PTASes for (subset) TSP in minor-free graphs

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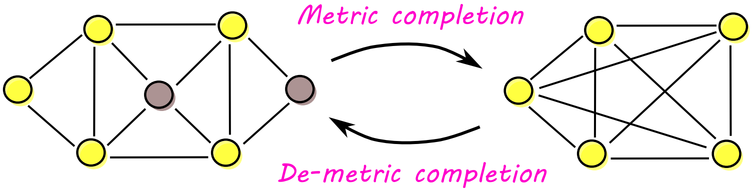
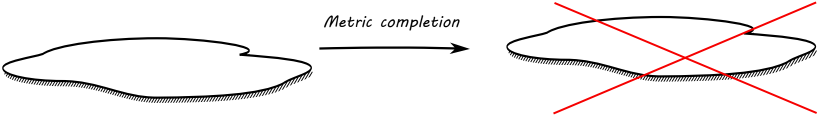
Subset TSP

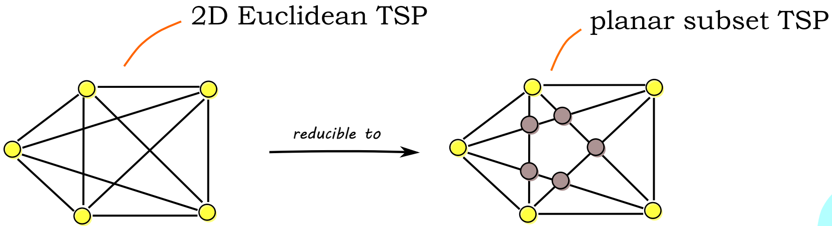
Definition

Given an edge weighted graph G and a set $S \subseteq V(G)$, find a shortest tour going through all vertices of S.

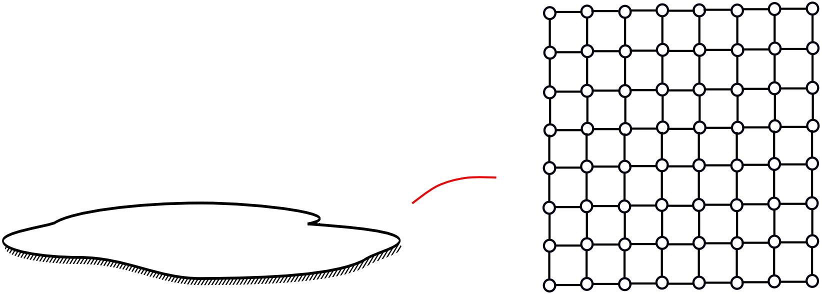
Subset TSP vs TSP



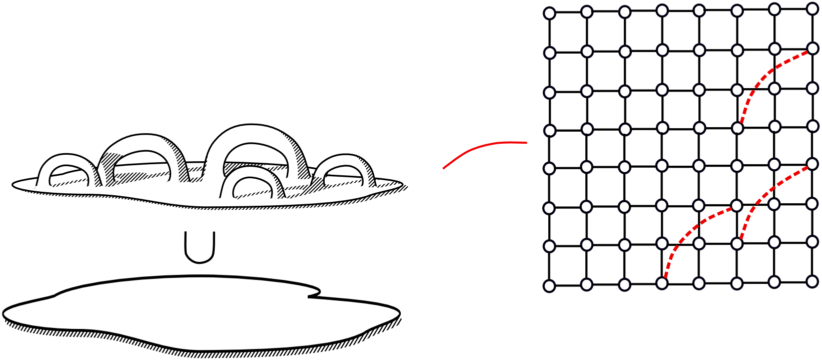




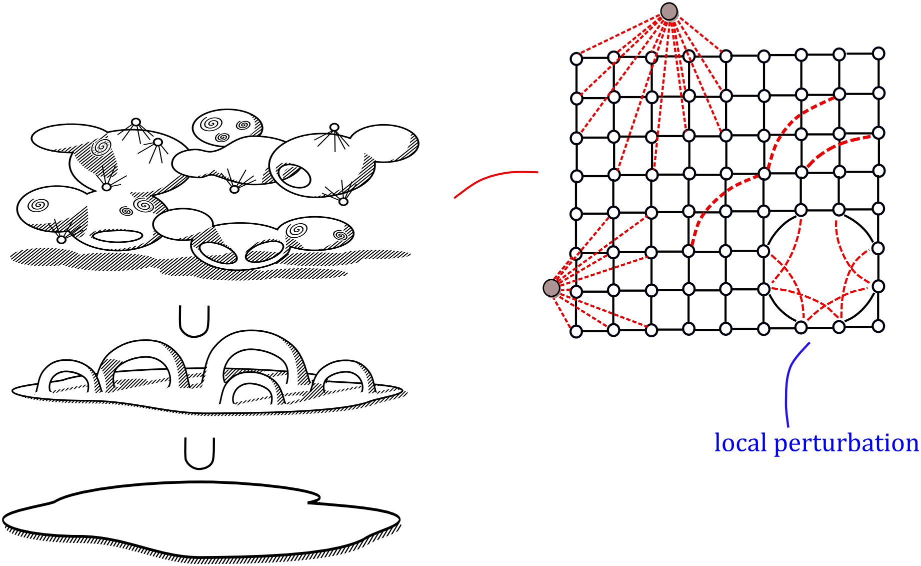
Planar graphs



Bounded genus graphs



Minor-free graphs



Polynomial Time Approximation Scheme

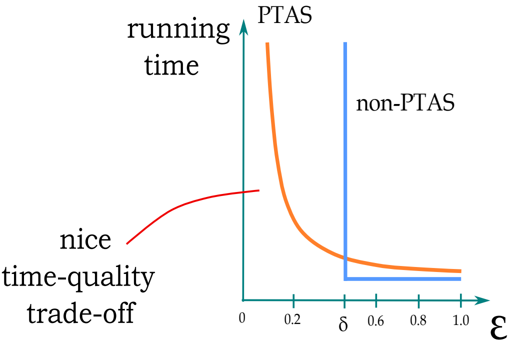
fixed < 1

$O(n^{1/\varepsilon})$

PTAS: find $(1+\varepsilon)$OPT in poly-time

$O(2^{1/\varepsilon} \cdot n)$

Efficient PTAS



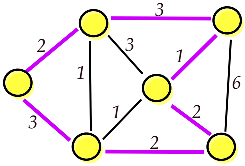
Main Results

Theorem (BLW17)

