}^{N-1}\left(\Gamma^{-}_{0j}p_j - \Gamma^{+}_{oj}p_0\right)$$

$$\dot{p}_{N-1} = \sum_{j=0}^{N-2}\left(\Gamma^{+}_{jN-1}p_j - \Gamma^{-}_{jN-1}p_{N-1}\right)$$

$$\dot{p}_k = -\left(\sum_{j=0}^{k-1}\bar{\Gamma}_{jk} + \sum_{j=k+1}^{N-1}\Gamma_{kj}\right)p_k + \left(\sum_{j=0}^{k-1}\Gamma_{jk} + \sum_{j=k+1}^{N-1}\bar{\Gamma}_{kj}\right)p_j \quad \forall k = 0, \ldots, N-1$$

Equivalent to a Lindblad master equation ?



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?

Lindblad dissipators: $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$

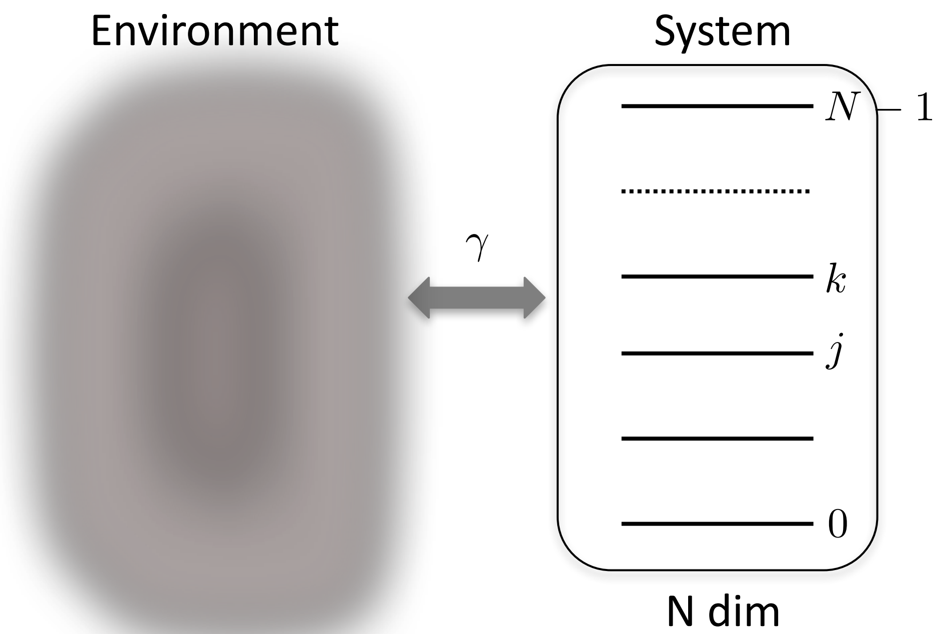
- Lindblad

$$\dot{p}_k = -\left(\sum_{j<k}\Gamma^-_{jk} + \sum_{j>k}\Gamma^+_{kj}\right) p_k + \left(\sum_{j<k}\Gamma^+_{jk} + \sum_{j>k}\Gamma^-_{kj}\right) p_j$$

- Reset

$$\begin{aligned}\dot{p}_k &= \gamma\,(\sigma_{kk} - p_k) \\ &= \gamma\left(\sigma_{kk}\sum_{j=0}^{N-1} p_j - p_k\right) \\ &= \gamma\,(\sigma_{kk} - 1)\,p_k + \gamma\,\sigma_{kk}\left(\sum_{j<k} p_j + \sum_{j>k} p_j\right)\end{aligned}$$

Equivalent to a Lindblad master equation ?



