Does the presenter work?

Binary symmetric Ramsey classes via semigroup-valued metric spaces

Completing edge-labelled graphs

Matěj Konečný

Faculty of Mathematics any Physics, Charles University

Unifying Themes in Ramsey Theory

Joint work with Jan Hubička and Jaroslav Nešetřil

Michael Kompatscher, colorized

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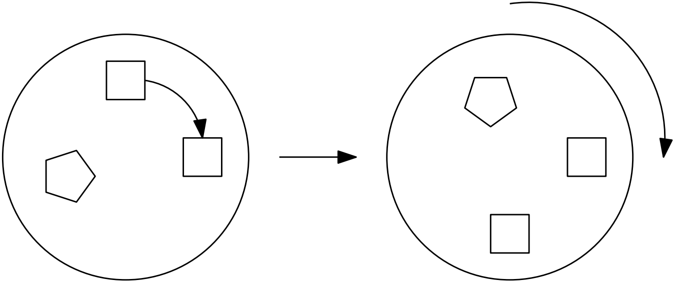
Homogeneous structures

A (not necessarily relational) structure **A** is homogeneous if every isomorphism of its finite substructures can be extended to an automorphism of **A**.



Definition

A class of finite structures $\mathcal{C}$ is an amalgamation class if it is the class of all finite substructures of some homogeneous structure.

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Example (Countably infinite homogeneous graphs, Lachlan–Woodrow 1980)

If **G** is a countably infinite homogenous graph, then **G** or its complement $\overline{\mathbf{G}}$ is one of the following:

1. the random (Rado) graph,
2. the generic K_n-free graph for $3 \leq n < \infty$,
3. the disjoint union of infinitely many K_n's for $1 \leq n \leq \omega$ or the disjoint union of finitely many K_ω's.

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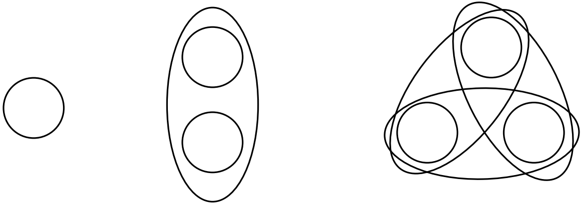
Example

The class $\mathcal{M}_\mathbb{Z}$ of all finite integer-valued metric spaces is an amalgamation class, the corresponding homogeneous structure is called the integer Urysohn's space.

Ramsey classes

Class $\mathcal{K}$ of finite structures has the Ramsey property (is Ramsey) if for every $\mathbf{A}, \mathbf{B} \in \mathcal{K}$ there is $\mathbf{C} \in \mathcal{K}$ such that for every colouring $c\colon \binom{\mathbf{C}}{\mathbf{A}} \to \{0, 1\}$ there is $\mathbf{B}' \in \binom{\mathbf{C}}{\mathbf{B}}$ such that $c|_{\mathbf{B}'}$ is constant.



A **B** **C**

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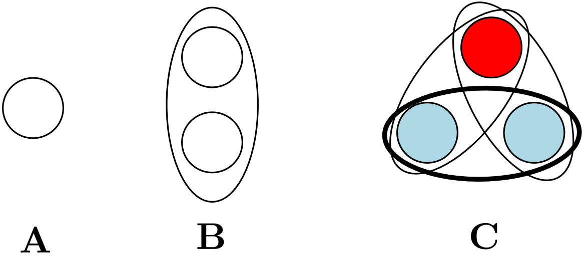

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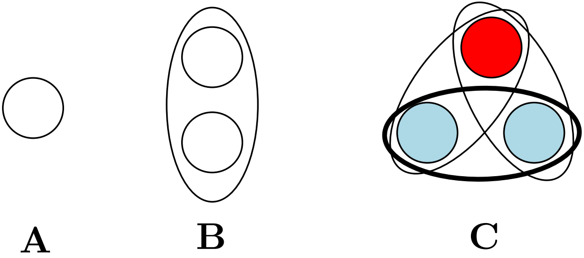

Theorem (Nešetřil, 2005)

Every Ramsey class with JEP is an amalgamation class.