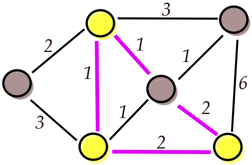
There is an efficient PTAS for TSP in minor-free graphs

Theorem (Le18)

There is a PTAS for Subset TSP in minor-free graphs



Efficient PTAS $O(2^{1/\varepsilon} \cdot n)$

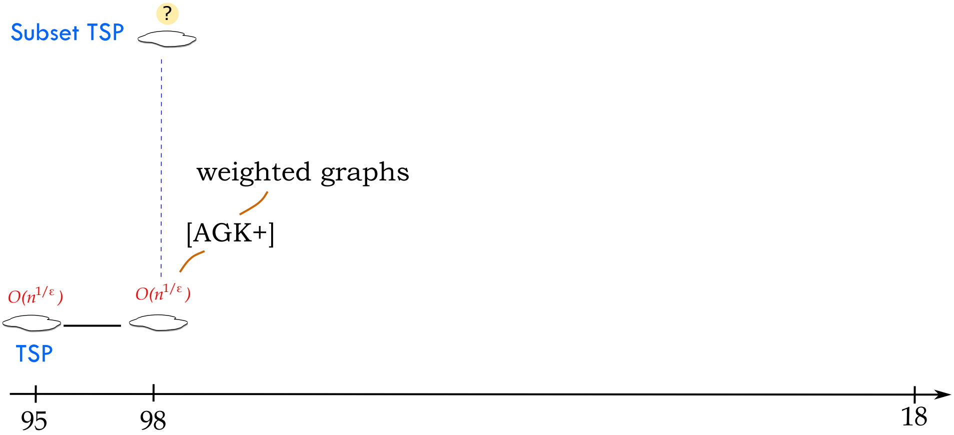


PTAS $O(n^{1/\varepsilon})$

History

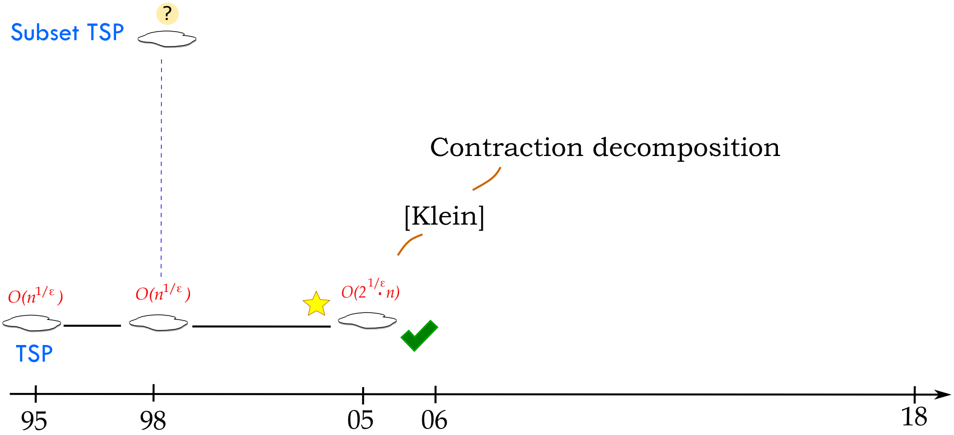
[GKP] M. Grigni, E. Koutsoupias, and C. H. Papadimitriou, "*An approximation scheme for planar graph TSP,*" FOCS'95.

History



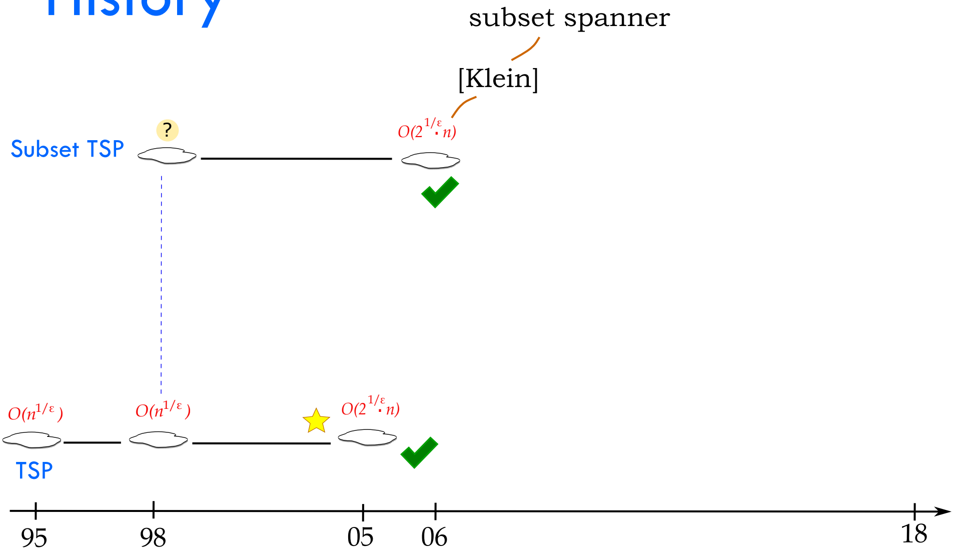
[AGK+] S. Arora, M. Grigni, D. R. Karger, P. N. Klein, A. Woloszyn "*A polynomial-time approximation scheme for weighted planar graph TSP,*" SODA'98.

