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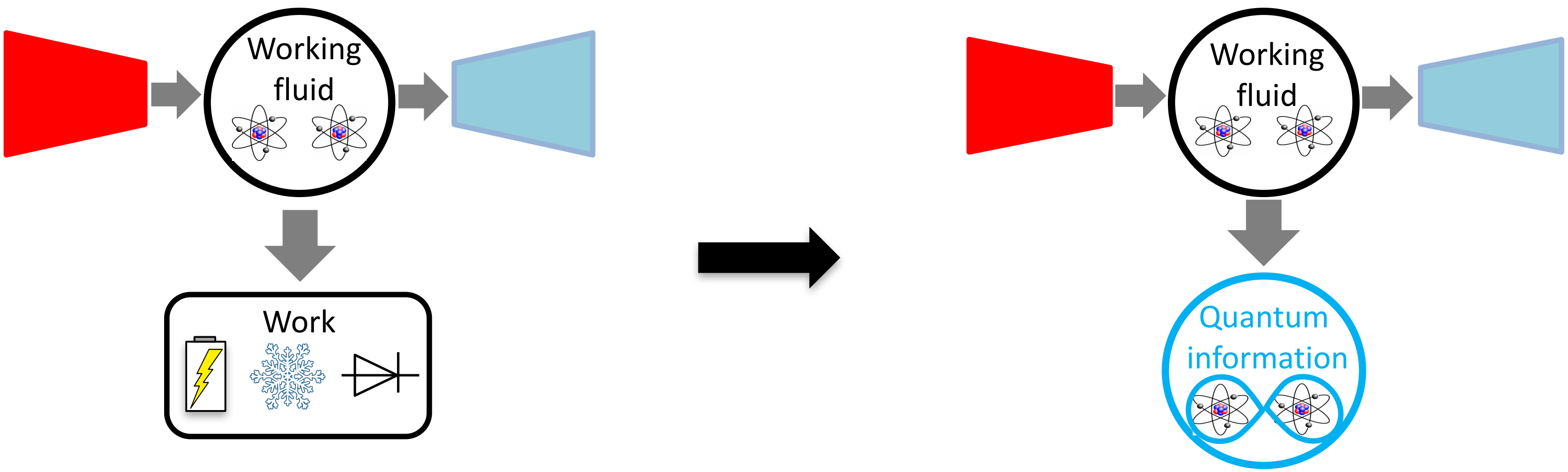
Autonomous nanoscale entanglement engines

Géraldine Haack
Junior group leader, Swiss Prima starting grant



The reset master equation: thermodynamics and open questions

BIRS workshop, Banff, 22.08.2019

"Charge and Energy Transfer Processes: Open Problems in Open Quantum Systems"

FNSNF

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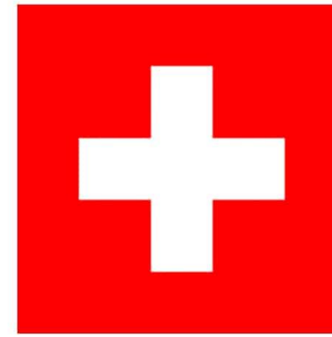
Quantum Correlations Group (Prof. Nicolas Brunner)

Quantum Information Theory & Quantum Thermodynamics



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Nicolas Brunner
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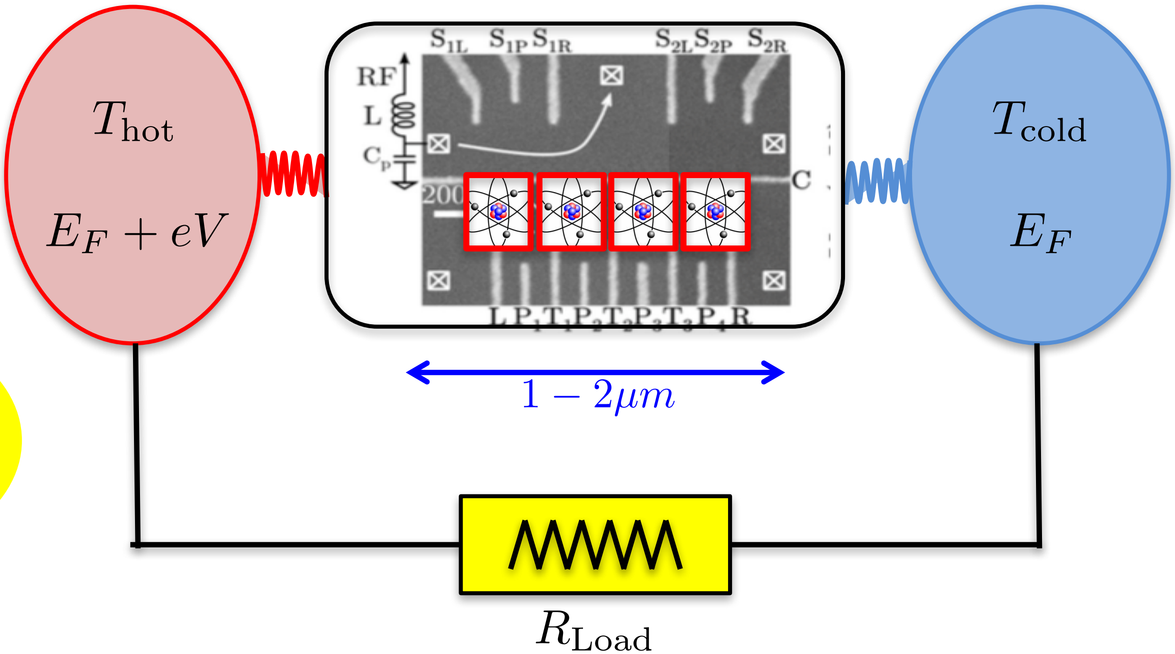
Jonatan B. Brask
(DTU, Denmark)

Ralph Silva (ETH Zürich), Marti Perarnau-Llobet (TU Munich), Patrick P. Potts (Lund Sweden)

Exploiting quantum properties of the working fluid



Correa, Kosloff
Hanna, Nitzan,
…

Typical outputs : Power (heat engine), cooling ratio (fridge)

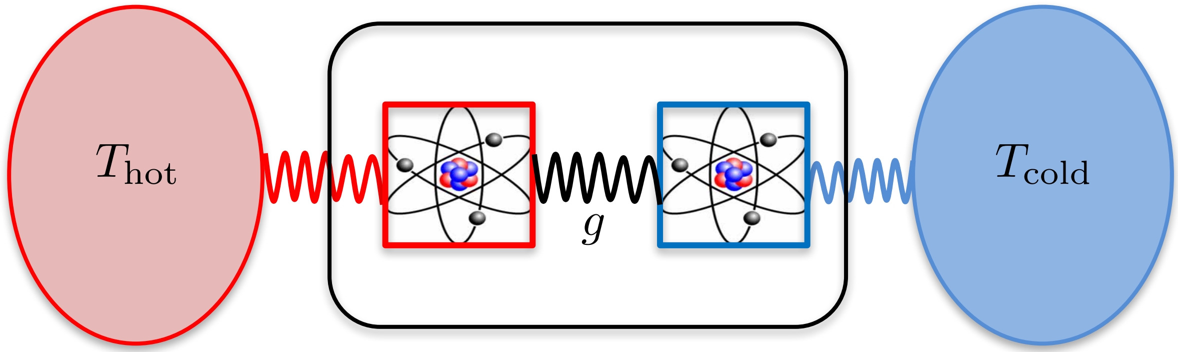
Take advantage of quantum properties to enhance efficiency

Haack, Giazotto, arXiv:1905.12672, Chiaracane et al., arXiv:1908.05139

Can we use thermal machines to generate quantum resources?

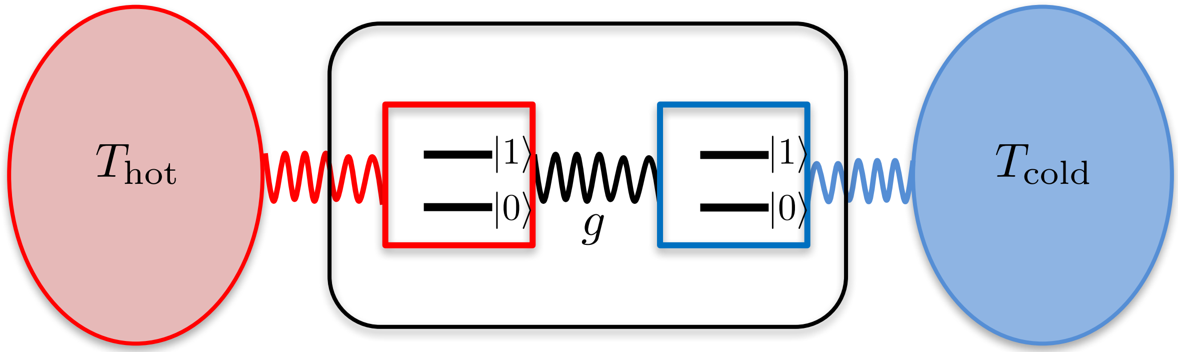
Entanglement? Multipartite entanglement?

Steady-state entanglement quantum engine



Brask, Haack, Brunner, Huber, NJP 17 (2015)

Steady-state entanglement quantum engine



Brask, Haack, Brunner, Huber, NJP 17 (2015)

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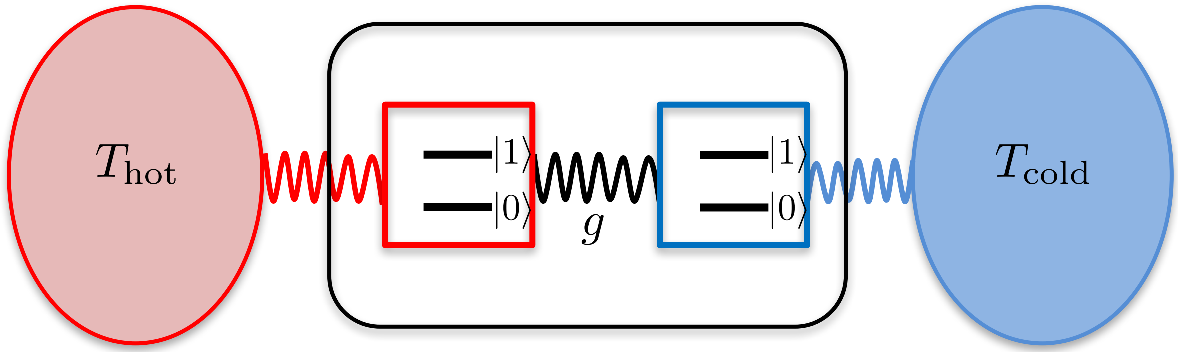
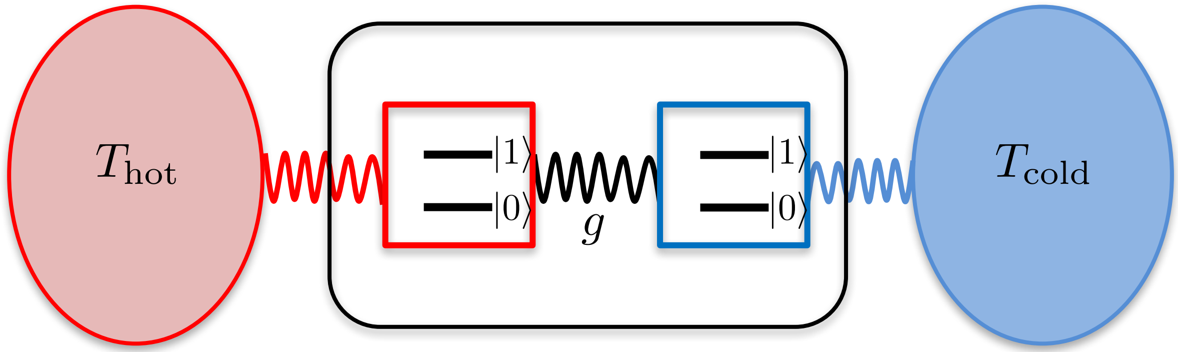


$$H_s = E\big(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2\big)$$

$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

Brask, Haack, Brunner, Huber, NJP 17 (2015)

Steady-state entanglement quantum engine



$$H_s = E\big(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2\big)$$

$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

- Time-independent interaction Hamiltonian, time-independent bath couplings

  ⟶ Thermodynamics: no work, only heat exchange

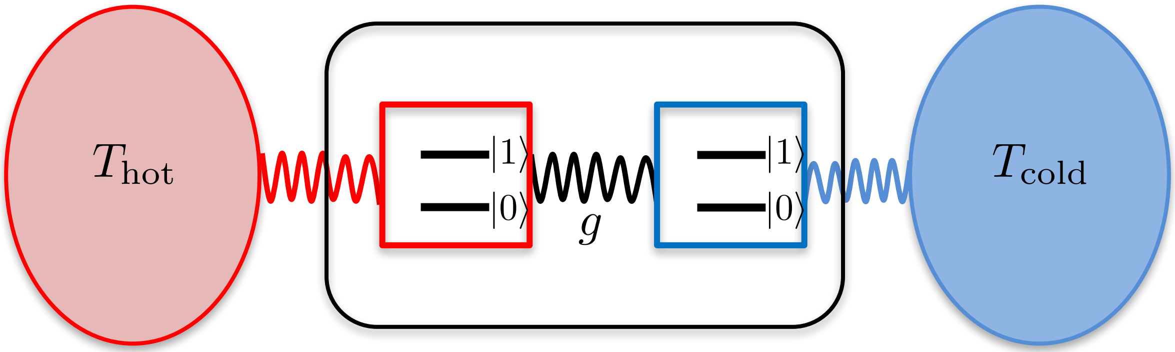
- Autonomous quantum thermal machine

- Ground state is a product state when $g < E$ (weak inter-qubit coupling)

- Solve master equation to obtain the steady-state solution

Brask, Haack, Brunner, Huber, NJP 17 (2015)

Dynamics



$$H_s = E\big(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2\big)$$

$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

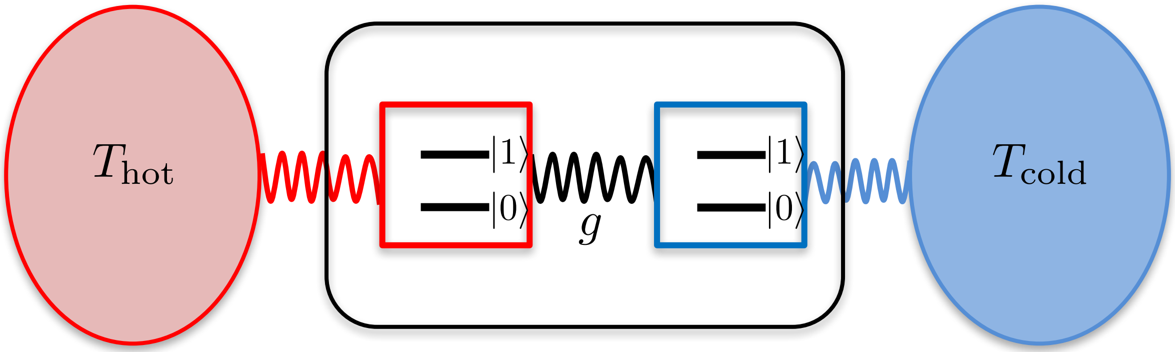
- Probabilistic reset: $\rho(t+dt) = -i[H_s, \rho(t)]\,dt + \gamma dt\,\tau + (1-\gamma dt)\rho(t)$

  Thermal state $\tau = r|0\rangle\langle 0| + (1-r)|1\rangle\langle 1|$

  Ground state population $r = \dfrac{1}{1+e^{-E/(k_B T)}}$

Brask, Haack, Brunner, Huber, NJP 17 (2015)