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Reset state not specified

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Identification for the populations:

$j < k$	$j > k$
$\gamma\sigma_{kk} = \Gamma^{+}_{jk}$	$\gamma\sigma_{jj} = \Gamma^{+}_{kj}$
$\gamma\sigma_{jj} = \Gamma^{-}_{jk}$	$\gamma\sigma_{kk} = \Gamma^{-}_{kj}$

- Lindblad

$$\dot{p}_k = -\left(\sum_{j<k}\Gamma^{-}_{jk} + \sum_{j>k}\Gamma^{+}_{kj}\right)p_k + \left(\sum_{j<k}\Gamma^{+}_{jk} + \sum_{j>k}\Gamma^{-}_{kj}\right)p_j$$

- Reset

$$\begin{aligned}\dot{p}_k &= \gamma\,(\sigma_{kk} - p_k)\\ &= \gamma\left(\sigma_{kk}\sum_{j=0}^{N-1}p_j - p_k\right)\\ &= \gamma\,(\sigma_{kk} - 1)\,p_k + \gamma\,\sigma_{kk}\left(\sum_{j<k}p_j + \sum_{j>k}p_j\right)\end{aligned}$$

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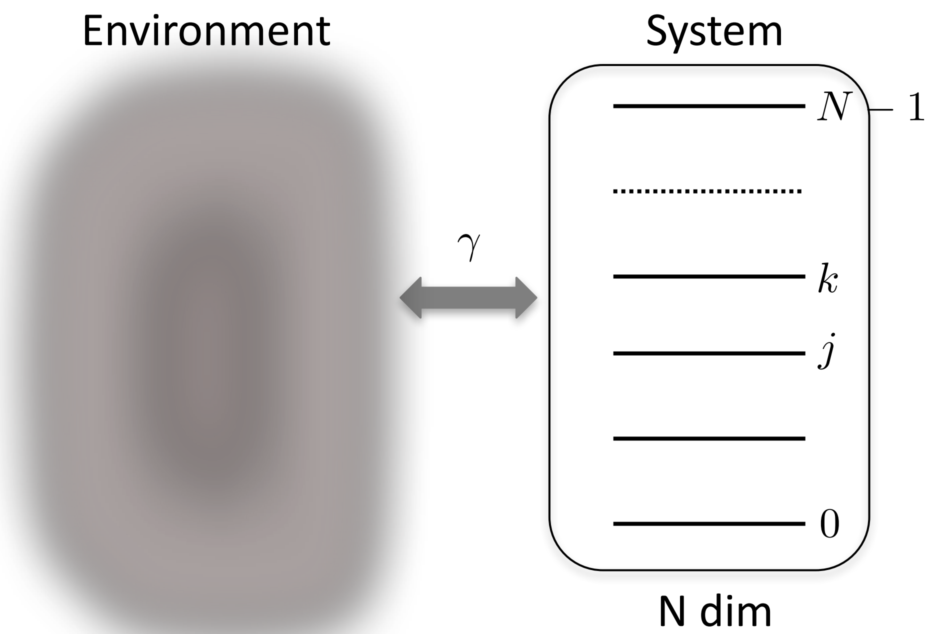

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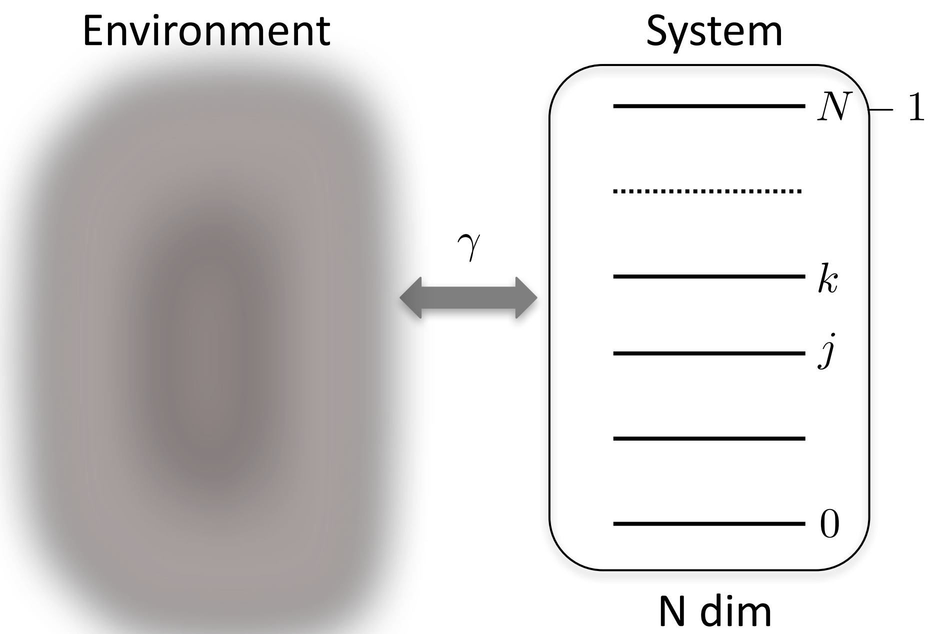
- For the coherences ?