History



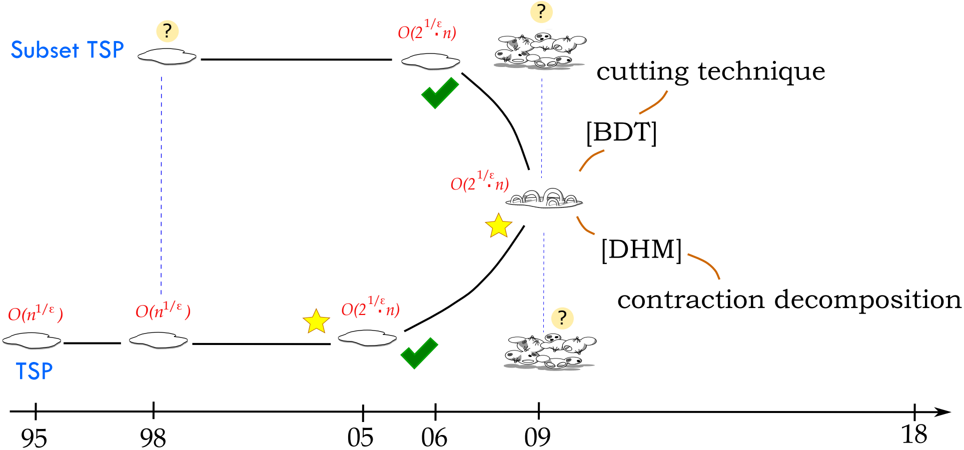
[Klein] P. N. Klein, "*A linear-time approximation scheme for planar weighted tsp*", FOCS'05.

History



[Klein] P. N. Klein, "*A Subset Spanner for Planar Graphs, with Application to Subset TSP*" STOC'06.

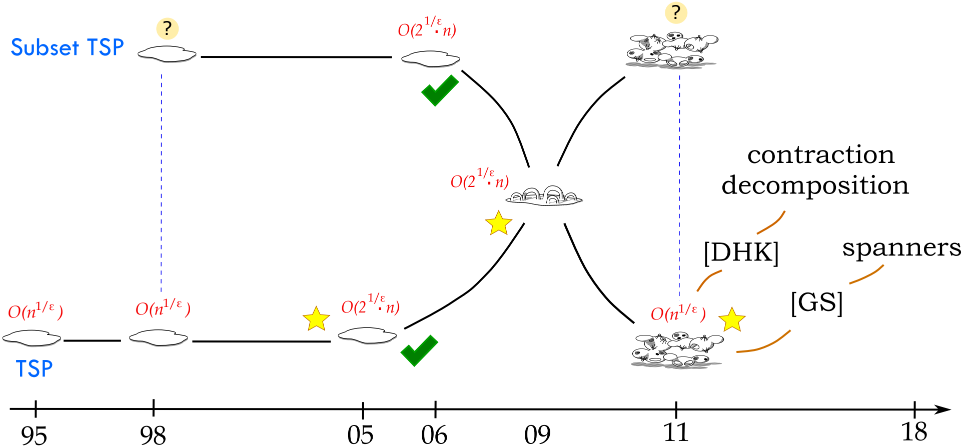
History



[BDT] M. Borradaile, E. Demaine, and S. Tazari, "*A polynomial-time approximation scheme for subset connectivity problems in bounded genus graphs*", STACS'09.

[DHM] E. Demaine, M. Hajiaghayi and B. Mohar, "*Approximation algorithms via contraction decomposition*", SODA''07.

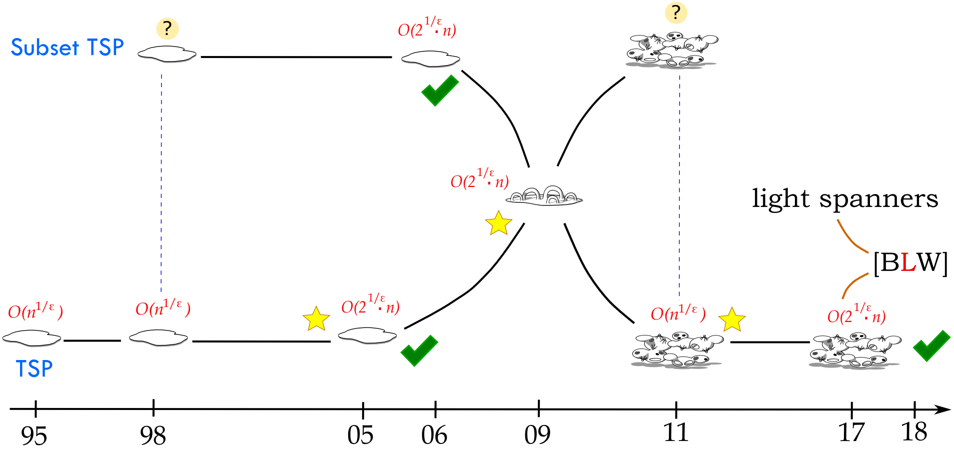
History



[BDT] E. Demaine, M. Hajiaghayi and K. Kawarabayashi, "*Contraction decomposition in H-minor-free graphs and algorithmic applications*", STOC'11.

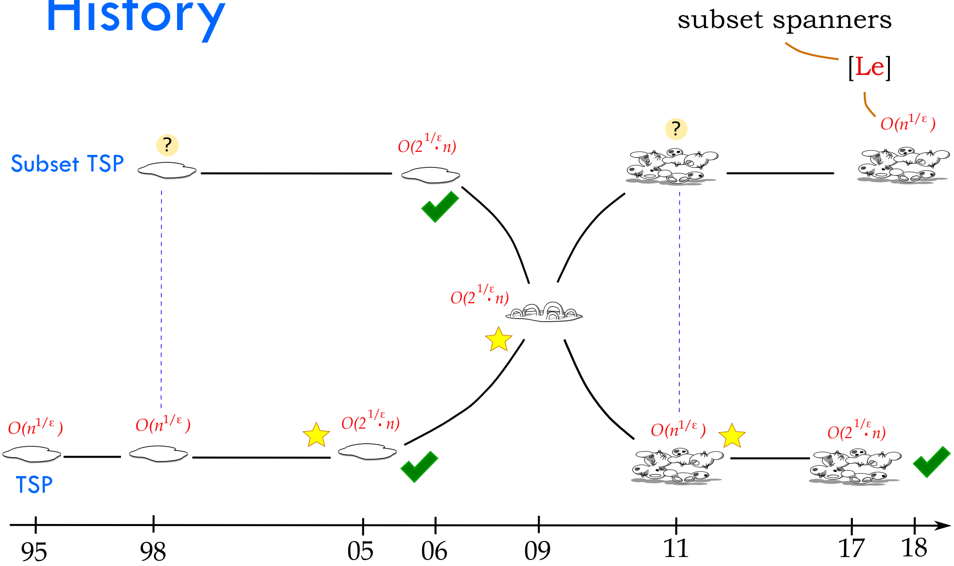
[GS] M. Grigni, and P. Sissokho, "*Light spanners and approximate TSP in weighted graphs with forbidden minors*", SODA''02.