Dynamics



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Brask, Haack, Brunner, Huber, NJP 17 (2015)

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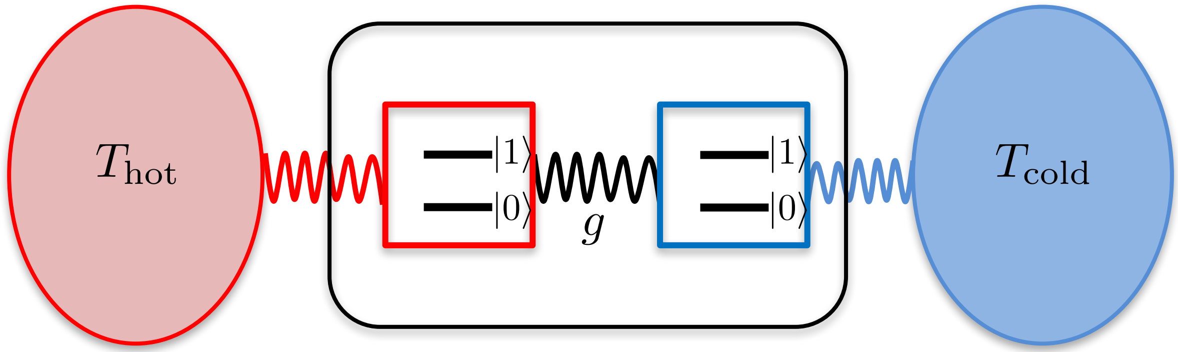

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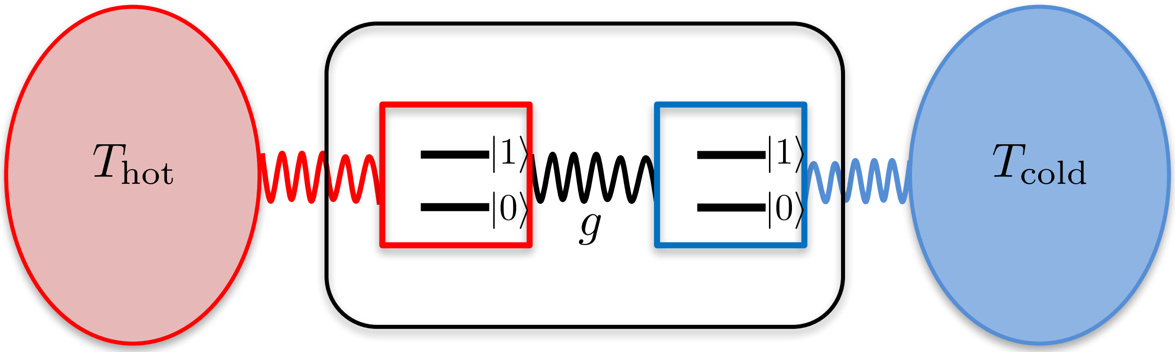
- For two qubits: $\dot{\rho}(t) = -i[H_s + H_{int}, \rho(t)] + \gamma_h\,(\tau_h \otimes \mathrm{Tr}_h\rho(t) - \rho(t)) + \gamma_c\,(\mathrm{Tr}_c\rho(t) \otimes \tau_c - \rho(t))$

Dynamics



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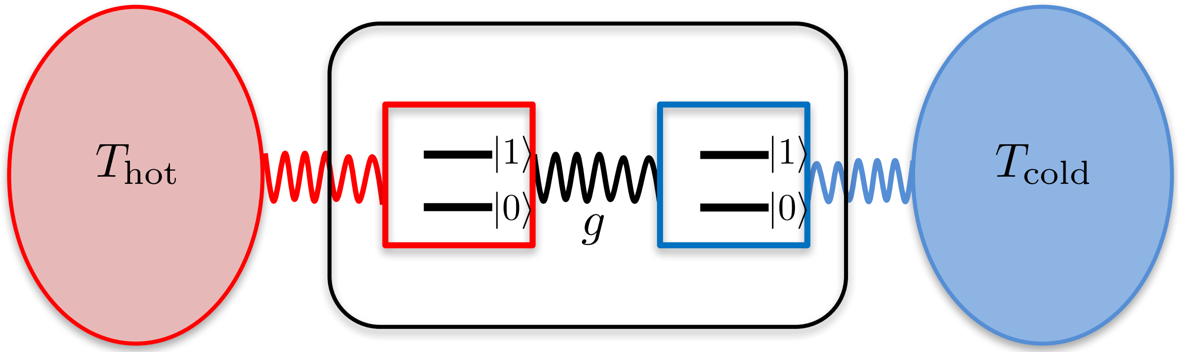
⇨ talk by C. Koch on optimal control for reset

Steady-state entanglement quantum engine



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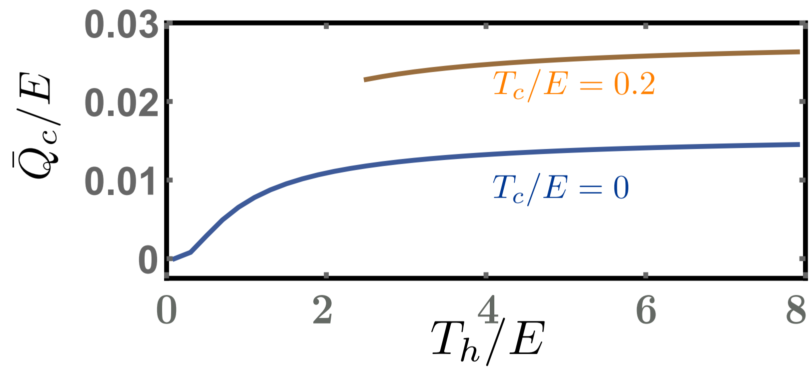
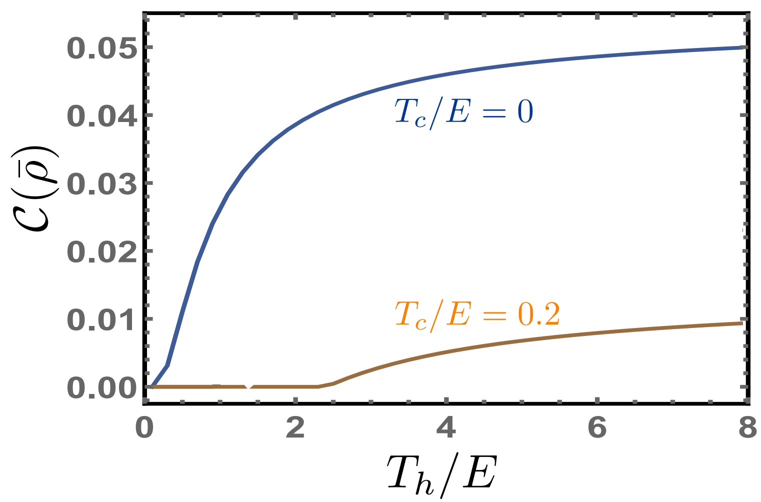
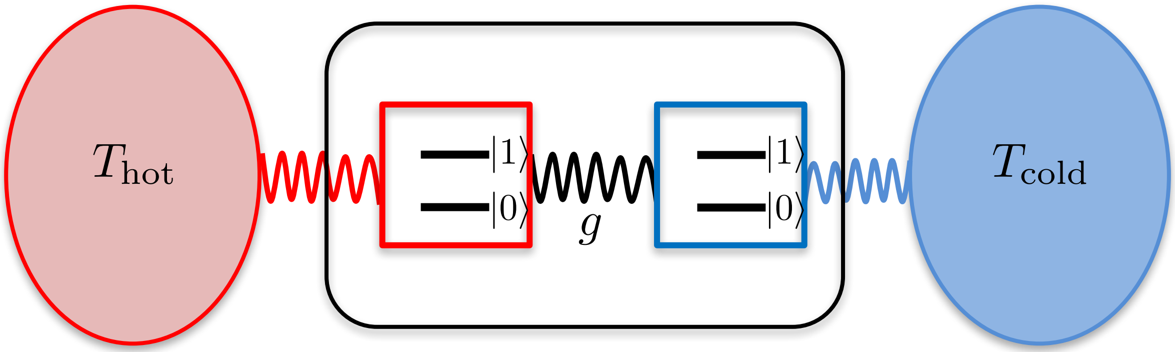




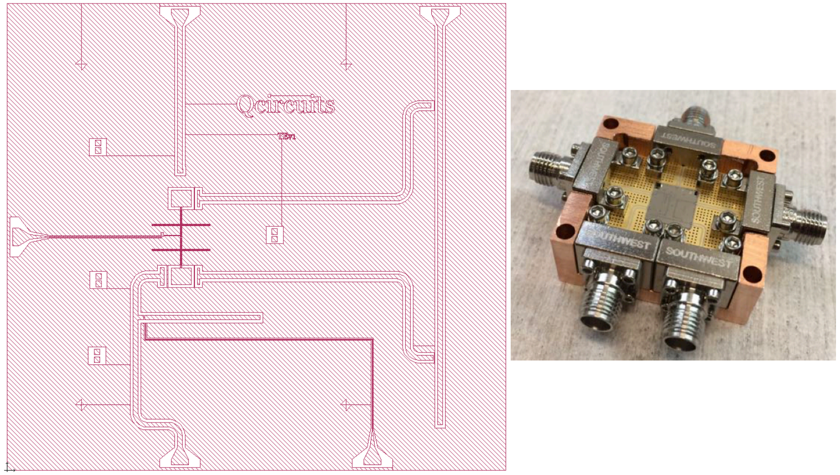
Steady-state heat flow sustains the generation of entanglement

Brask, Haack, Brunner, Huber, NJP 17 (2015)

Steady-state entanglement quantum engine



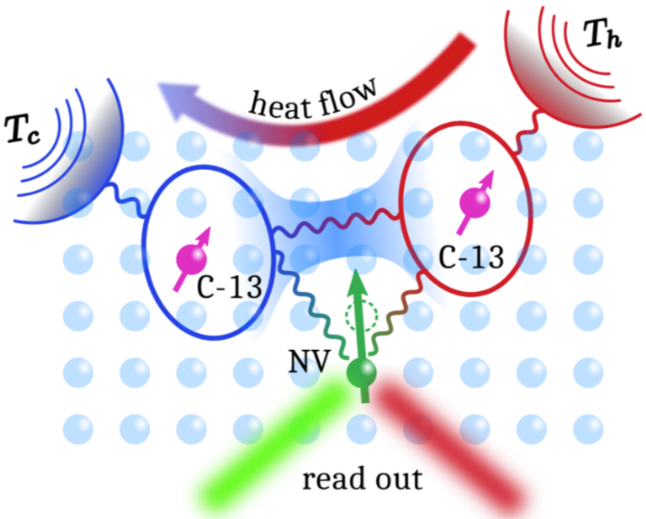
Circuit QED platform
Huard group (ENS Lyon, France)



Thermal baths : Spectral density of current noise

$$\propto E\,\mathrm{Re}[Y(E)]\frac{1}{1-e^{-E/(k_B T)}}$$

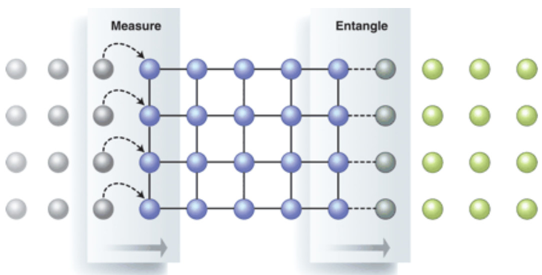
NV centres
Houck and Berg-Sorensen (DTU, Denmark)



(Proposal)

Multipartite entanglement?

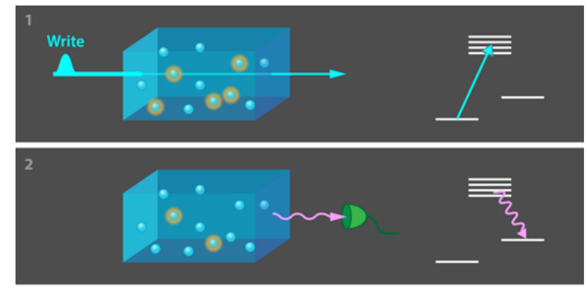
Cluster states



O'Brien, Science 318 (2007)

Quantum Computing

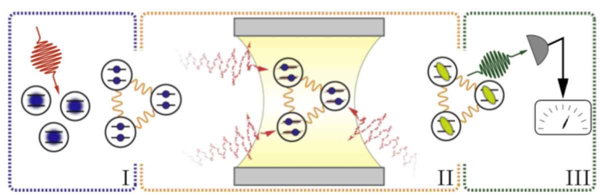
Dicke states / W-states



Nunn, Physics 10 (2017)

Quantum memories

GHZ states



Haase *et al.*, NJP 20 (2018)

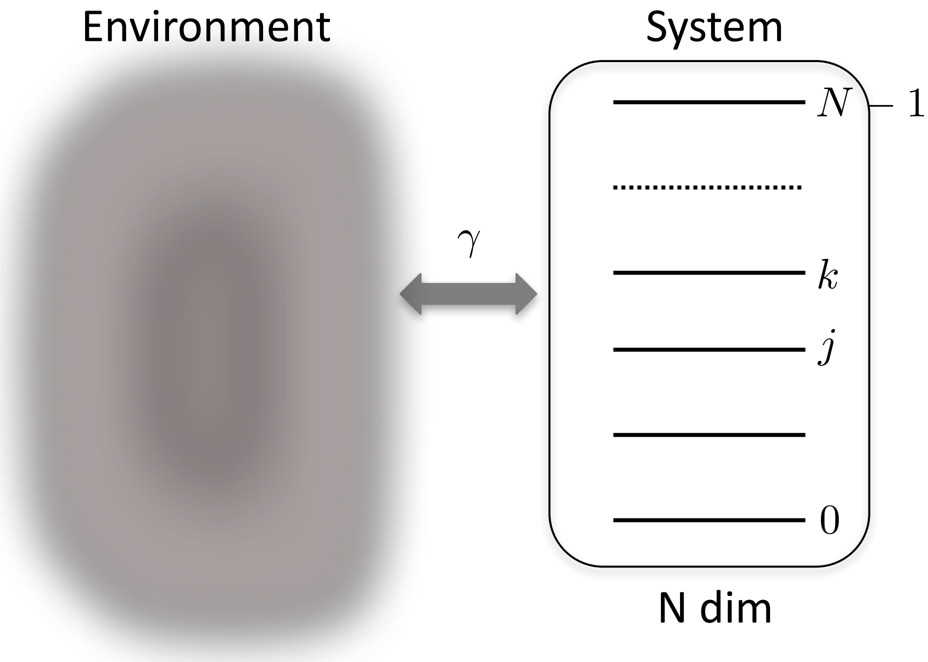
Quantum metrology

Which quantum states can be generated via an autonomous thermal machine?

Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)
Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

The reset evolution equation

Open questions on open quantum systems



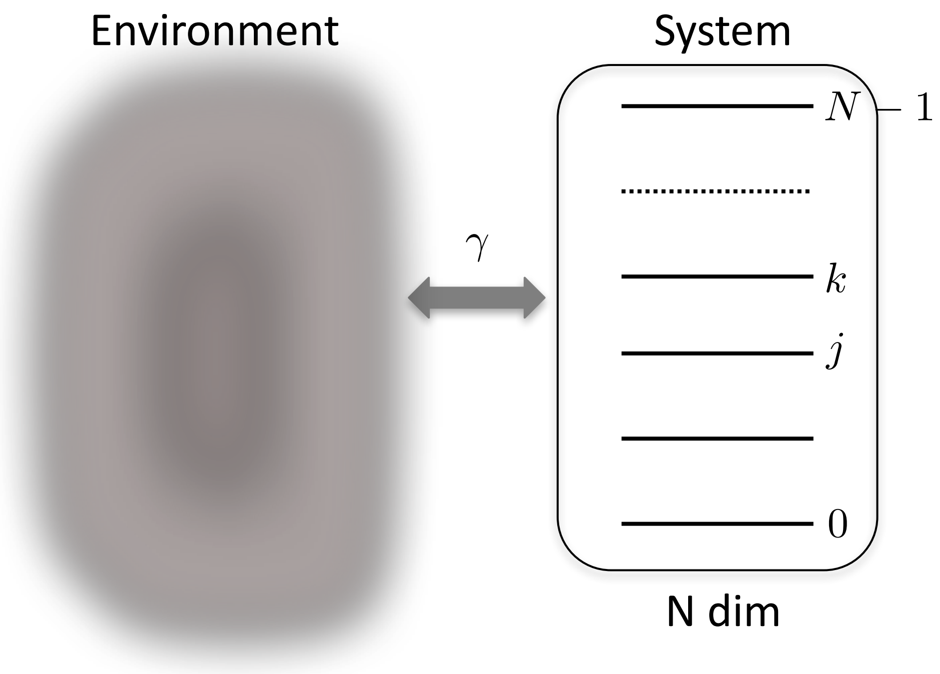
$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$$

Reset state not specified

$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\sigma - \rho(t))$$

The reset evolution equation

Open questions on open quantum systems



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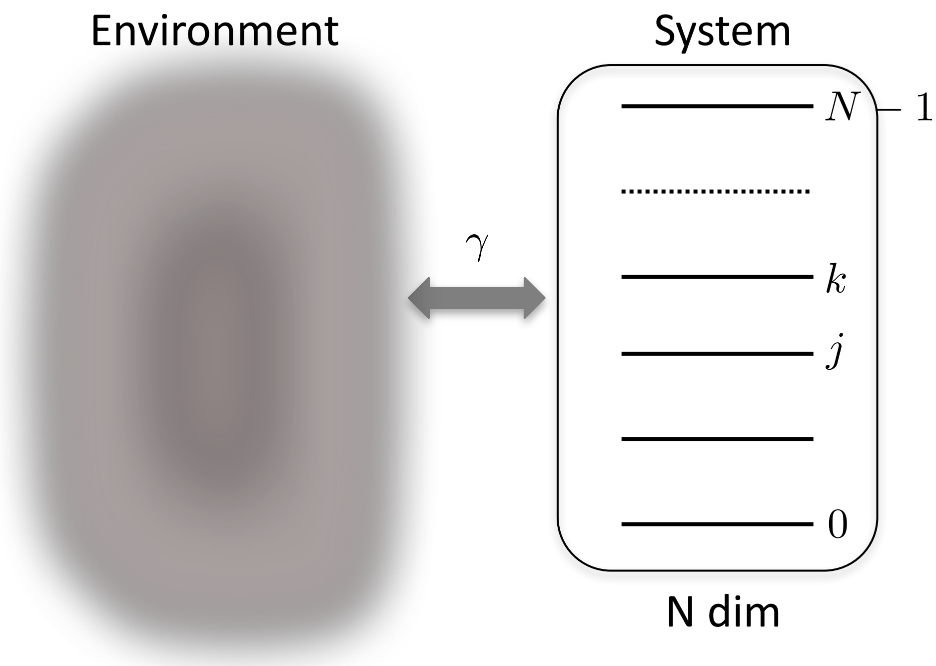
- Linearity ?
- Equivalent to a Lindblad-type ME ?
- Relevant for experiments ?
- Are the laws of thermodynamics valid ? For any reset state ?

The reset evolution equation

Open questions on open quantum systems



$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$$

Reset state not specified

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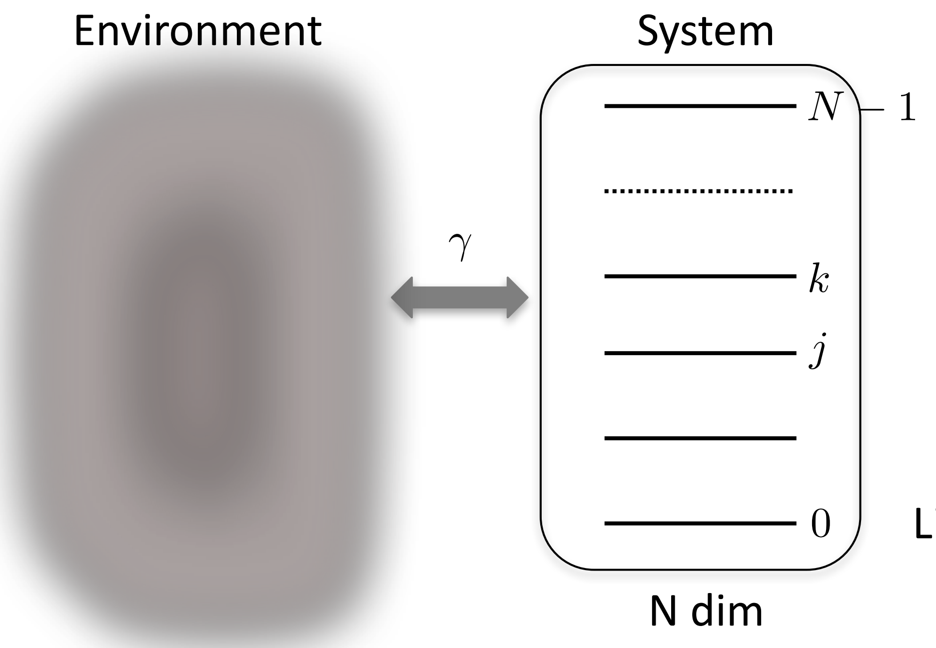
- Linearity ? $\dot{\rho}(t) = \gamma\,(\sigma\,\mathrm{Tr}[\rho(t)] - \rho(t))$ ⟶ $\frac{d}{dt}(a\rho_1 + b\rho_2) = \ldots = a\frac{d\rho_1}{dt} + b\frac{d\rho_2}{dt}$
- Equivalent to a Lindblad-type ME ?
- Relevant for experiments ?
- Are the laws of thermodynamics valid ? For any reset state ?

Equivalent to a Lindblad master equation ?



$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$$

Reset state not specified

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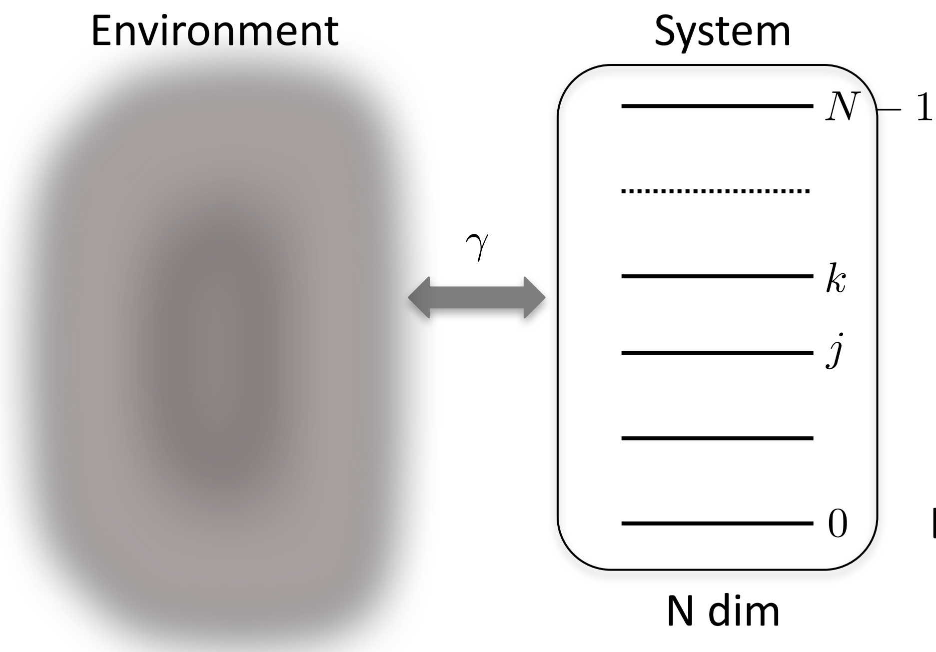
?

Lindblad dissipators: $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$

Equivalent to a Lindblad master equation ?



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?

Lindblad dissipators: $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$

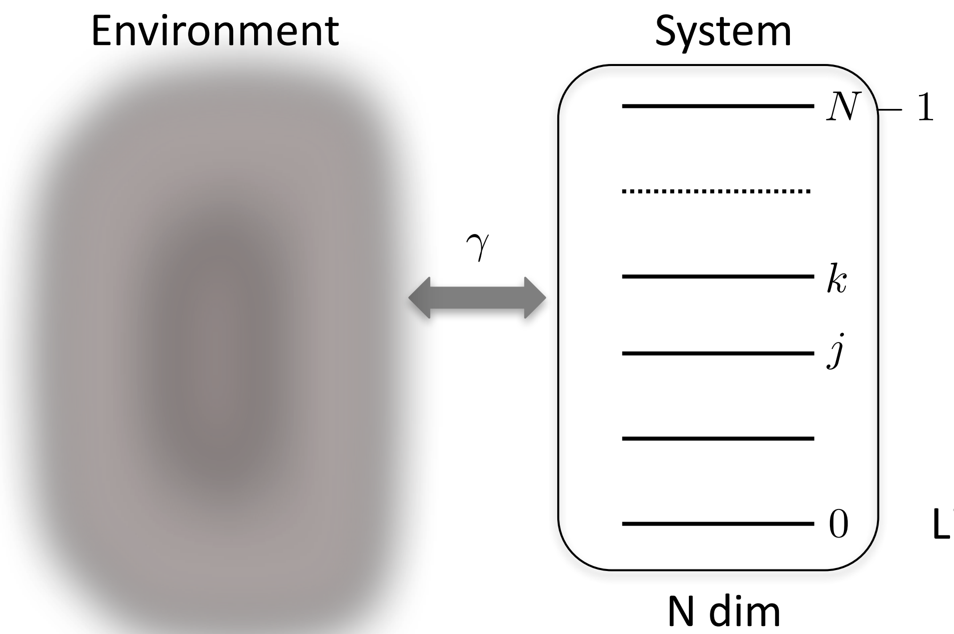
- Rate equations for the populations for Lindblad dissipators

$$\dot{p}_0 = \sum_{j>0}^{N-1} \left(\Gamma^-_{0j} p_j - \Gamma^+_{oj} p_0\right)$$

$$\dot{p}_{N-1} = \sum_{j=0}^{N-2} \left(\Gamma^+_{jN-1} p_j - \Gamma^-_{jN-1} p_{N-1}\right)$$

$$\dot{p}_k = -\left(\sum_{j=0}^{k-1} \bar{\Gamma}_{jk} + \sum_{j=k+1}^{N-1} \Gamma_{kj}\right) p_k + \left(\sum_{j=0}^{k-1} \Gamma_{jk} + \sum_{j=k+1}^{N-1} \bar{\Gamma}_{kj}\right) p_j \qquad \forall k = 0, \ldots, N-1$$

Equivalent to a Lindblad master equation ?



$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$$

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?

Lindblad dissipators: $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$

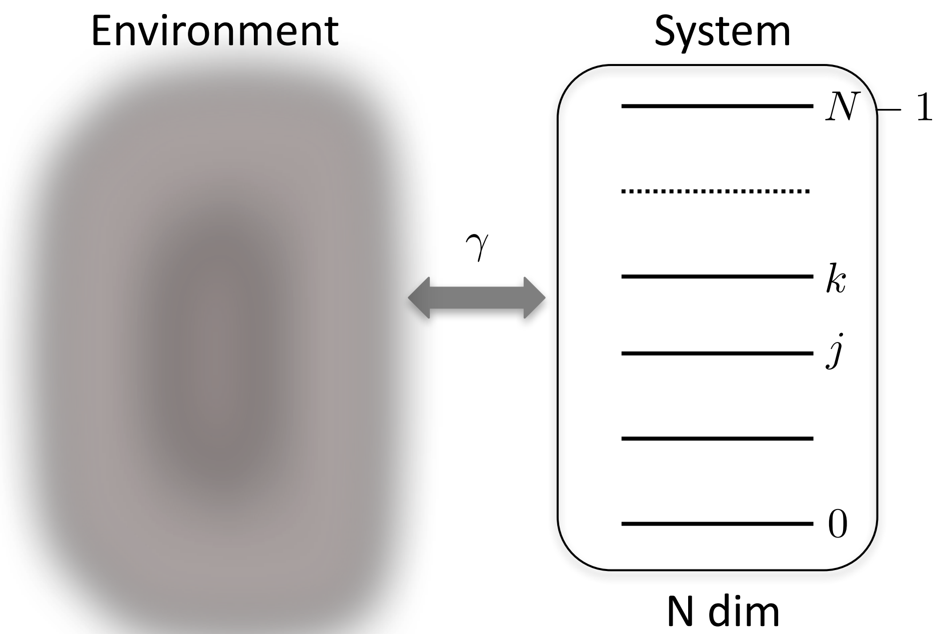
- Lindblad

$$\dot{p}_k = -\left(\sum_{j<k}\Gamma^-_{jk} + \sum_{j>k}\Gamma^+_{kj}\right)p_k + \left(\sum_{j<k}\Gamma^+_{jk} + \sum_{j>k}\Gamma^-_{kj}\right)p_j$$

- Reset

$$\begin{aligned}\dot{p}_k &= \gamma\,(\sigma_{kk} - p_k)\\ &= \gamma\left(\sigma_{kk}\sum_{j=0}^{N-1}p_j - p_k\right)\\ &= \gamma\,(\sigma_{kk}-1)\,p_k + \gamma\,\sigma_{kk}\left(\sum_{j<k}p_j + \sum_{j>k}p_j\right)\end{aligned}$$

Equivalent to a Lindblad master equation ?