  Reset: $\dot{p}_{jk} = \gamma\Big(\sigma_{jk} - \rho_{jk}\Big)$

  Lindblad : … → Need an additional pure dephasing channel! $\Gamma^{\phi}_{jk}\mathcal{D}[\sigma^{jk}_{z}]\rho$

Equivalent to a Lindblad master equation ?



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$$j < k \qquad j > k$$

- For the coherences ?

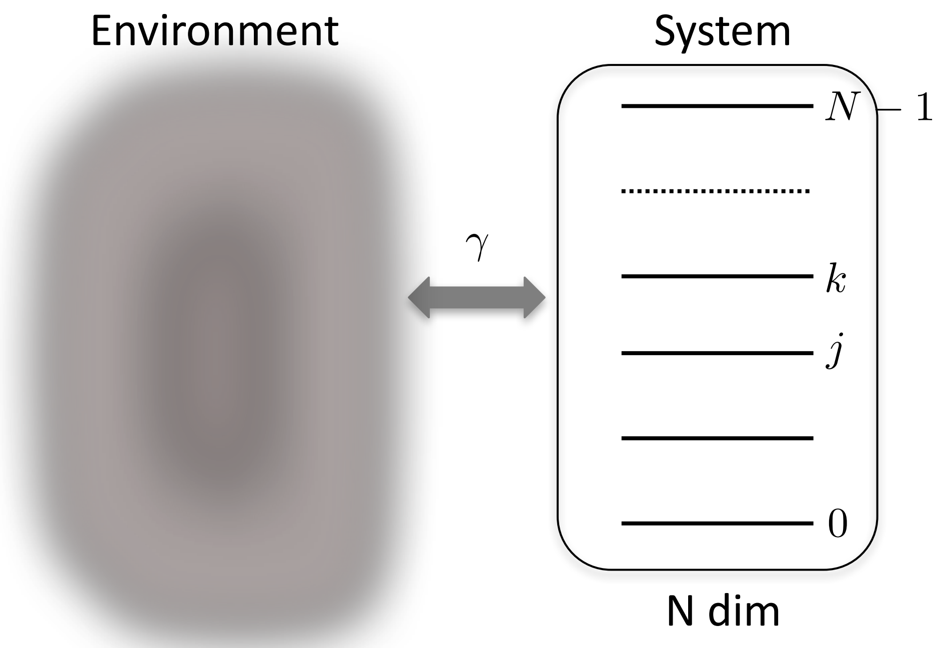
  Reset: $\dot{p}_{jk} = \gamma\Big(\sigma_{jk} - \rho_{jk}\Big)$

  Lindblad : … → Need an additional pure dephasing channel! $\Gamma^{\phi}_{jk}\mathcal{D}[\sigma_z^{jk}]\rho$

Reset ME : Lindblad-type ME + additional dephasing channel

Relevant for experiments!