History



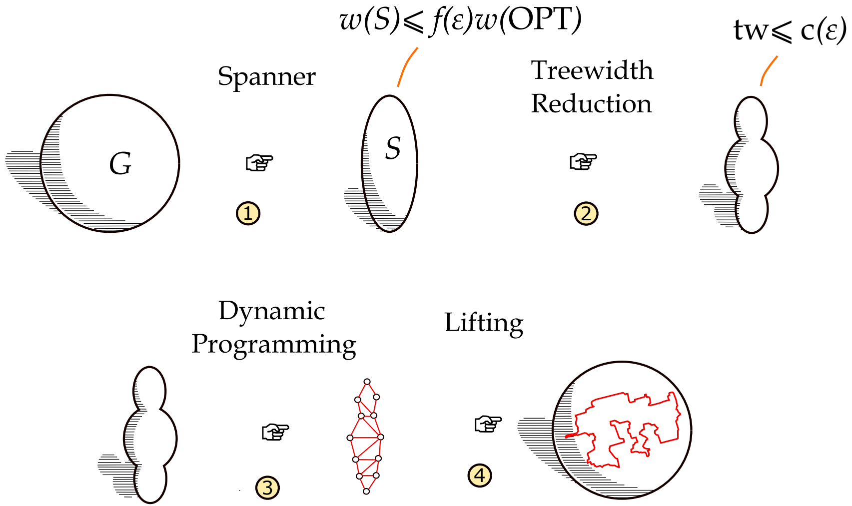
[BLW] G. Borradaile, H. Le, and C. Wulff-Nilsen, "*Minor-free graphs have light spanners,*" FOCS'17.

History



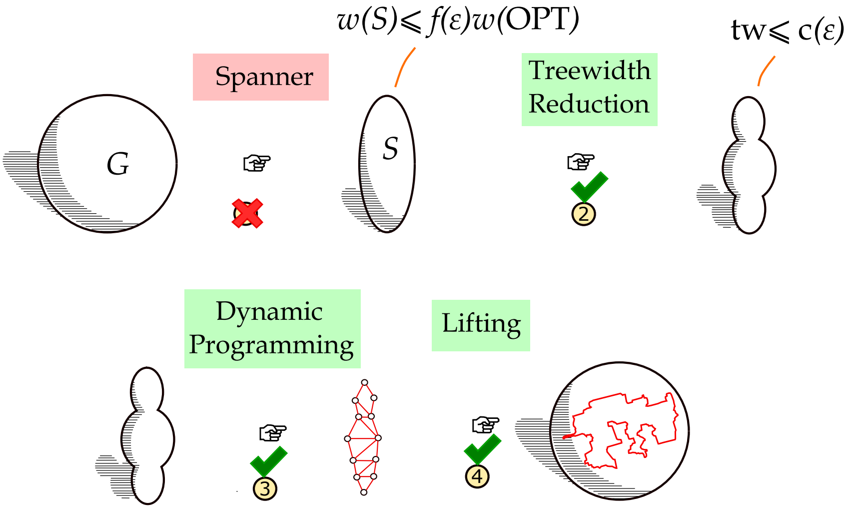
[Le] H. Le, "*A PTAS for subset TSP in minor-free graphs*", preprint 2018.

Contraction Decomposition



[Klein] P. N. Klein, "*A linear-time approximation scheme for planar weighted tsp*", FOCS'05.

Contraction Decomposition



[Klein] P. N. Klein, "*A linear-time approximation scheme for planar weighted tsp*", FOCS'05.

Spanners

TSP spanners

A spanning subgraph S is an $(1+\varepsilon)$-spanner of G iff:

$$d_G(u,v) \leqslant d_S(u,v) \leqslant (1+\varepsilon)d_G(u,v) \quad \forall\ u \neq v \in V(G)$$

Subset TSP Spanners

A subgraph S is a subset spanner of G for a set of terminals $T \subseteq V(G)$ iff:

$$d_G(u,v) \leqslant d_S(u,v) \leqslant (1+\varepsilon)d_G(u,v) \quad \forall\ u \neq v \in T$$

Spanners

Theorem [BLW17]

Any minor-free graph G has an $(1+\varepsilon)$-spanner S s.t:

$$w(S) \leqslant \tilde{O}\left(\frac{1}{\varepsilon^3}\right) w(\text{MST})$$

Theorem [Le18]

Any minor-free graph G and any set T of k terminals has a subset spanner S s.t:

$$w(S) \leqslant O\left(\frac{1}{\text{poly}(\varepsilon)} \log(k)\right) w(\text{ST})$$

MST = Minimum Spanning Tree

ST = Minimum Steiner Tree for T

Spanners

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MST = Minimum Spanning Tree

ST = Minimum Steiner Tree for T

Greedy spanners

GreedySpanner(G, w, ε)

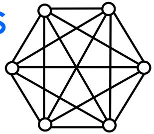
$S \leftarrow \varnothing$
Sort edges of G in ⇈ of weight
For each edge uv in sorted order
  if $(1+\varepsilon)w(uv) < d_S(u,v)$
    $S \leftarrow S \cup \{uv\}$
return S

S

$w(S) > w(MST)$

How much bigger?