$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$$

Reset state not specified

$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\sigma - \rho(t))$$

Identification for the populations:

$$\gamma\sigma_{kk} = \Gamma^{+}_{jk} \qquad \gamma\sigma_{jj} = \Gamma^{-}_{jk} \qquad j<k$$

$$\gamma\sigma_{jj} = \Gamma^{+}_{kj} \qquad \gamma\sigma_{kk} = \Gamma^{-}_{kj} \qquad j>k$$

- Lindblad

$$\dot{p}_k = -\left(\sum_{j<k}\Gamma^{-}_{jk} + \sum_{j>k}\Gamma^{+}_{kj}\right)p_k + \left(\sum_{j<k}\Gamma^{+}_{jk} + \sum_{j>k}\Gamma^{-}_{kj}\right)p_j$$

- Reset

$$\begin{aligned}\dot{p}_k &= \gamma\,(\sigma_{kk} - p_k) \\ &= \gamma\left(\sigma_{kk}\sum_{j=0}^{N-1} p_j - p_k\right) \\ &= \gamma\,(\sigma_{kk} - 1)\,p_k + \gamma\,\sigma_{kk}\left(\sum_{j<k} p_j + \sum_{j>k} p_j\right)\end{aligned}$$

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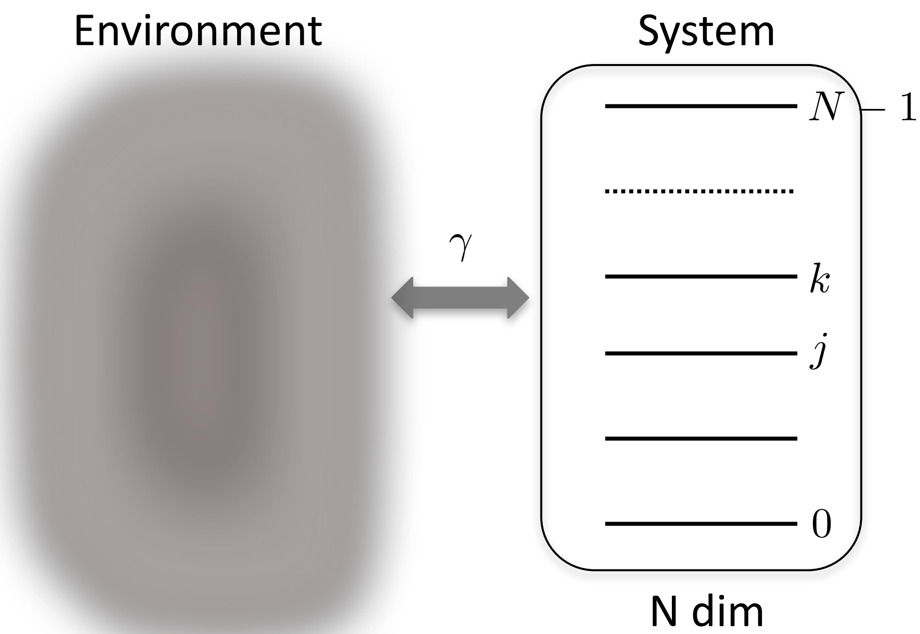

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$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\left(\sigma - \rho(t)\right)$$

Identification for the populations:

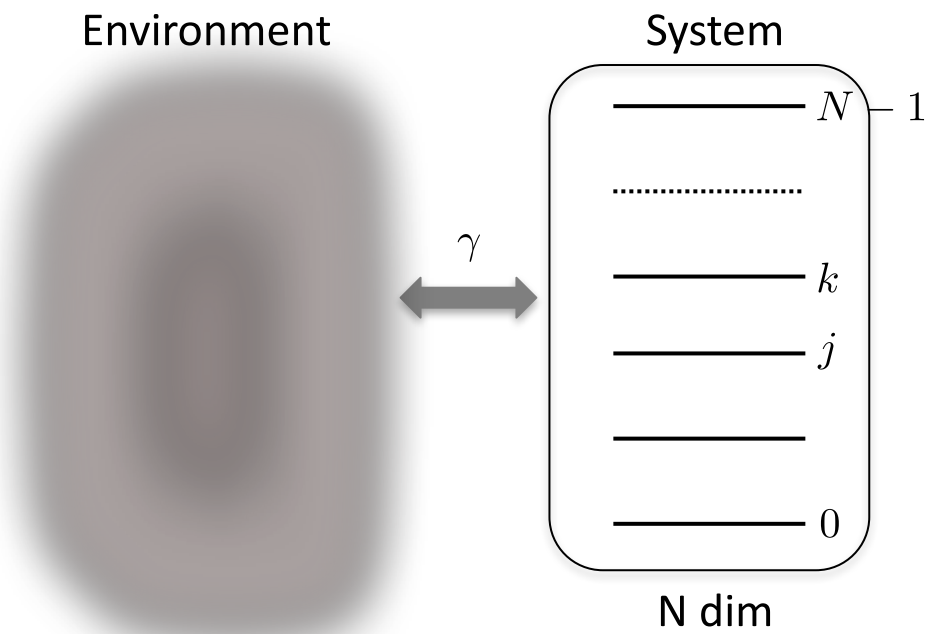
$j < k$	$j > k$
$\gamma\sigma_{kk} = \Gamma^{+}_{jk}$	$\gamma\sigma_{jj} = \Gamma^{+}_{kj}$
$\gamma\sigma_{jj} = \Gamma^{-}_{jk}$	$\gamma\sigma_{kk} = \Gamma^{-}_{kj}$

- For the coherences ?

  Reset: $\dot{p}_{jk} = \gamma\Big(\sigma_{jk} - \rho_{jk}\Big)$

  Lindblad : … → Need an additional pure dephasing channel! $\Gamma^{\phi}_{jk}\mathcal{D}[\sigma_z^{jk}]\rho$

Equivalent to a Lindblad master equation ?



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- For the coherences ?

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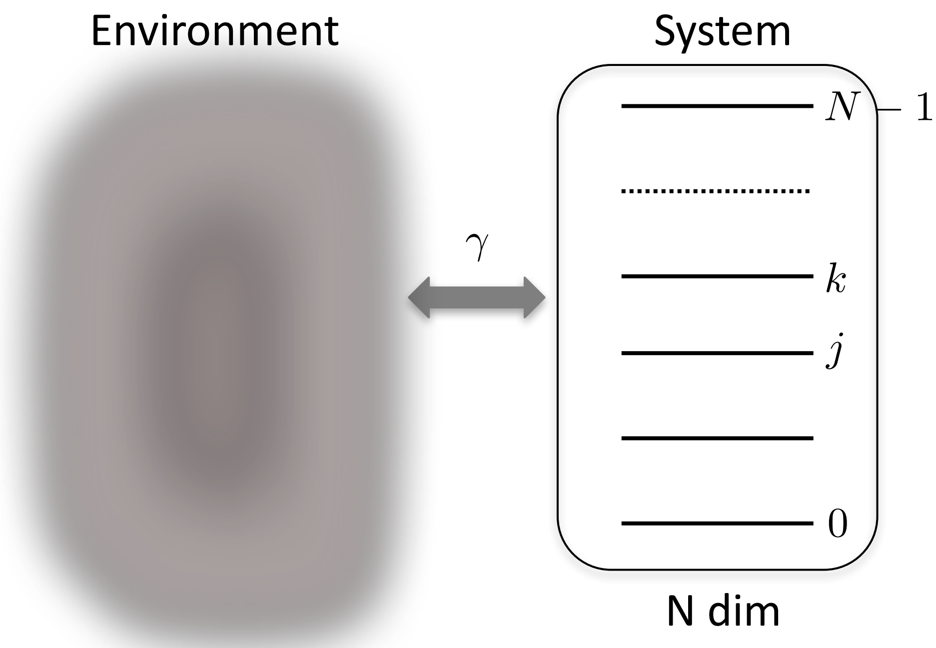
Lindblad : … → Need an additional pure dephasing channel! $\Gamma^{\phi}_{jk}\mathcal{D}[\sigma_z^{jk}]\rho$

Reset ME : Lindblad-type ME + additional dephasing channel

Relevant for experiments!

Local detailed balance



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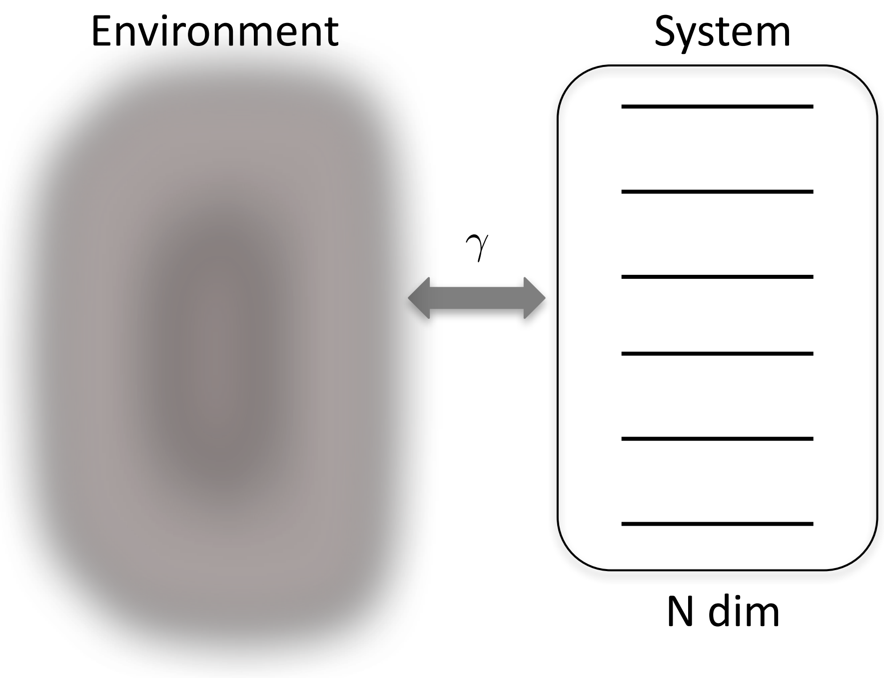
Identification for the populations:

$j < k$	$j > k$
$\gamma\sigma_{kk} = \Gamma^{+}_{jk}$	$\gamma\sigma_{jj} = \Gamma^{+}_{kj}$
$\gamma\sigma_{jj} = \Gamma^{-}_{jk}$	$\gamma\sigma_{kk} = \Gamma^{-}_{kj}$

$$\frac{\Gamma^{+}_{jk}}{\Gamma^{-}_{jk}} = e^{-E_{jk}/k_B T} \;\Rightarrow\; \sigma_{kk} = \frac{e^{-E_k/(k_B T)}}{\mathcal{Z}}, \quad \sigma_{jk} = 0$$

Reset state must be thermal to satisfy local detailed balance

First law of thermodynamics



$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$$

Reset state not specified

$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\sigma - \rho(t))$$

$$\dot{E}(t) = \dot{Q}(t)$$

- Energy change

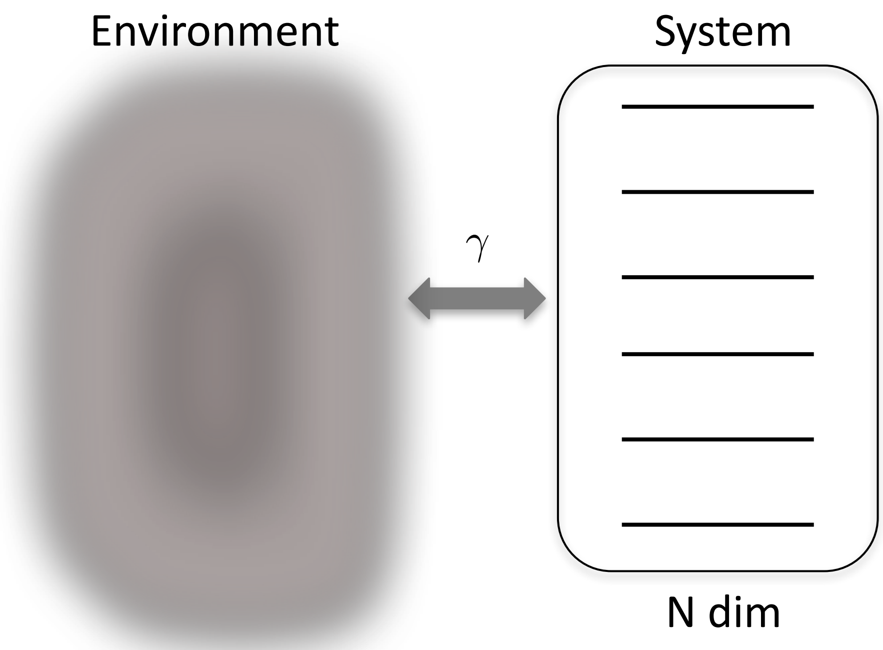
$$\dot{E}(t) = \sum_k E_k(t)\dot{p}_{kk}(t)$$
$$= \gamma \sum_k E_k(t)(\sigma_{kk} - p_{kk}).$$

- Heat flow

$$\dot{Q}(t) = \sum_k \sum_{j<k} (E_k - E_j)(\Gamma^+_{jk} p_j - \Gamma^-_{jk} p_k)$$
$$= \dots$$
$$= \dot{E}(t)$$

Identification

First law of thermodynamics



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Reset state not specified

$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\sigma - \rho(t))$$

$$\dot{E}(t) = \dot{Q}(t)$$

No constraint on the reset state

- Energy change

$$\dot{E}(t) = \sum_k E_k(t)\dot{p}_{kk}(t)$$

$$= \gamma \sum_k E_k(t)(\sigma_{kk} - p_{kk}).$$

- Heat flow

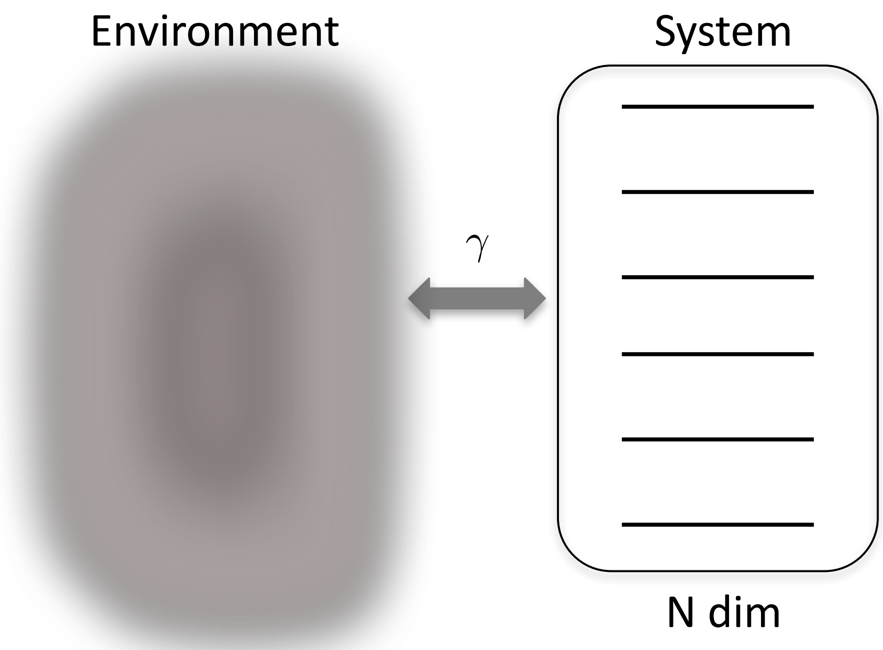
$$\dot{Q}(t) = \sum_k \sum_{j<k} (E_k - E_j)(\Gamma^+_{jk} p_j - \Gamma^-_{jk} p_k)$$

$$= \dots$$

Identification

$$= \dot{E}(t)$$

Second law of thermodynamics



$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$$

Reset state not specified

$$\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\sigma - \rho(t))$$

$$\dot{E}(t) = \dot{Q}(t)$$

No constraint on the reset state

- Entropy change

$$\dot{S}(t) = \frac{d}{dt}\left(-\mathrm{Tr}(\rho \ln \rho)\right) = -\mathrm{Tr}(\dot{\rho} \ln \rho) = -\gamma \mathrm{Tr}\Big((\sigma - \rho)\ln \rho\Big) = \gamma \underbrace{\mathcal{D}(\sigma||\rho)}_{>0} + \gamma\Big(S(\sigma) - S(\rho)\Big)$$

Entropy production   Entropy flow ?

Can the entropy flow be written in terms of heat flow ?
Does it necessarily imply a thermal reset state ?

Conclusion and outlook

For autonomous entanglement engines, reset ME turned out to be very useful

- Steady-state entanglement between two qubits

  Brask, Haack, Brunner, Huber, NJP 17 (2015).