EPPA

Class $\mathcal{K}$ of finite structures has EPPA (extension property for partial automorphisms, also Hrushovski property) if for every $\mathbf{A} \in \mathcal{K}$ there is $\mathbf{B} \in \mathcal{K}$ such that $\mathbf{A} \subseteq \mathbf{B}$ and every partial automorphism of $\mathbf{A}$ extends to an automorphism of $\mathbf{B}$.

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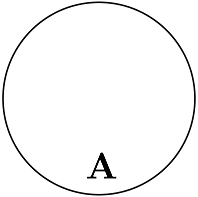

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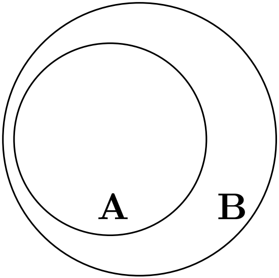

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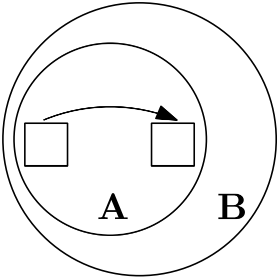

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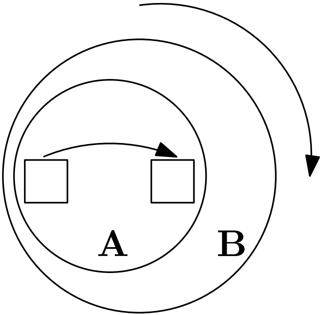

Examples of binary symmetric Ramsey classes

- All *ordered* graphs and K_n-free graphs (Nešetřil–Rödl 1977).
- *Convexly ordered* metric spaces with distances from $S \subseteq \mathbb{R}^{>0}$ whenever they form an amalgamation class (Sauer 2013; Hubička–Nešetřil 2016).
- *Convexly ordered* generalised metric spaces with distances from a linearly ordered monoid (Conant 2015; Hubička–K–Nešetřil 2017).
- Cherlin's *ordered* metrically homogeneous graphs (Cherlin 2011; AB-WHHKKKP 2017).
- *Convexly ordered* lattice-like nested equivalences (Braunfeld 2017).

The completion problem for edge-labelled graphs

L is a set, $\mathcal{C}$ a class of finite **complete** L-edge-labelled graphs. For $\mathbf{G}$ an L-edge-labelled graph, can we add the remaining edges and their labels to get $\overline{\mathbf{G}} \in \mathcal{C}$?

We call such $\overline{\mathbf{G}}$ a completion of $\mathbf{G}$. If $\mathrm{Aut}(\mathbf{G}) = \mathrm{Aut}(\overline{\mathbf{G}})$, $\overline{\mathbf{G}}$ is an automorphism-preserving completion of $\mathbf{G}$

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Example (Graphs)

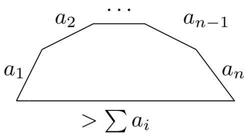
$L = \{N, E\}$, $\mathcal{G}$ is the class of all finite complete L-edge-labelled graphs. Then every $\mathbf{G}$ has an automorphism-preserving completion in $\mathcal{G}$.

Metric spaces

$L = \mathbb{R}^{>0}$, $\mathbf{M} \in \mathcal{M}_{\mathbb{R}>0} \iff$ every triangle of $\mathbf{M}$ satisfies the triangle inequality.

Proposition

$\mathbf{G}$ *has a completion in* $\mathcal{M}_{\mathbb{R}>0} \iff$ *it contains no non-metric cycle:*



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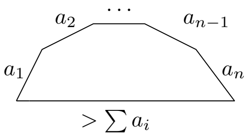

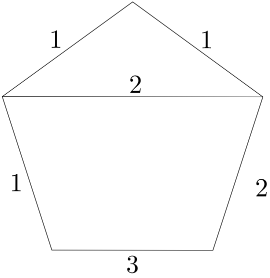

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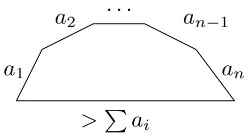