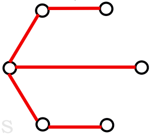
Old question dated back to 90s



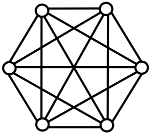
KruskalMST(G, w)

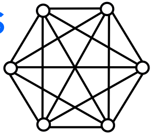
$S \leftarrow \varnothing$
Sort edges of G in ⇈ of weight
For each edge uv in sorted order
  if $S \cup \{uv\}$ is acyclic
    $S \leftarrow S \cup \{uv\}$
return S

MST



Greedy spanners





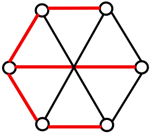
GREEDYSPANNER(G, w, ε)

- $S \leftarrow \emptyset$
- Sort edges of G in ⇈ of weight
- For each edge uv in sorted order
  - if $(1+\varepsilon)w(uv) < d_S(u,v)$
    - $S \leftarrow S \cup \{uv\}$
- return S

KRUSKALMST(G, w)

- $S \leftarrow \emptyset$
- Sort edges of G in ⇈ of weight
- For each edge uv in sorted order
  - if $S \cup \{uv\}$ is acyclic
    - $S \leftarrow S \cup \{uv\}$
- return S

S

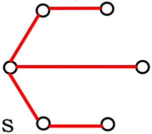


$w(S) > w(MST)$

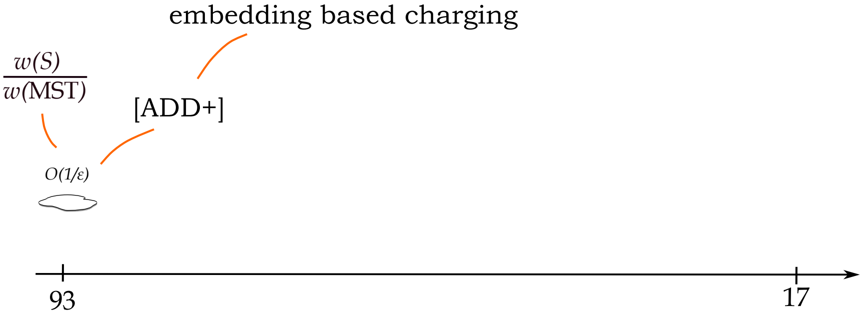
How much bigger?

Old question dated back to 90s

MST

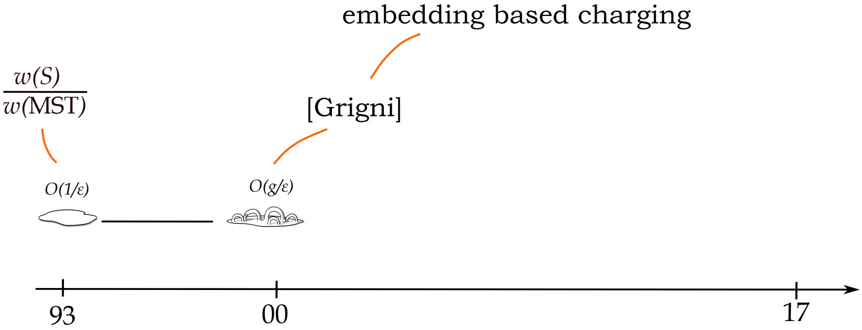


History



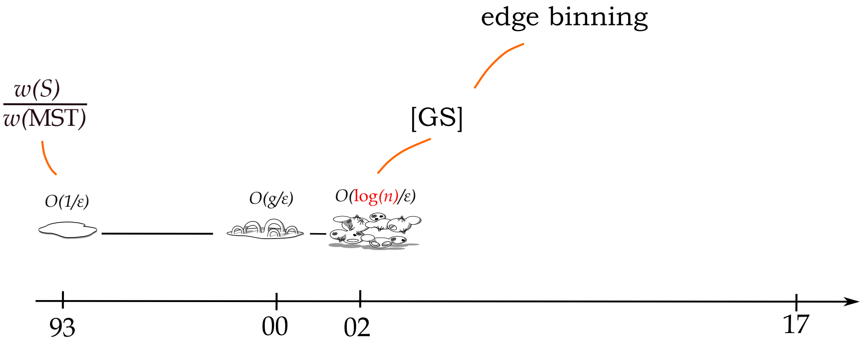
[ADD+] I. Althöfer, G. Das, D Dobkin, D. Joseph, and J. Soares, "*On sparse spanners of weighted graphs*", Discrete Computational Geometry, 1993.

History



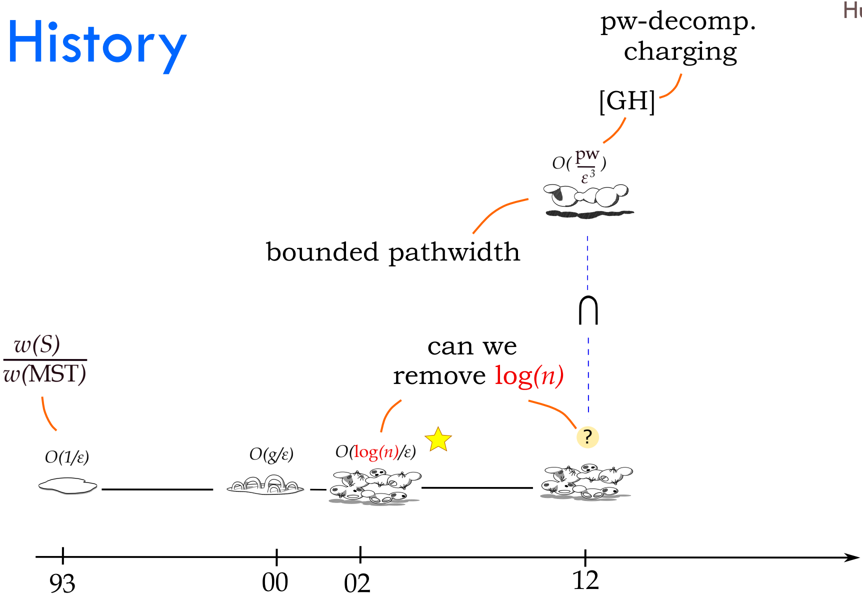
[Grigni] M. Grigni, “*Approximate TSP in graphs with forbidden minors*", ICAL'00.