- Generalization to multipartite entanglement (cluster & GHZ & W states)

  Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)

  Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

Reset ME

Equivalent to a Lindblad-type ME + pure dephasing channel

Local detailed balance only verified for a thermal reset state

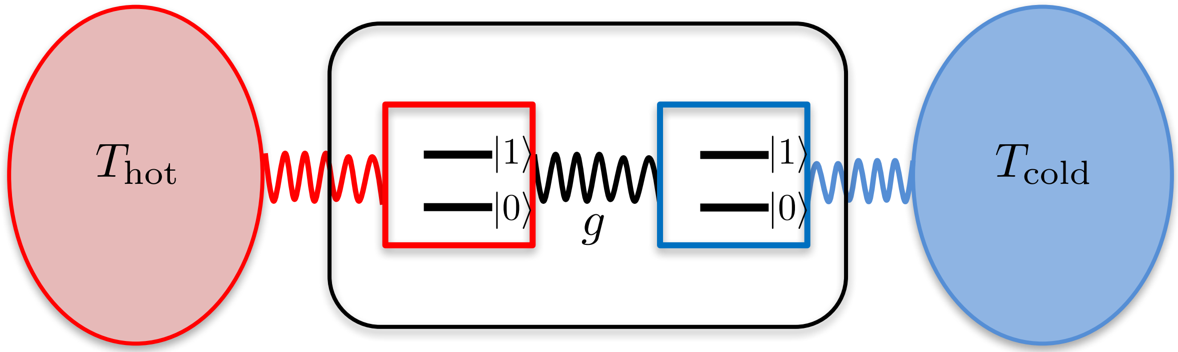
First law of thermodynamics seems to not required a thermal state

Second law ?

Non-Markovianity ? Engineered environment ?

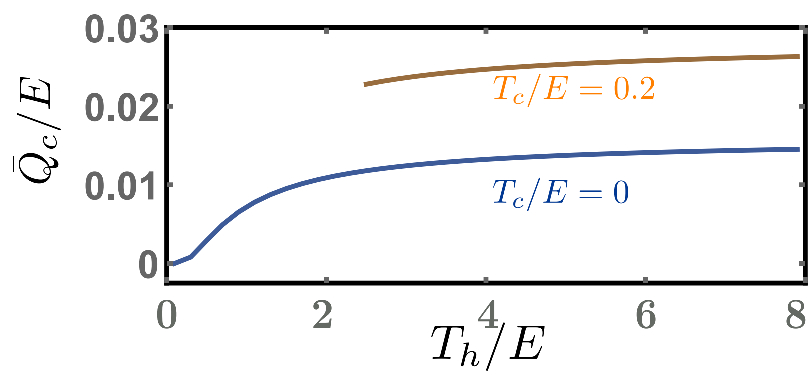
Haack et al., in preparation (2019)

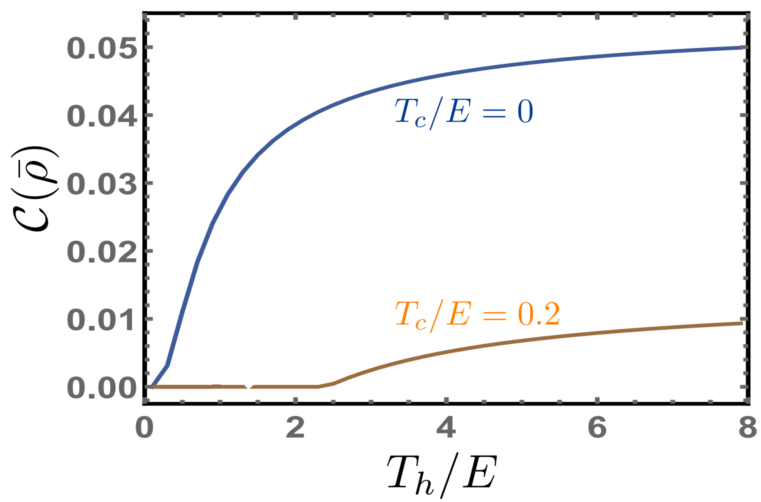
Steady-state entanglement quantum engine



$$H_s = E\big(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2\big)$$

$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

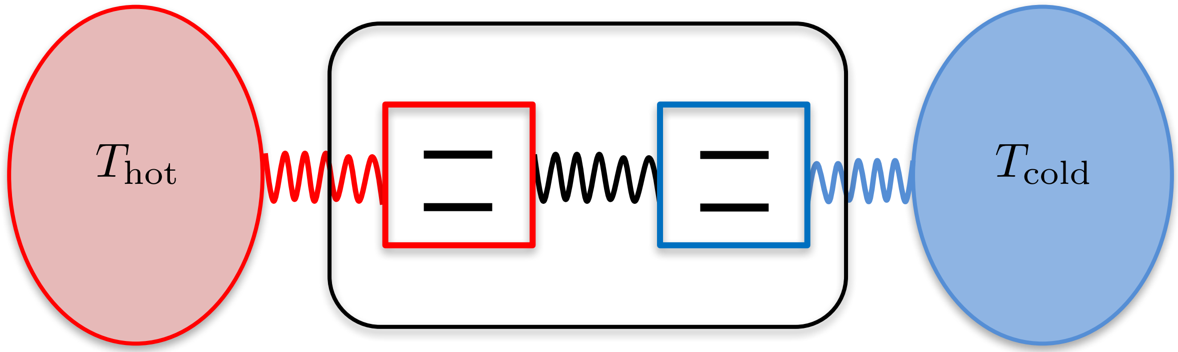




Do incoherent couplings to thermal baths limit the amount of entanglement?

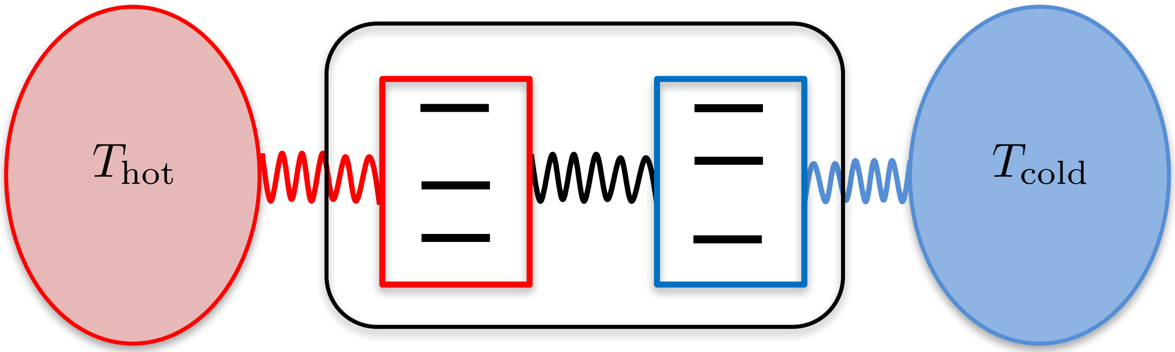
Brask, Haack, Brunner, Huber, NJP 17 (2015)

Heralded entanglement quantum engine



Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)

Heralded entanglement quantum engine



Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)

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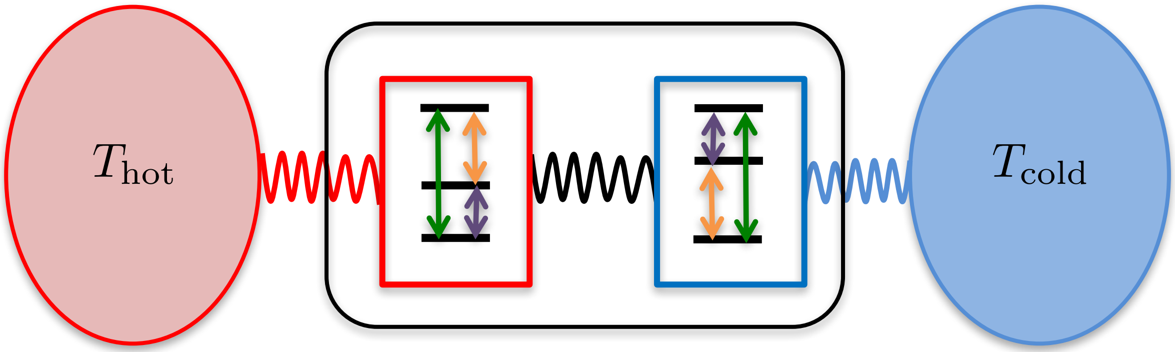
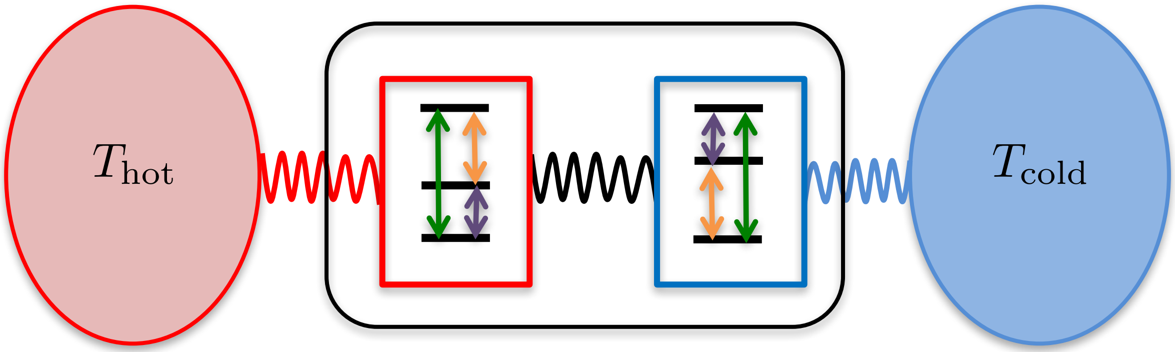


$$H_{int} = g_1 |02\rangle\langle 20| + g_2 |11\rangle\langle 20| + g_3 |11\rangle\langle 02| + h.c.$$

- Dynamics : Reset master equation
- Steady state solution : weakly entangled steady state

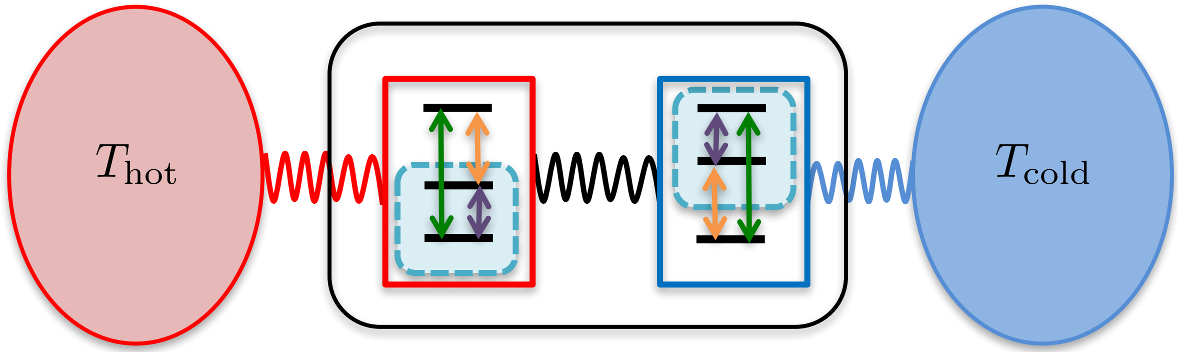
Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)

Heralded entanglement quantum engine



Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)

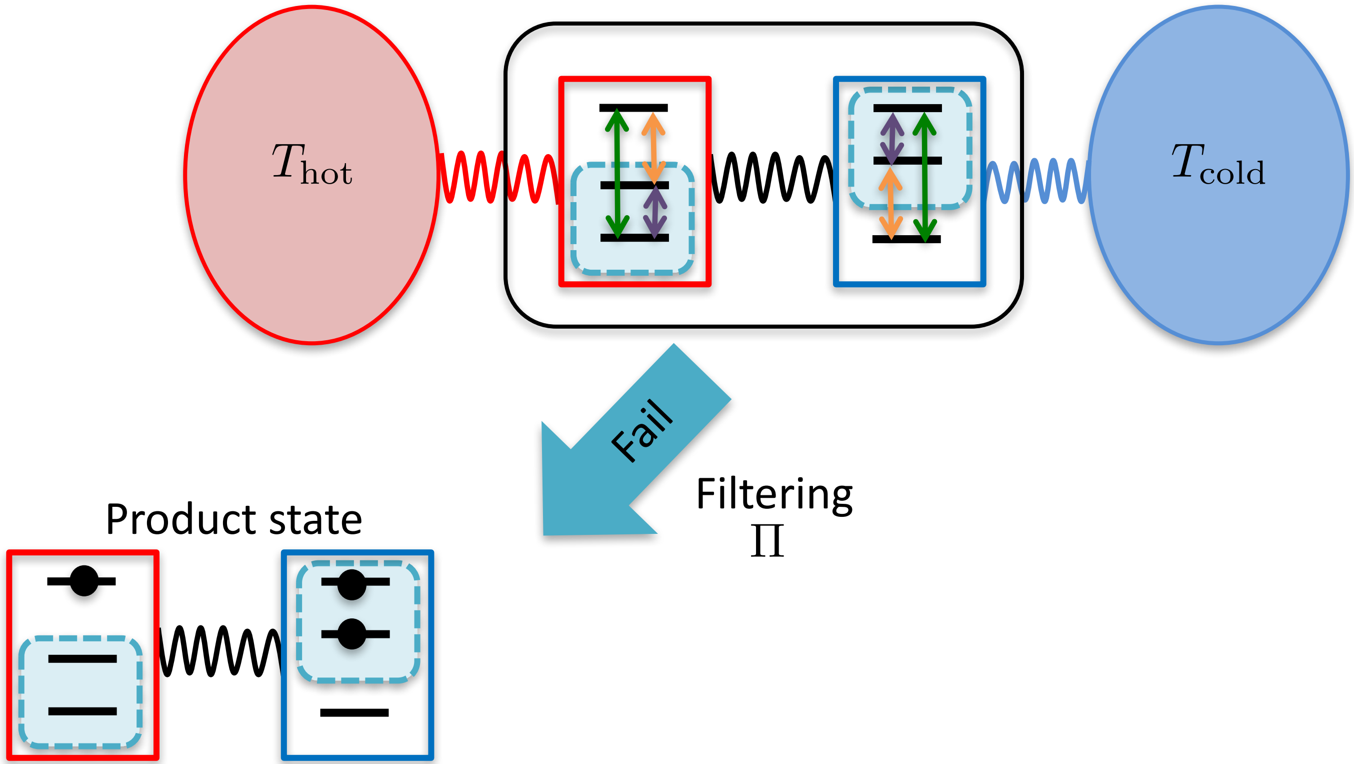
Heralded entanglement quantum engine



Filtering
Π

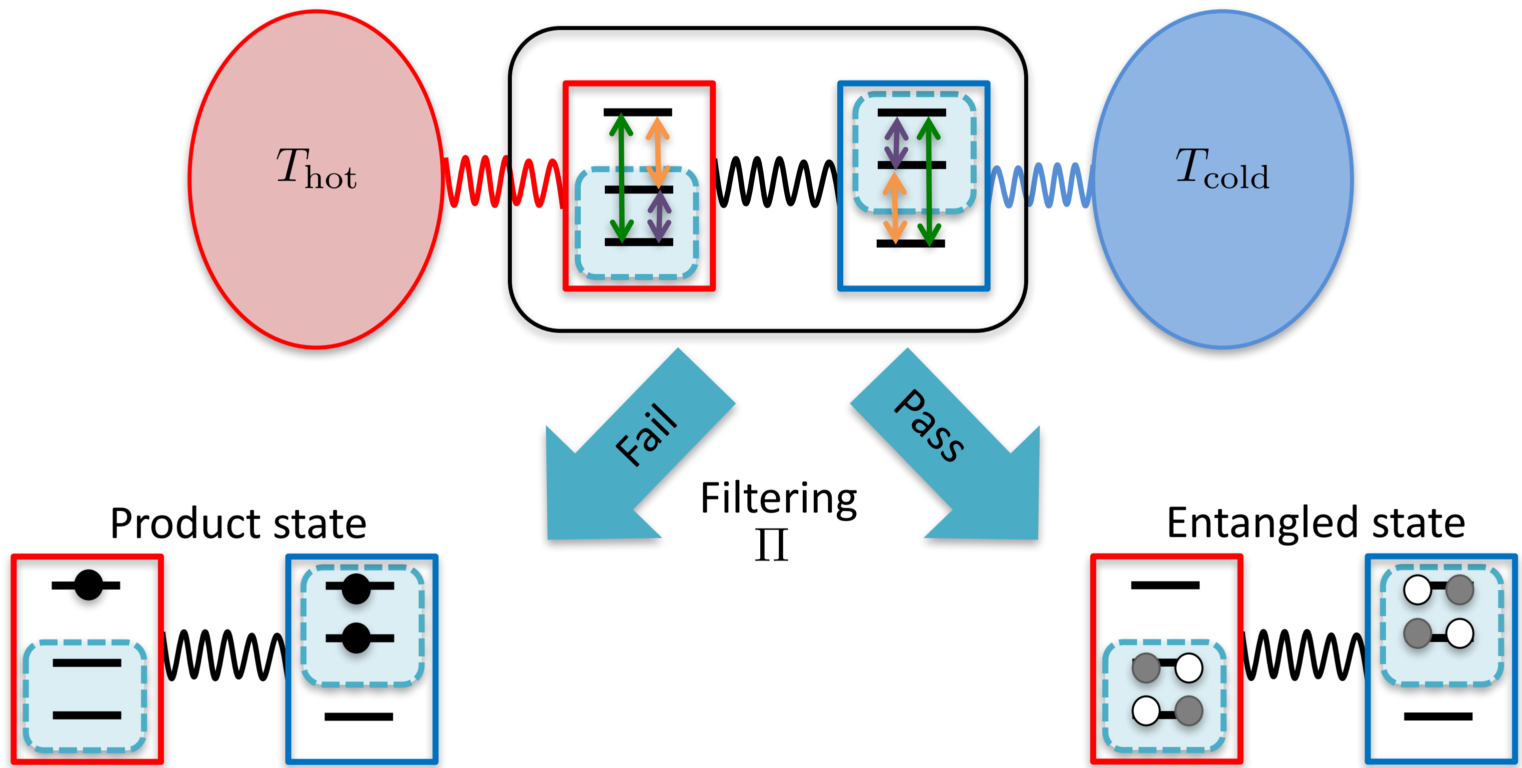
Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)