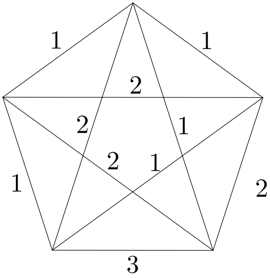

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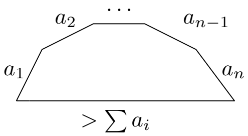

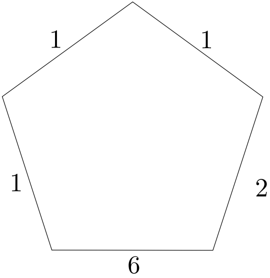

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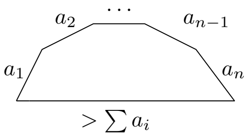

Shortest path completion

$\mathbf{G} = (V, E, \ell)$, $\ell \colon E \to L$. Define $d \colon \binom{V}{2} \to L$ as

$$d(x, y) := \min\{\|\mathbf{P}\| : \mathbf{P} \text{ is a path from } x \text{ to } y \text{ in } \mathbf{G}\},$$

where $\|\mathbf{P}\|$ is the sum of labels of $\mathbf{P}$.
We call $\overline{\mathbf{G}} = (V, \binom{V}{2}, d)$ the shortest path completion of $\mathbf{G}$.

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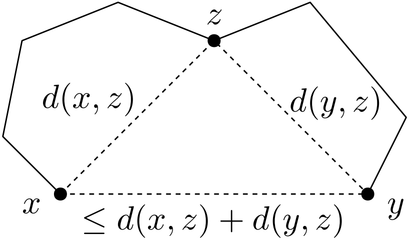
Lemma

$\overline{\mathbf{G}} = (V, \binom{V}{2}, d) \in \mathcal{M}_{\mathbb{R}>0}$.

Proof.



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Corollary

Let $\mathcal{O}$ be the family of all non-metric L-edge-labelled cycles. Then $\mathbf{G}$ has an automorphism-preserving completion in $\mathcal{M}_{\mathbb{R}>0}$ if and only if $\mathbf{G} \in \mathrm{Forb}(\mathcal{O})$, that is, there is no $\mathbf{O} \in \mathcal{O}$ with a homomorphism $\mathbf{O} \to \mathbf{G}$.

Why should one care?

Definition (Finite obstacles)

$\mathcal{K}$ is an amalgamation class of finite L-structures. $\mathcal{K}$ has finite obstacles if there is a family $\mathcal{O}$ of L-structures s.t.

1. for every finite $S \subseteq L$ there are only finitely many S-structures in $\mathcal{O}$,
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Theorem (Herwig–Lascar, 2000)

Every relational amalgamation class which has finite obstacles and an automorphism-preserving completion has EPPA.

Theorem (Hubička–Nešetřil, 2015; imprecise statement)

Every "reasonable" ordered amalgamation class whose orderless reduct has finite obstacles is Ramsey.

EPPA for metric spaces

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Theorem (Solecki, 2005; Vershik, 2007)

$\mathcal{M}_{\mathbb{R}>0}$ *has EPPA.*

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Proof.

- If $S = \{s_1 < s_2 < \ldots < s_k\} \subset \mathbb{R}^{>0}$ then each non-metric cycle with edges from S has at most $\lceil \frac{s_k}{s_1} \rceil$ vertices.
- The shortest path completion preserves automorphisms.

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Advertisement

A combinatorial proof of the extension property for partial isometries (Hubička–K–Nešetřil, 2018, arXiv:1807.10976). 7 pages.

Examples of classes "with finite obstacles"

(after eliminating imaginaries)

- All graphs and K_n-free graphs
- Metric spaces with distances from $S \subseteq \mathbb{R}^{>0}$ whenever they form an amalgamation class
- Generalised metric spaces with distances from a linearly ordered monoid
- Cherlin's metrically homogeneous graphs
- Lattice-like nested equivalences

$\mathfrak{M}$-valued metric spaces

Definition (Partially ordered commutative semigroup (POCS))

$\mathfrak{M} = (M, \oplus, \preceq)$ is a partially ordered commutative semigroup if:

1. $(M, \oplus)$ is a commutative and associative operation,
2. $(M, \preceq)$ is a partial order, and
3. $\oplus$ is monotone in $\preceq$ ($a \preceq b \Rightarrow a \oplus c \preceq b \oplus c$).