History



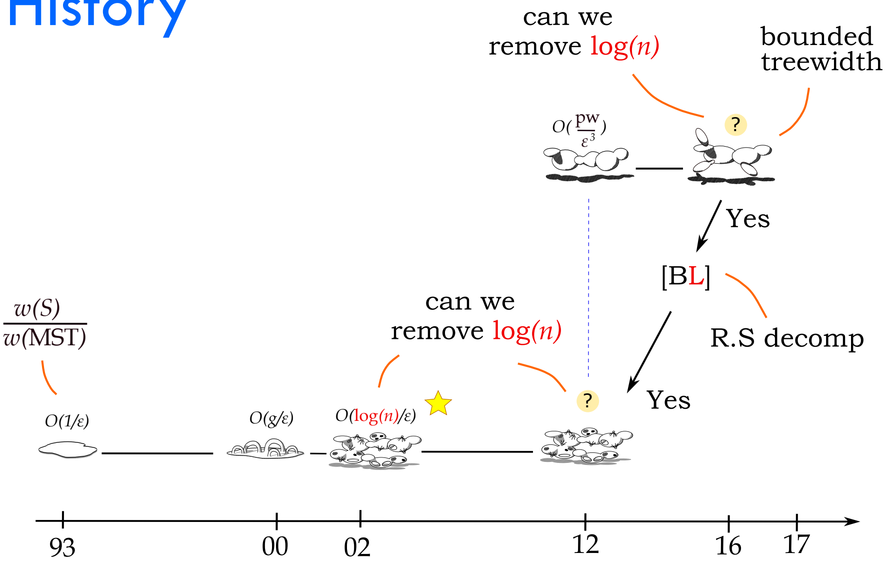
[GS] M. Grigni, and P. Sissokho, "*Light spanners and approximate TSP in weighted graphs with forbidden minors*", SODA''02.

History



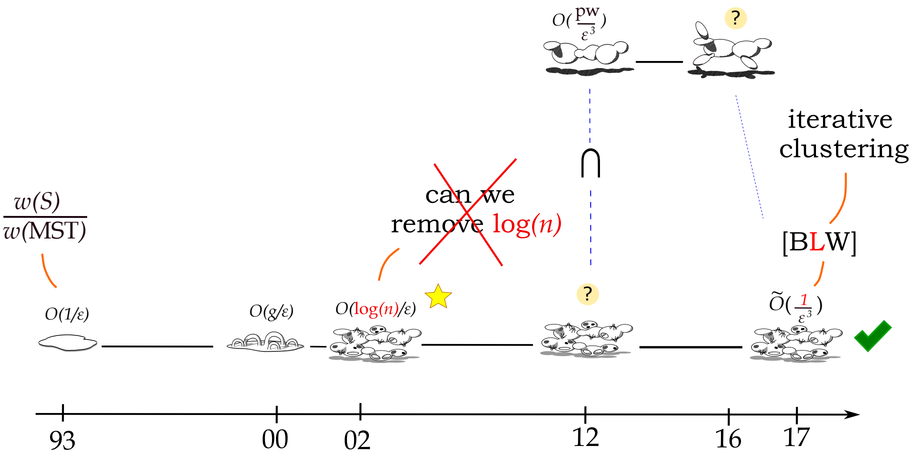
[GH] M. Grigni and H. Hung, " *Light spanners in bounded pathwidth graphs*", MFCS'12.

History



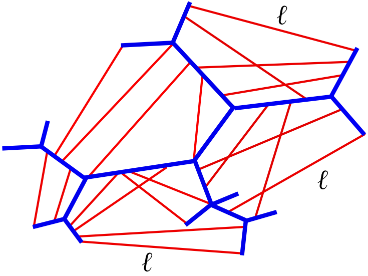
[BL] G. Borradaile, and H. Le, "*Light spanners for bounded treewidth graphs implies light spanners for H-minor-free graphs*", preprint 2017.

History



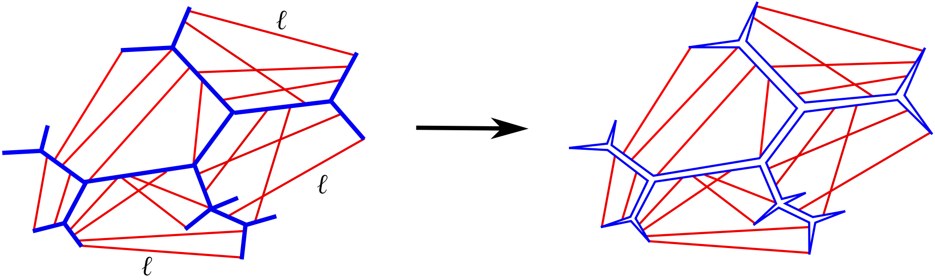
[BLW] G. Borradaile, H. Le, and C. Wulff-Nilsen, "*Minor-free graphs have light spanners,*" FOCS'17.