Heralded entanglement quantum engine

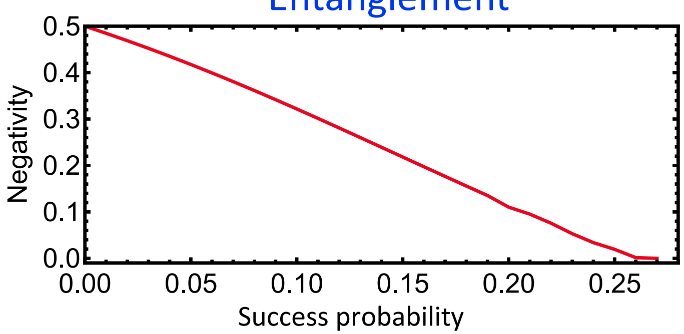


Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)

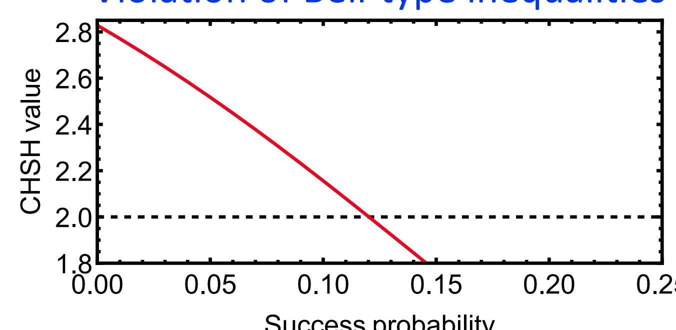
Heralded entanglement quantum engine



Entanglement



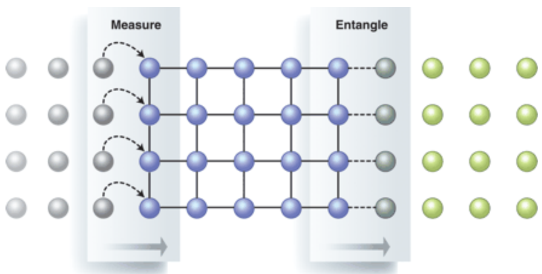
Violation of Bell-type inequalities



Tavakoli, Haack, Huber, Brunner, Brask, Quantum 2 (2018)

Multipartite entanglement?

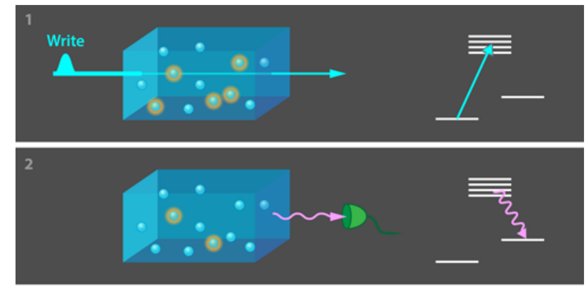
Cluster states



O'Brien, Science 318 (2007)

Quantum Computing

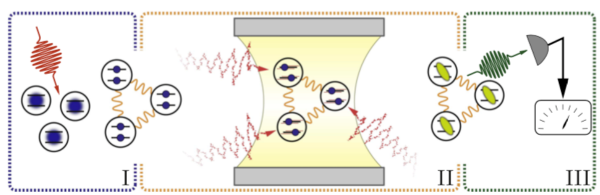
Dicke states / W-states



Nunn, Physics 10 (2017)

Quantum memories

GHZ states

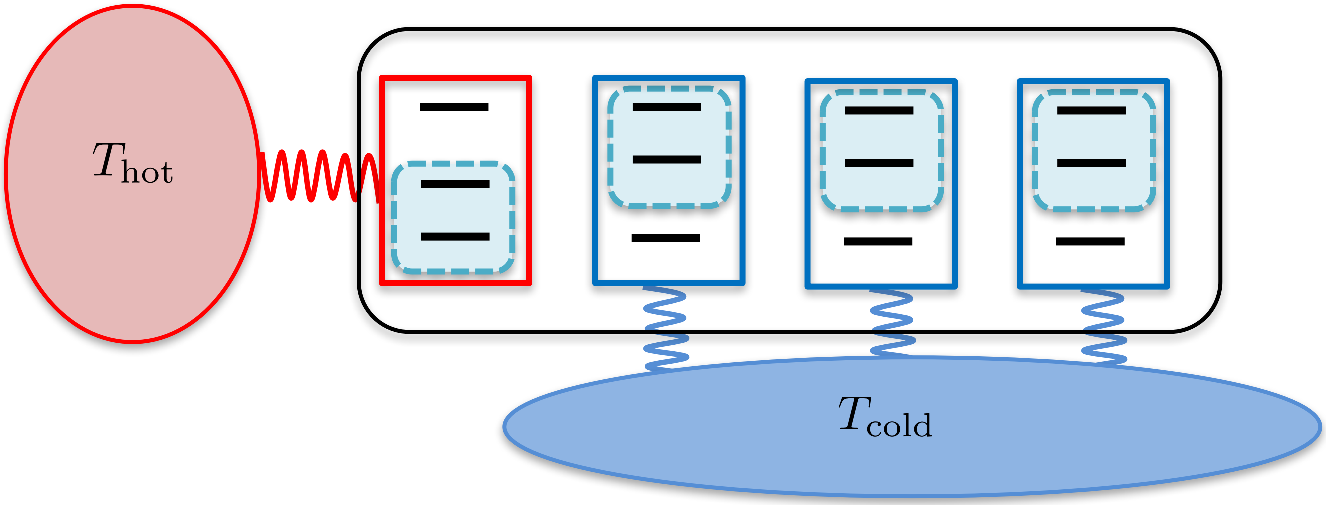


Haase *et al.*, NJP 20 (2018)

Quantum metrology

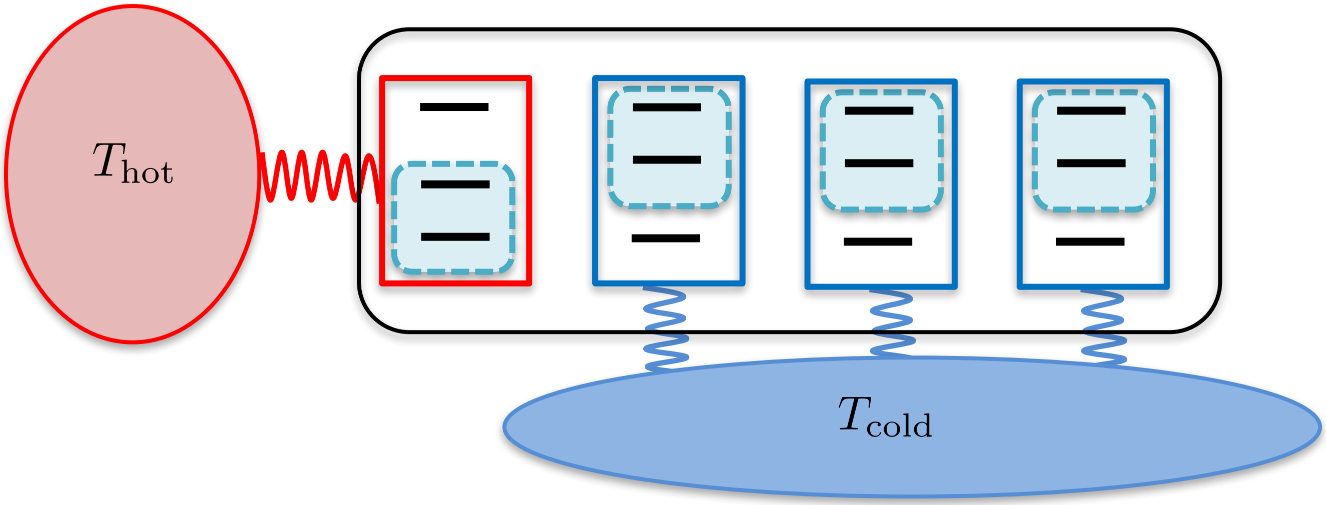
Which quantum states can be generated via an autonomous thermal machine?

Generalize to multipartite entanglement



Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

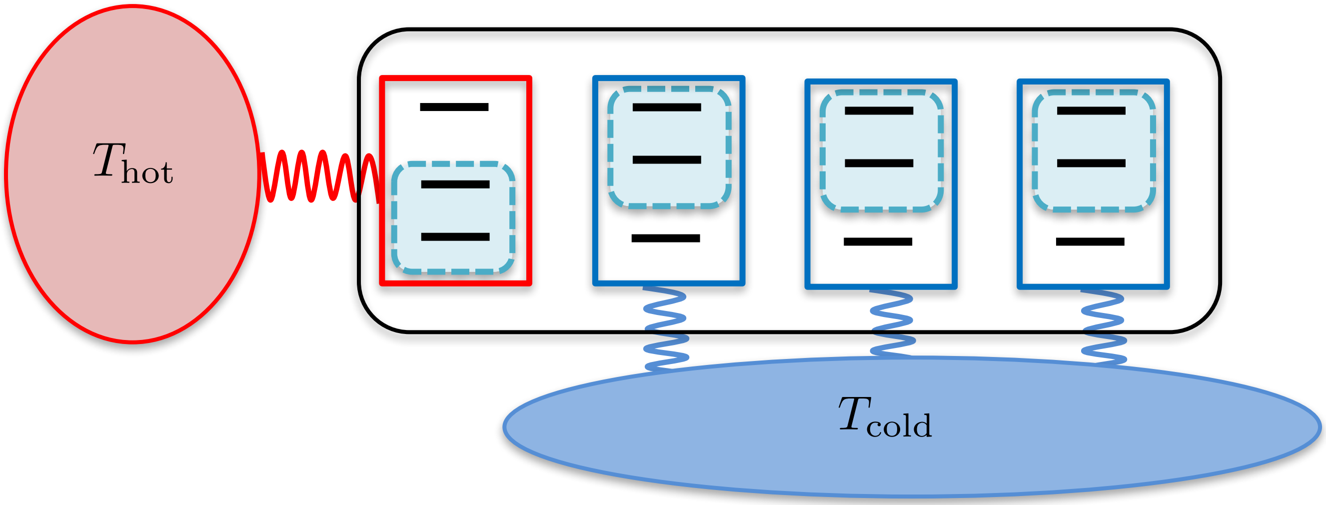
Generalize to multipartite entanglement



- Target state $|\Psi\rangle$
- $H_{\text{free}} = \sum_{k=1}^{N} \left(\sum_{l=1}^{2} \Delta_k^{(l)} |l\rangle_k \langle l| \right)$
- Discarded state for qutrit k : R_k
- $|R\rangle = |R_1, R_2, \ldots, R_k\rangle$
- $H_{int} = g\big(|R\rangle\langle\bar{\Psi}| + h.c.\big)$

Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

Generalize to multipartite entanglement



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Which target admits an entanglement engine?

$$[H_s, H_{int}] = 0$$

Which target can be generated?

$$\Pi \rho_{ss} \Pi \sim |\Psi\rangle\langle\Psi|$$

Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

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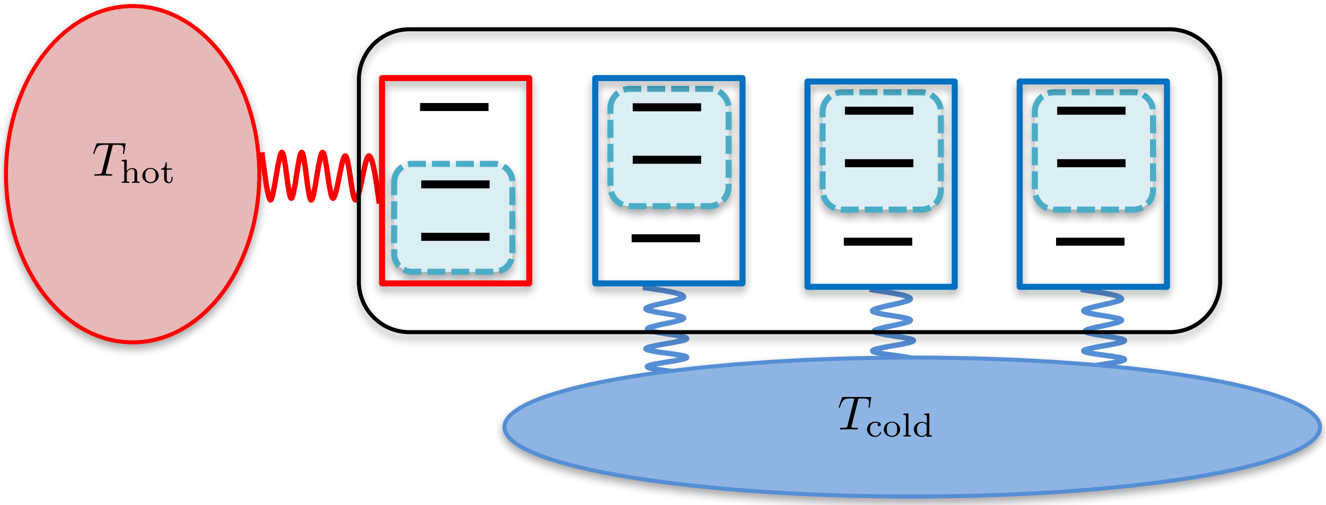

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Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