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Definition ($\mathfrak{M}$-valued metric space)

Let $\mathfrak{M} = (M, \oplus, \preceq)$ be a POCS. A complete $\mathfrak{M}$-edge-labelled graph $\mathbf{G} = (V, d)$ with $d\colon \binom{V}{2} \to \mathfrak{M}$ is an $\mathfrak{M}$-valued metric space if for every $x \neq y \neq z \in V$ it holds that

$$d(x, y) \oplus d(y, z) \succeq d(x, z).$$

We denote by $\mathcal{M}_{\mathfrak{M}}$ the class of all finite $\mathfrak{M}$-valued metric spaces.

Examples

- If $\mathfrak{M} = (\mathbb{R}^{>0}, +, \leq)$ then $\mathcal{M}_{\mathfrak{M}}$ is the class of all finite metric spaces.
- If $\mathfrak{M} = (\mathbb{N}, \cdot, |)$, then the $\mathfrak{M}$-valued metric spaces are the divisibility metric spaces.
- If $\Lambda = (\Lambda, \vee, \leq)$ is a distributive lattice, then Λ-valued metric spaces are Braunfeld's nested equivalences.
- If $S \subseteq \mathbb{R}^{>0}$ is one of Sauer's subset, $x \oplus_S y := \sup\{s \in S : s \leq x + y\}$ and $\mathfrak{M} = (S, \oplus_S, \leq)$, then $\mathcal{M}_{\mathfrak{M}}$ is one of Sauer's classes.

Our result

Theorem

Let $\mathfrak{M} = (M, \oplus, \preceq)$ *be a POCS and let* $\mathcal{F}$ *be a "sufficiently nice" family of* $\mathfrak{M}$*-edge-labelled cycles. Then the class* $\mathcal{M}_{\mathfrak{M}} \cap \mathrm{Forb}(\mathcal{F})$ *is an amalgamation class, has EPPA and a precompact Ramsey expansion.*

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Shortest path completion for $\mathfrak{M}$

$\mathbf{G} = (V, E, \ell)$ is a finite $\mathfrak{M}$-edge-labelled graph. Define

$$d(x, y) := \inf_{\preceq}\{\|\mathbf{P}\| : \mathbf{P} \text{ is a path from } x \text{ to } y \text{ in } \mathbf{G}\},$$

where $\|\mathbf{P}\|$ is the $\oplus$-sum of the labels of $\mathbf{P}$. We call $\overline{\mathbf{G}} = (V, \binom{V}{2}, d)$ the $\mathfrak{M}$-shortest path completion of $\mathbf{G}$.

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Definition (Sufficiently nice family $\mathcal{F}$)

$\mathcal{F}$ is sufficiently nice if it ensures that the shortest path completion is defined, behaves well w.r.t. it and for every finite $S \subseteq \mathfrak{M}$ there are only finitely many S-edge-labelled cycles in $\mathcal{F}$.

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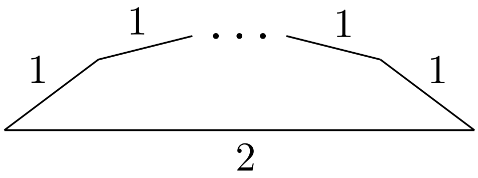
Let $\mathfrak{M} = (M, \oplus, \preceq)$ be a POCS and let $\mathcal{F}$ be a "sufficiently nice" family of $\mathfrak{M}$-edge-labelled cycles. Then the class $\mathcal{M}_{\mathfrak{M}} \cap \mathrm{Forb}(\mathcal{F})$ is an amalgamation class, has EPPA and a precompact Ramsey expansion.

Example

Let $\mathfrak{M} = (\{1, 2\}, \max, \leq)$ be an ultrametric space. Then the completion problem for $\mathcal{M}_{\mathfrak{M}}$ is not easy.

Proof.