Warm up: equi-long case



spanner edges have the same length ℓ

Warm up: equi-long case



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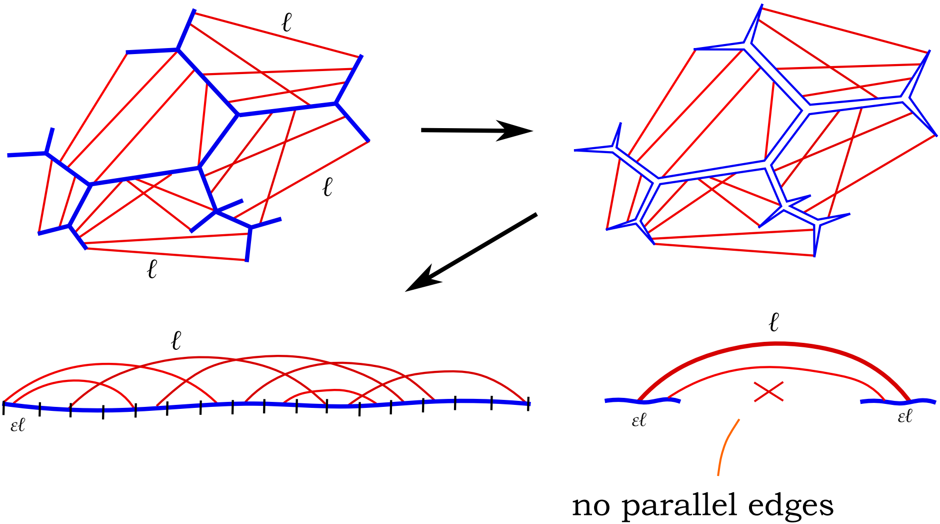
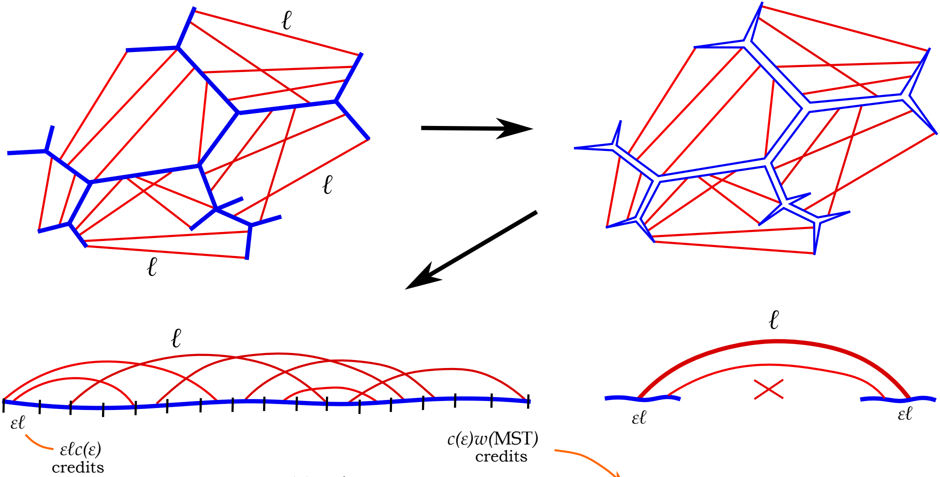


Warm up: equi-long case

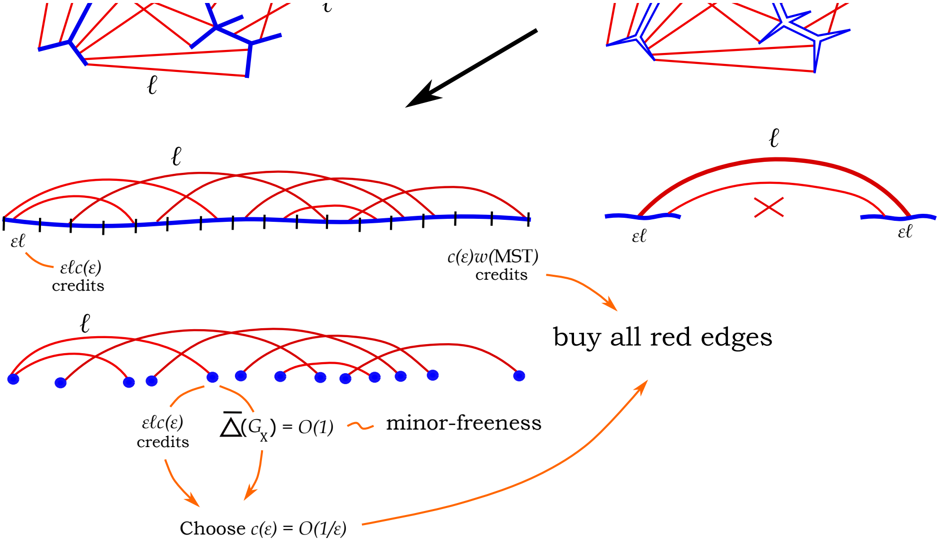


assume MST edges have length *1*

buy all red edges

give each MST edge $c(\varepsilon)$ credits

Warm up: equi-long case



Warm up:

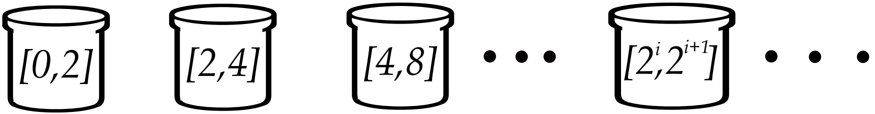
Lemma

$w(S) \leqslant O(\frac{1}{\varepsilon})w(\mathrm{MST})$ if every edge of S has length $\Theta(\ell)$

Theorem [GH]

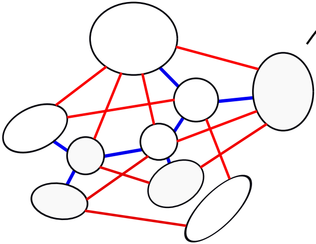
$w(S) \leqslant O(\frac{\log n}{\varepsilon})w(\mathrm{MST})$ if G is minor-free

$\log(n)$ bins



depedency between bins?