Local detailed balance



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$\gamma\sigma_{kk} = \Gamma^{+}_{jk}$	$\gamma\sigma_{jj} = \Gamma^{+}_{kj}$
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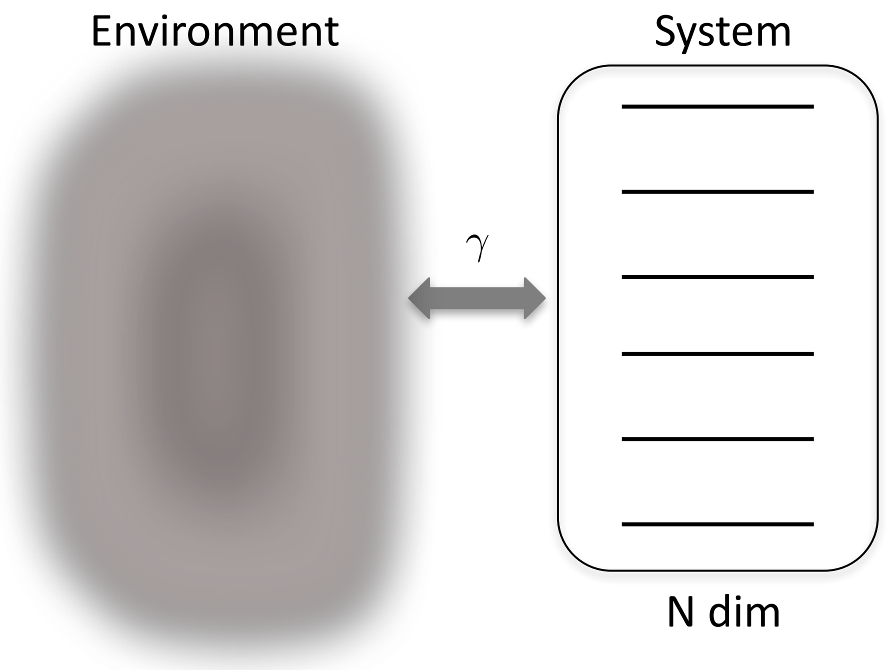
$$\frac{\Gamma^{+}_{jk}}{\Gamma^{-}_{jk}} = e^{-E_{jk}/k_B T} \quad \Rightarrow \quad \sigma_{kk} = \frac{e^{-E_k/(k_B T)}}{\mathcal{Z}}, \qquad \sigma_{jk} = 0$$

Reset state must be thermal to satisfy local detailed balance

First law of thermodynamics



$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$$

Reset state not specified

$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\sigma - \rho(t))$$

$$\dot{E}(t) = \dot{Q}(t)$$

- Energy change

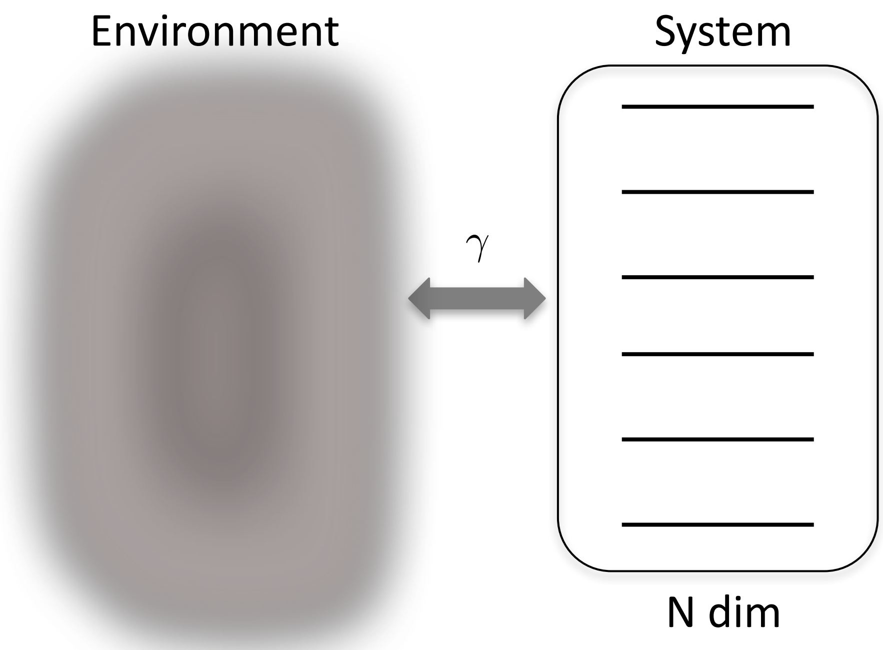
$$\dot{E}(t) = \sum_k E_k(t)\dot{p}_{kk}(t)$$
$$= \gamma \sum_k E_k(t)(\sigma_{kk} - p_{kk}).$$