Proof overview

1. Prove strong amalgamation and completion — infinite $\mathcal{O}$
2. Study semigroup structure
   2.1 "Blocks" (subsets defining equivalences)
   2.2 Approximation of block maxima
3. Elminate imaginaries
   3.1 Define an expansion
   3.2 Prove strong amalgamation and completion — finite $\mathcal{O}$ (with unary functions)
   3.3 EPPA (**See Honza's talk!**)
4. Introduce an order (like Braunfeld)
   4.1 Define an expansion
   4.2 Prove strong amalgamation and completion
   4.3 Prove Ramsey and the expansion property
5. Applications

Corollaries

Class	**Ramsey**	**EPPA**
S-metric spaces	HN16	new (part Conant15)
Conant's generalized metric spaces	HKN18	new (part Conant15)
Braunfeld's nested equivalences	Braunfeld17	new
Metrically homogeneous graphs	AB-WHHKKKP 2017	AB-WHHKKKP 2017
Cherlin's 4-edge-labelled graphs	new using Li 2018+	new using Li 2018+
Divisibility metric spaces, ...	new	new

Remarks

SIR

Shortest path completion gives a SIR (stationary independence relation). Tent and Ziegler used SIR to prove bounded simplicity of the automorphism group of the Urysohn space.

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Henson constraints

One can forbid cliques consisting of irreducible distances (i.e. not created by the shortest path completion).

Conjectures

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Let $\mathcal{C}$ be an amalgamation class of M-edge-labelled graphs and assume that $\mathcal{C}$ has finite obstacles. Then there is a partially ordered commutative semigroup $\mathfrak{M} = (M, \oplus, \preceq)$ and a "sufficiently nice" family $\mathcal{F}$ of $\mathfrak{M}$-edge-labelled cycles such that $\mathcal{C} = \mathcal{M}_{\mathfrak{M}} \cap \mathrm{Forb}(\mathcal{F})$.

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Conjecture

Every triangle-constrained strong amalgamation class of (finitely-)edge-labelled graphs where all definable equivalence relations have infinite index corresponds to a semigroup-valued metric space as above.

Conjectures

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Let $\mathcal{C}$ be an amalgamation class of M-edge-labelled graphs and assume that $\mathcal{C}$ has finite obstacles. Then there is a partially ordered commutative semigroup $\mathfrak{M} = (M, \oplus, \preceq)$ and a "sufficiently nice" family $\mathcal{F}$ of $\mathfrak{M}$-edge-labelled cycles such that $\mathcal{C} = \mathcal{M}_{\mathfrak{M}} \cap \mathrm{Forb}(\mathcal{F})$.

Conjecture

Every triangle-constrained strong amalgamation class of (finitely-)edge-labelled graphs where all definable equivalence relations have infinite index corresponds to a semigroup-valued metric space as above.

Weak evidence

Probably this holds for primitive classes with at most 5 labels.

The end!

Thank you for your attention
Questions?

Can you show more pictures of Michael?

Sure.

Tell me more about the family $\mathcal{F}$

For example all odd cycles up to some diameter in $\mathcal{M}_{\mathbb{Z}}$ work.

1. $\mathcal{F}$ needs to be closed on homomorphisms.
2. $\mathcal{F}$ needs to be closed on metrically adding edges.
3. $\mathcal{F}$ needs to be closed on the inverse steps of the shortest path completion.
4. $\mathcal{F}$ needs to ensure that all infima encountered by the SPC in $\mathrm{Forb}(\mathcal{F})$ exist (if $\inf(a, b)$ is not defined, all cycles constisting of $\mathbf{P}_1$ and $\mathbf{P}_2$ such that $\|\mathbf{P}_1\| = a$ and $\|\mathbf{P}_2\| = b$ must be forbidden).
5. $\mathcal{F}$ needs to ensure that all infima encountered distribute with the addition.

Which structures are not semigroup-valued?

Great question!

1. Bipartite metric spaces of finite diameter.
2. Bipartite metric spaces of infinite diameter.
3. Other things with equivalences with finitely many classes (or of finite index).
4. Non-strong amalgamation classes like the antipodal spaces.

What do you mean by eliminating imaginaries?

We add new vertex for every equivalence class corresponding to every proper meet-irreducible block, link its original vertices to it by unary functions and link the added ball vertices representing classes in inclusion by other unary functions.