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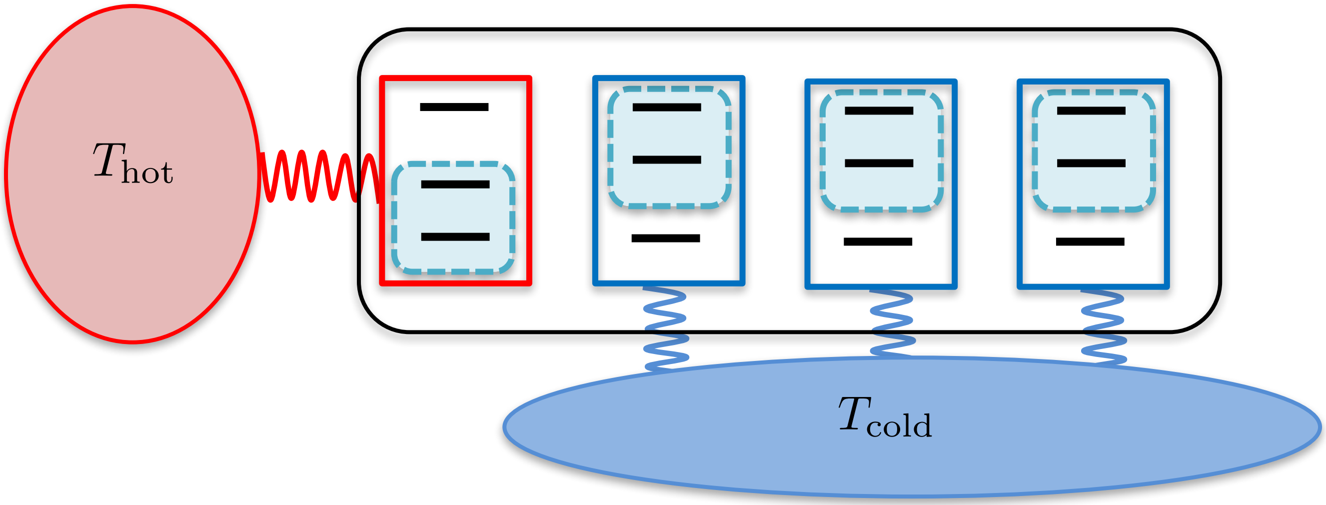

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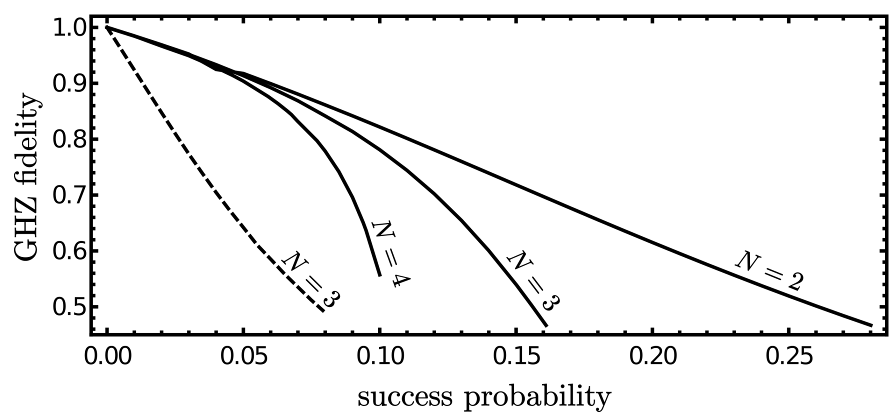
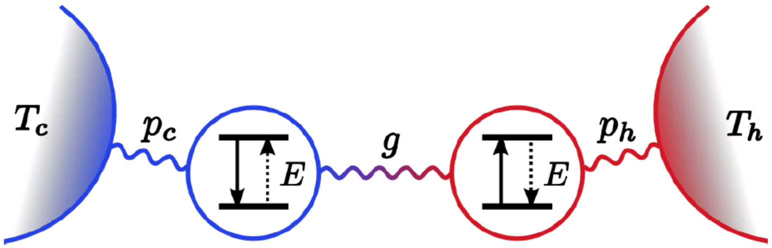
For a GHZ state



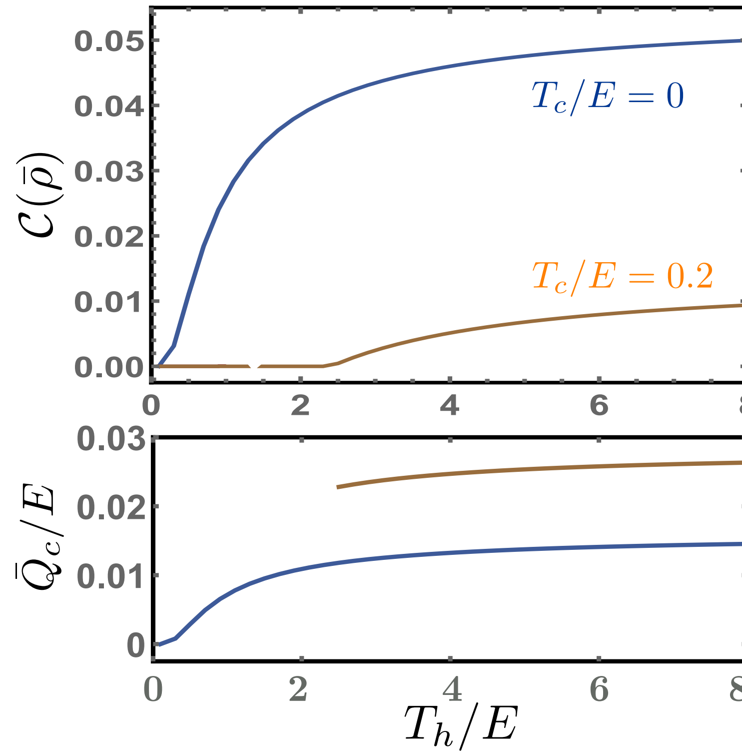
Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

Thermal steady-state entanglement

- Most basic model



$H_{int} = g\left(|01\rangle\langle 10| + h.c.\right)$



- Analytic steady-state

$$\bar{\rho} = \begin{pmatrix} X & 0 & 0 & 0 \\ 0 & X & X & 0 \\ 0 & X & X & 0 \\ 0 & 0 & 0 & X \end{pmatrix} = \gamma\left[p_c p_h \tau_c \otimes \tau_h + \frac{2g^2}{(p_c+p_h)^2}(p_c\tau_c + p_h\tau_h)^{\otimes 2} + \frac{g p_c p_h (r_c - r_h)}{p_c + p_h}\mathcal{Y}\right]$$

- Concurrence (measure of entanglement) : Given by the eigenvalues of $R = \sqrt{\sqrt{\bar{\rho}}\tilde{\bar{\rho}}\sqrt{\bar{\rho}}}$

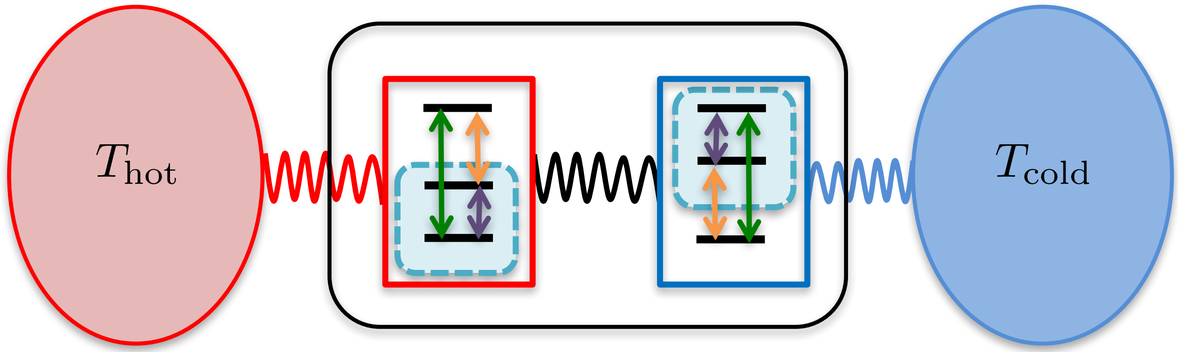
Wooters, PRL (2001)

- Heat flow: $\bar{Q}_c = p_c E \langle 1|\bar{\rho}_c - \tau_c|1\rangle$

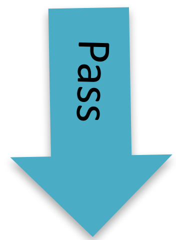
(Parameters optimization for each temp. bias)

J. B. Brask, G. Haack et al., NJP 17 (2015)

Heralded entanglement

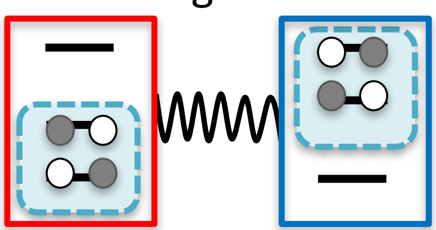


p_{succ}



$$\rho' = \frac{1}{p_{suc}}(\Pi_A \otimes \Pi_B)\bar{\rho}(\Pi_A \otimes \Pi_B)$$

Entangled state



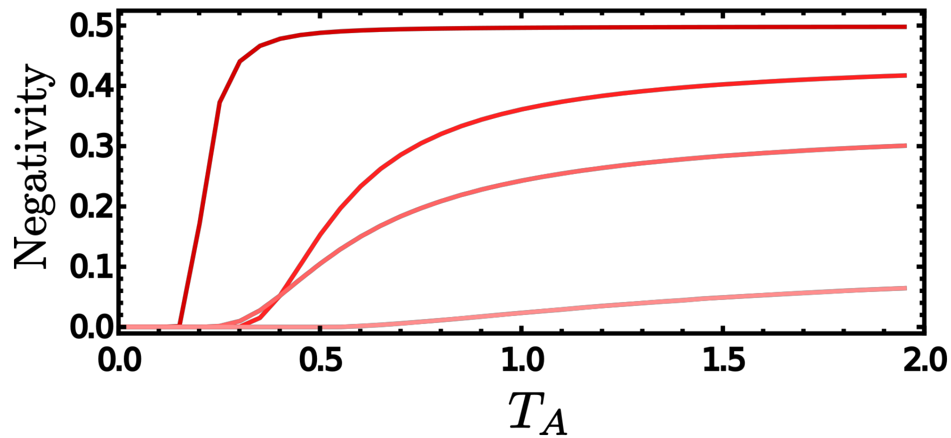
$$\rho' = \begin{pmatrix} \frac{p_h}{4p_h+6p_c} & 0 & 0 & 0 \\ 0 & \frac{p_h+3p_c}{4p_h+6p_c} & \frac{3p_c}{4p_h+6p_c} & 0 \\ 0 & \frac{3p_c}{4p_h+6p_c} & \frac{p_h+3p_c}{4p_h+6p_c} & 0 \\ 0 & 0 & 0 & \frac{p_h}{4p_h+6p_c} \end{pmatrix}.$$

$$T_h \to \infty\,, T_c = 0$$

$$p_c \gg p_h$$

Heralded entanglement

- Finite temperature



Top to bottom

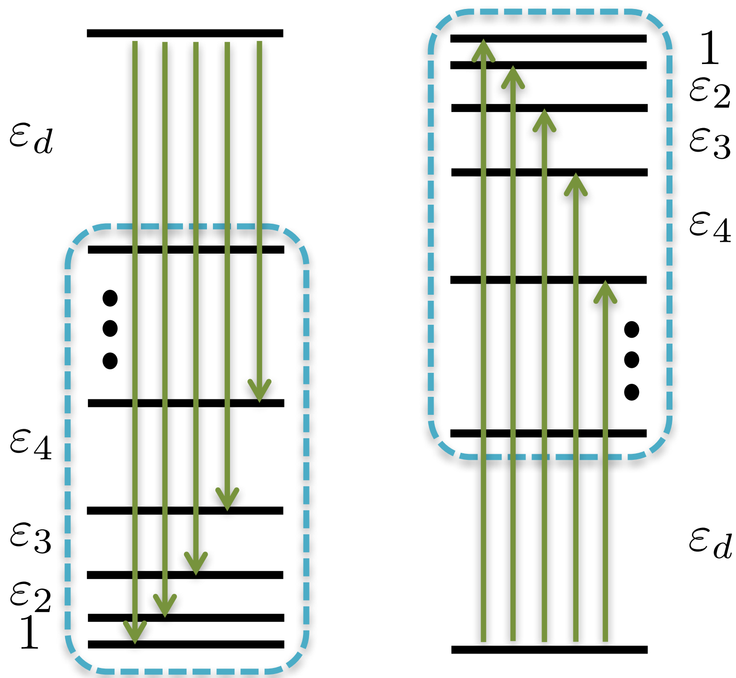
$(T_B, \epsilon) = (0.1, 1.5)$

$(T_B, \epsilon) = (0.2, 1.5)$

$(T_B, \epsilon) = (0.1, 0.5)$

$(T_B, \epsilon) = (0.1, 0.5)$

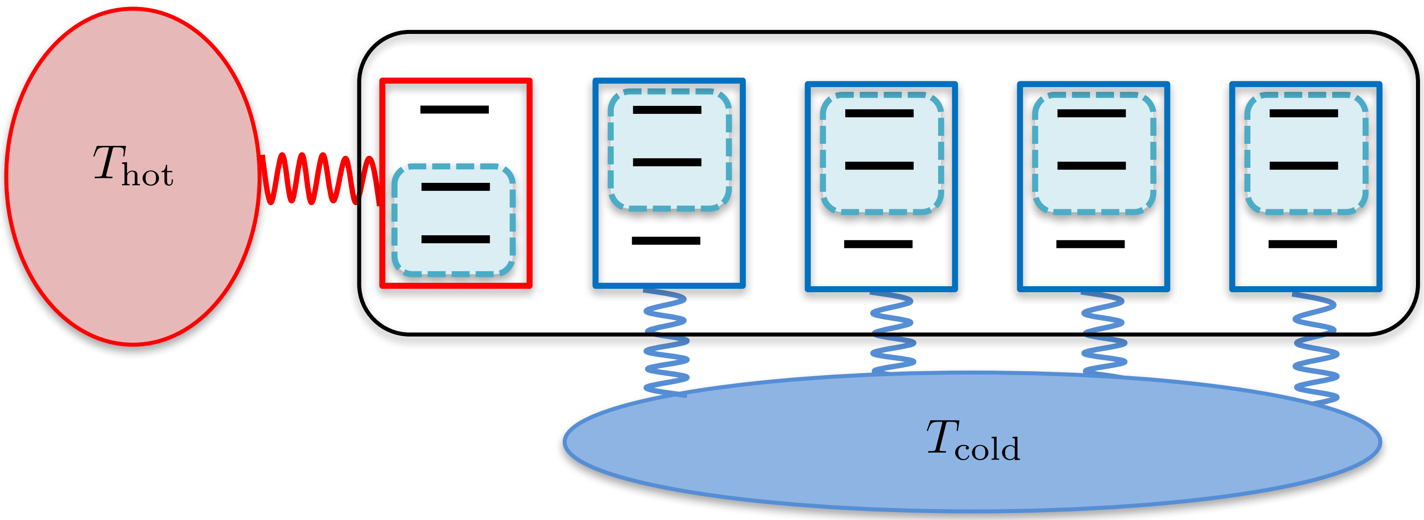
- Arbitrary dimension



$$H_{int} = \sum_{k=1}^{d} g_k \Big(|d, 0\rangle\langle k-1, d-k+1| + h.c. \Big)$$

$$|S_d\rangle = \frac{1}{\sqrt{d}} \sum_{k=1}^{d} |k-1, d-k\rangle.$$

Generalize to multipartite entanglement



- Target state $|\Psi\rangle$
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Which target admits an entanglement engine?

$$[H_s, H_{int}] = 0$$

To be determined: $|R\rangle, \Delta_k$

Which target can be generated?

$$\Pi \rho_{ss} \Pi \sim |\Psi\rangle\langle\Psi|$$

All the above!

Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

Proof

- Target state $|\Psi\rangle$
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$$\Pi\rho_{ss}\Pi \sim |\Psi\rangle\langle\Psi|$$

All the above!

1. Autonomy condition -> $|R\rangle, \Delta_k$

2. If this condition is satisfied for q hot qutrits and N-q cold qutrits, then it can also be satisfied for 1 hot qutrit and N-1 cold ones

3. In the limit $T_h \to \infty \,, T_c = 0$, $\rho' = \dfrac{\Pi\rho_{ss}\Pi}{\mathrm{Tr}\{\Pi\rho_{ss}\}} = |\bar{\Psi}\rangle\langle\bar{\Psi}|$

Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

Autonomy condition

- Target state $|\Psi\rangle$
- $H_{\text{free}} = \sum_{k=1}^{N} \left(\sum_{l=1}^{2} \Delta_k^{(l)} |l\rangle_k \langle l| \right)$
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To be determined: $|R\rangle, \Delta_k$

Which target can be generated?

$$\Pi\rho_{ss}\Pi \sim |\Psi\rangle\langle\Psi|$$

All the above!

$$|\bar{\Psi}\rangle = \sum_{\bar{n} \in S_{|\bar{\Psi}\rangle}} c_{\bar{n}} |\bar{n}\rangle$$

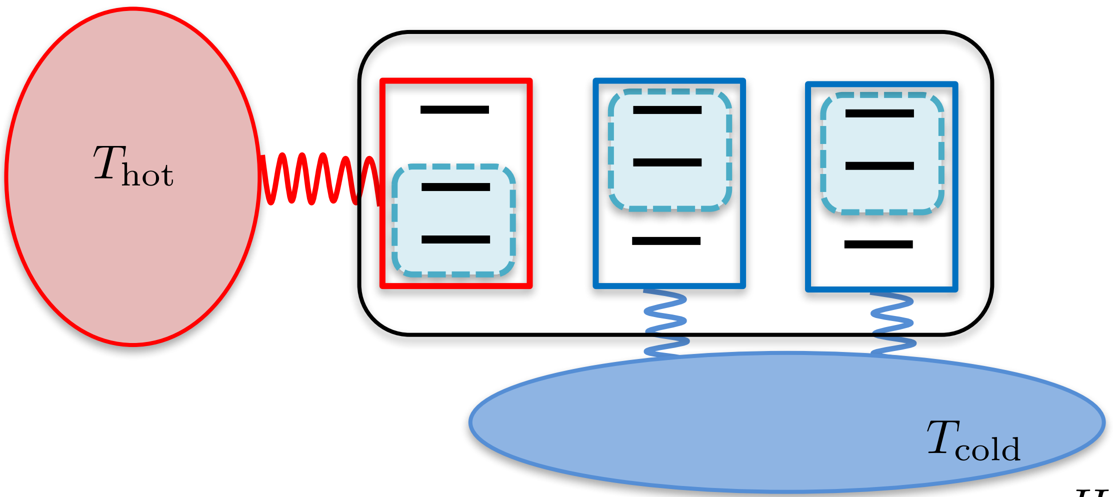
Both $|R\rangle$ and $|\bar{\Psi}\rangle$ are eigenstates of H_{free} with eigenvalues E_R and $E_{\bar{n}}$

$$[H_s, H_{int}] = 0 \quad \Leftrightarrow \quad E_{\bar{n}} = E_R$$

$$\frac{1}{2}\sum_{k=1}^{N}\left[R_k n_k \Delta_k^{(1)} + (2 - R_k)((1 - n_k)\Delta_k^{(1)} + n_k \Delta_k^{(2)})\right] - \frac{1}{2}\sum_{k=1}^{N}\left[R_k \Delta_k^{(2)}\right] = 0$$

Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

Example: GHZ states with N=3



$$|GHZ\rangle = |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|100\rangle + |011\rangle\right)$$

$$|\bar{\Psi}\rangle = \frac{1}{\sqrt{2}}\left(|111\rangle + |022\rangle\right)$$

$$|R\rangle = |200\rangle$$

$$H_{int} = g\big(|200\rangle\langle 111| + |200\rangle\langle 022| + h.c.\big)$$

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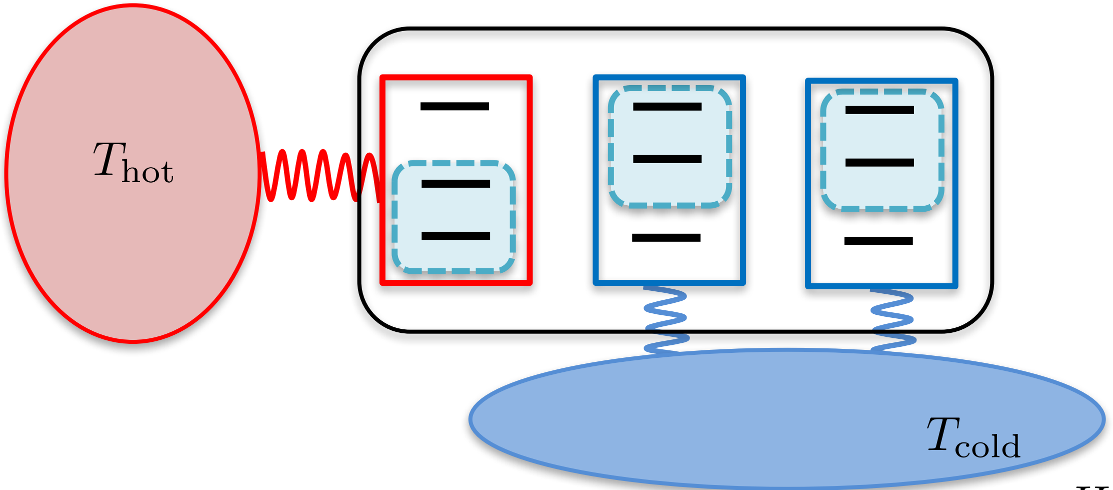

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$$|R\rangle = |200\rangle$$

$$H_{int} = g\big(|200\rangle\langle 111| + |200\rangle\langle 022| + h.c.\big)$$

1. Autonomy condition

$$E_R = \Delta_h^{(2)}$$

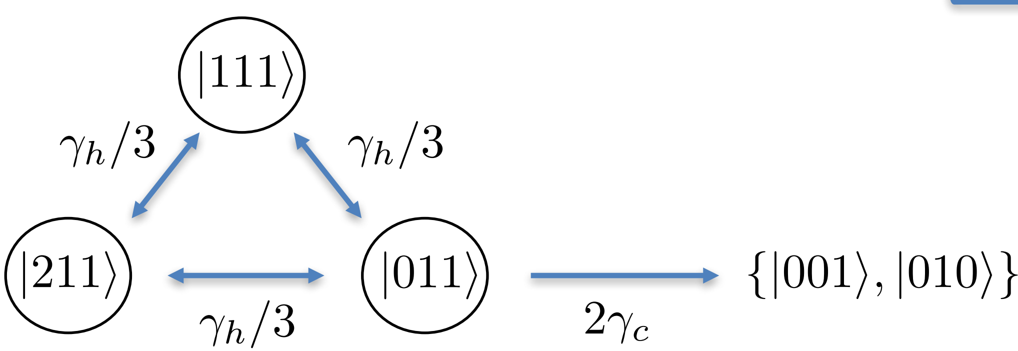
$$E_{111} = \Delta_h^{(1)} + 2\Delta_c^{(1)}$$

$$E_{022} = 2\Delta_c^{(2)}$$

$$\Delta_c^{(2)} = \Delta_h^{(2)}/2$$

$$\Delta_c^{(1)} = (\Delta_h^{(2)} - \Delta_h^{(1)})/2$$

3. Flow diagram

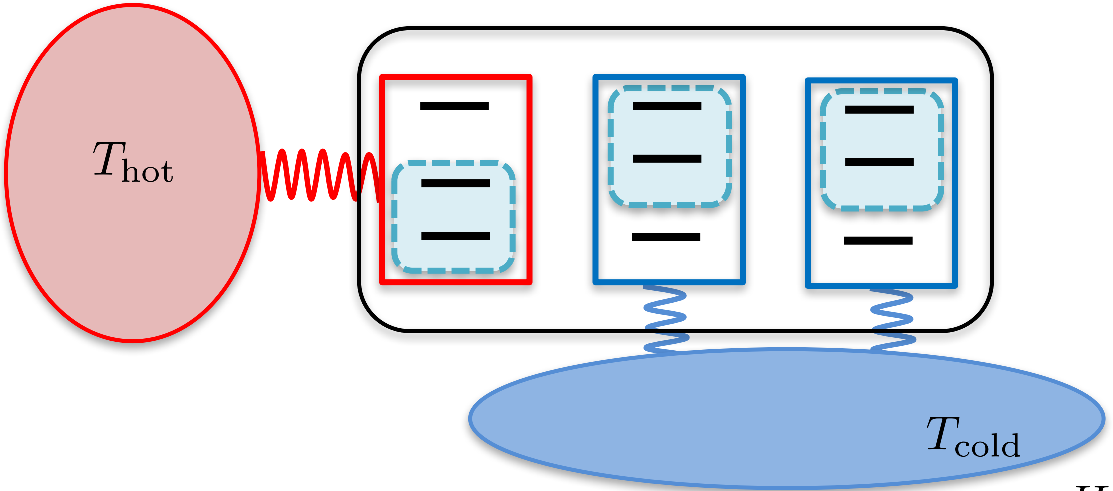


$$P_{011}\big(2\gamma_h/3 + 2\gamma_c\big) = \gamma_h/3\big(P_{111} + P_{211}\big)$$

Goes out — Comes in

Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

Example: GHZ states with N=3



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$$E_{022} = 2\Delta_c^{(2)}$$

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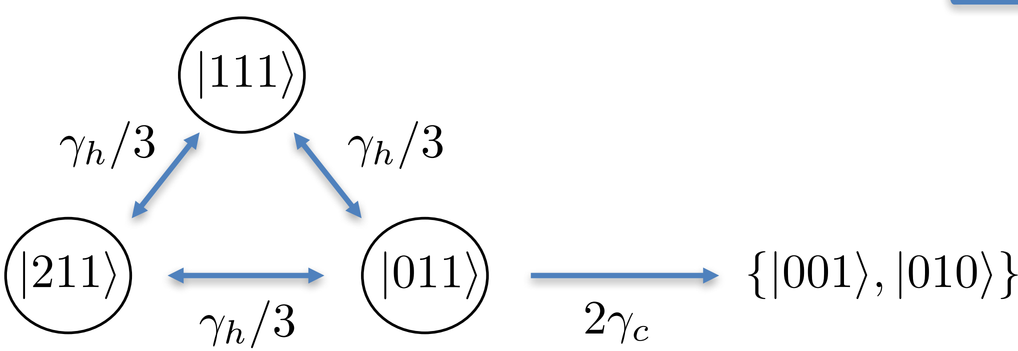
$$\Delta_c^{(1)} = (\Delta_h^{(2)} - \Delta_h^{(1)})/2$$

3. Flow diagram

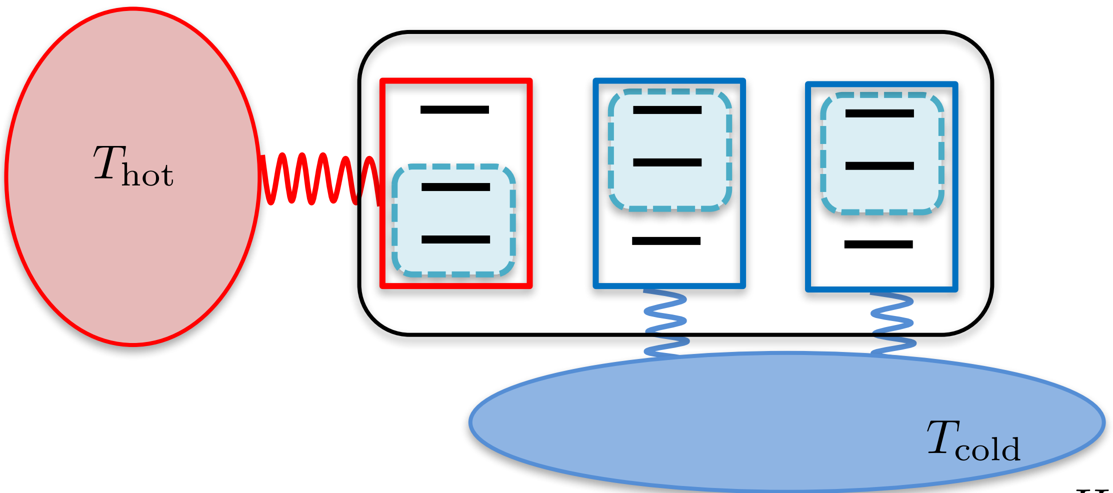


$$\underbrace{P_{011}\big(2\gamma_h/3 + 2\gamma_c\big)}_{\text{Goes out}} = \underbrace{\gamma_h/3\big(P_{111} + P_{211}\big)}_{\text{Comes in}}$$

$$\frac{P_o}{P_S} = \frac{\gamma_h}{\gamma_h + 6\gamma_c}$$

Tavakoli, Haack, Brunner, Brask, arXiv:1906.00022

Example: GHZ states with N=3



$$|GHZ\rangle = |\Psi\rangle = \frac{1}{\sqrt{2}}\big(|100\rangle + |011\rangle\big)$$

$$|\bar{\Psi}\rangle = \frac{1}{\sqrt{2}}\big(|111\rangle + |022\rangle\big)$$

$$|R\rangle = |200\rangle$$

$$H_{int} = g\big(|200\rangle\langle 111| + |200\rangle\langle 022| + h.c.\big)$$

In the filtered subspace, normalization:

$$\bar{P}_{111} + \bar{P}_{022} + \bar{P}_{011} + \bar{P}_{122} = 1$$

$$\bar{P}_S + \bar{P}_o = 1/2 \Leftrightarrow \bar{P}_S\big(1 + \frac{\bar{P}_o}{\bar{P}_S}\big) = 1/2$$

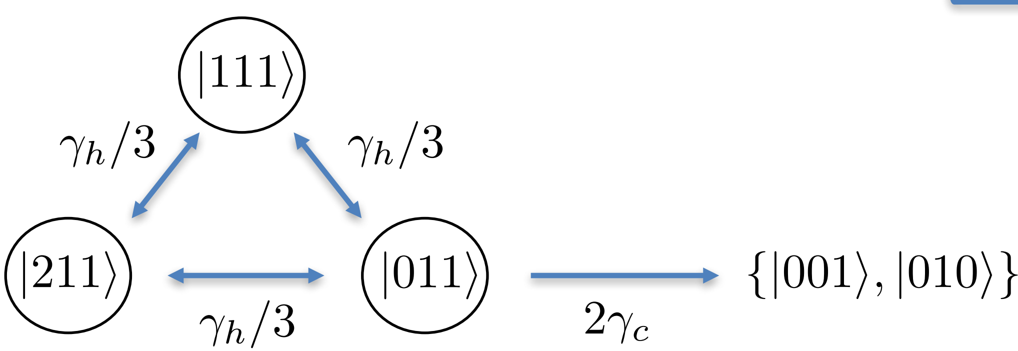
Ratio is conserved

$$\bar{P}_S \to \frac{1}{2} \qquad \gamma_h \ll \gamma_c$$

$$P_{011}\big(2\gamma_h/3 + 2\gamma_c\big) = \gamma_h/3\big(P_{111} + P_{211}\big)$$

Goes out — Comes in

$$\frac{P_o}{P_S} = \frac{\gamma_h}{\gamma_h + 6\gamma_c}$$



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