- Heat flow

$$\dot{Q}(t) = \sum_k \sum_{j<k} (E_k - E_j)(\Gamma^{+}_{jk} p_j - \Gamma^{-}_{jk} p_k)$$
$$= \dots$$
$$= \dot{E}(t)$$

Identification

First law of thermodynamics



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Reset state not specified

$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\sigma - \rho(t))$$

$$\dot{E}(t) = \dot{Q}(t)$$

No constraint on the reset state

- Energy change

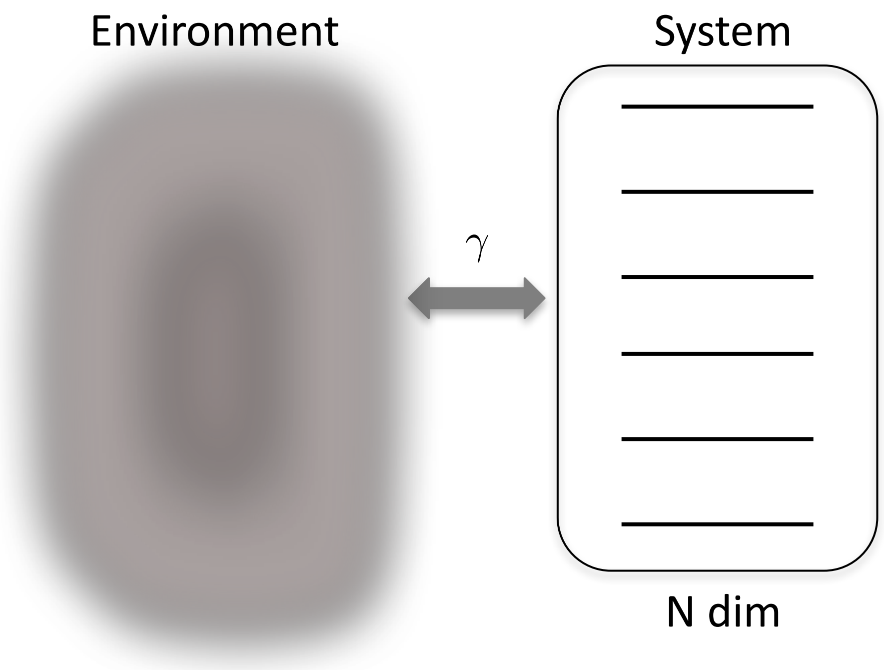
$$\dot{E}(t) = \sum_k E_k(t)\dot{p}_{kk}(t)$$
$$= \gamma \sum_k E_k(t)(\sigma_{kk} - p_{kk}).$$

- Heat flow

$$\dot{Q}(t) = \sum_k \sum_{j<k} (E_k - E_j)(\Gamma^+_{jk} p_j - \Gamma^-_{jk} p_k)$$
$$= \dots$$
$$= \dot{E}(t)$$

Identification

Second law of thermodynamics



$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$$

Reset state not specified

$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\sigma - \rho(t))$$

$$\dot{E}(t) = \dot{Q}(t)$$

No constraint on the reset state

- Entropy change

$$\dot{S}(t) = \frac{d}{dt}\left(-\mathrm{Tr}(\rho \ln \rho)\right) = -\mathrm{Tr}(\dot{\rho} \ln \rho) = -\gamma \mathrm{Tr}\Big((\sigma - \rho)\ln \rho\Big) = \gamma \underbrace{\mathcal{D}(\sigma||\rho)}_{>0} + \gamma\Big(S(\sigma) - S(\rho)\Big)$$

Entropy production Entropy flow ?

Can the entropy flow be written in terms of heat flow ?
Does it necessarily imply a thermal reset state ?

Conclusion and outlook

For autonomous entanglement engines, reset ME turned out to be very useful

- Steady-state entanglement between two qubits

  Brask, Haack, Brunner, Huber, NJP 17 (2015).

- Generalization to multipartite entanglement (cluster & GHZ & W states)

  Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)
  Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

Reset ME

Equivalent to a Lindblad-type ME + pure dephasing channel

Local detailed balance only verified for a thermal reset state

First law of thermodynamics seems to not required a thermal state

Second law ?

Non-Markovianity ? Engineered environment ?

Haack et al., in preparation (2019)