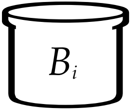
Iterative clustering

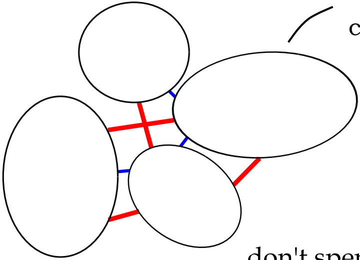


$c(\varepsilon)d_i$ credits

$d_i \sim \frac{1}{\varepsilon^{i-1}}$

$(\frac{1}{\varepsilon^{i-1}}, \frac{1}{\varepsilon^{i}}]$

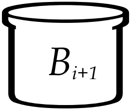




$c(\varepsilon)d_{i+1}$ credits

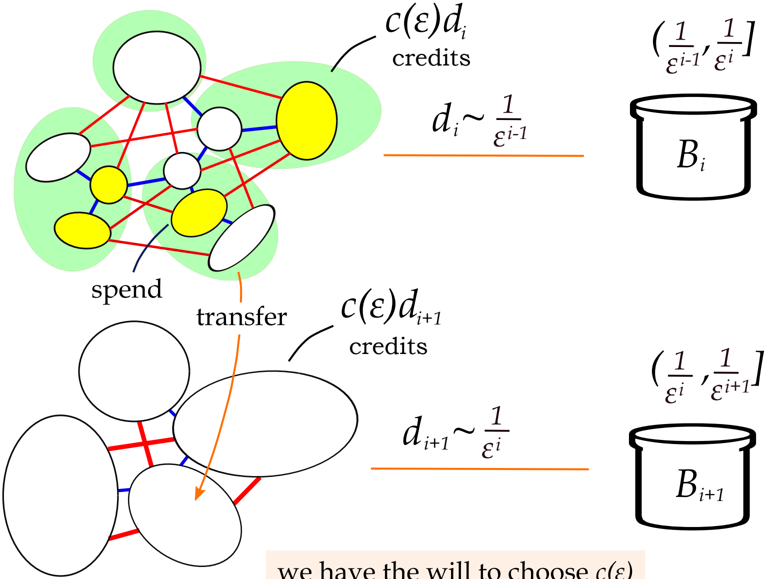
$d_{i+1} \sim \frac{1}{\varepsilon^{i}}$

$(\frac{1}{\varepsilon^{i}}, \frac{1}{\varepsilon^{i+1}}]$



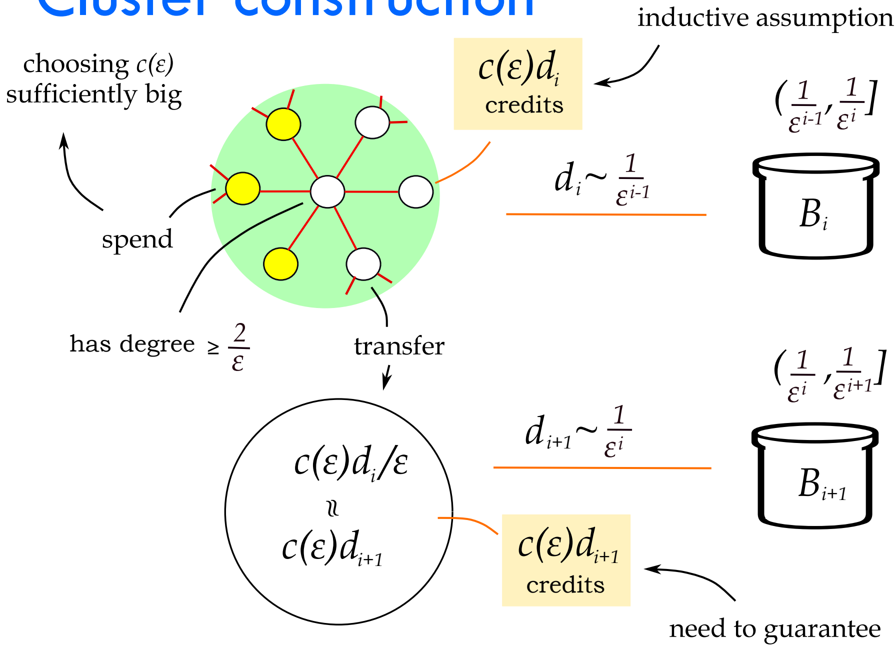
don't spend all cluster credits to pay for each level

Iterative clustering

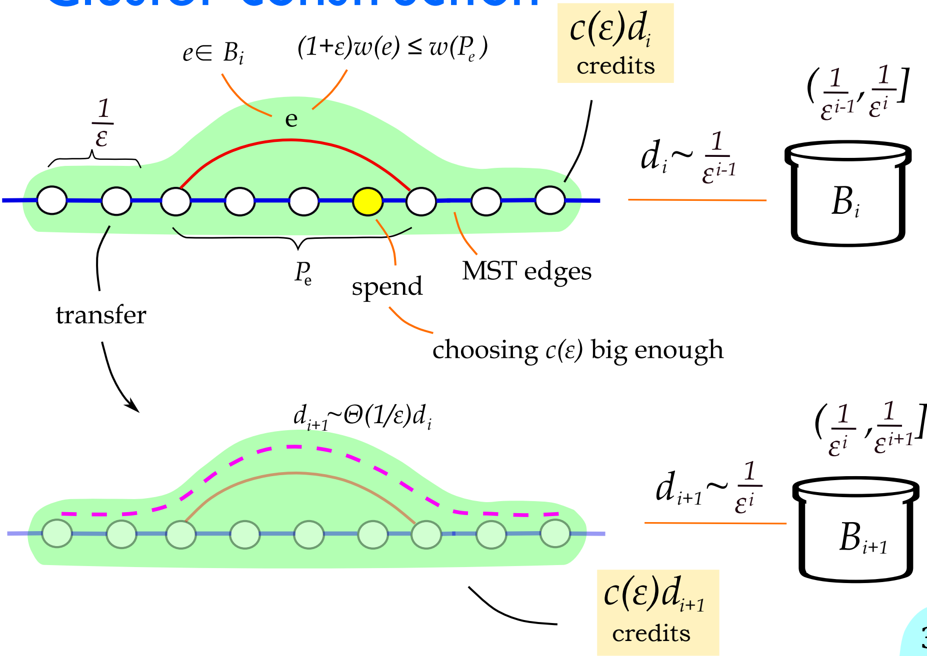


we have the will to choose $c(\varepsilon)$

Cluster construction



Cluster construction



Spanners

Theorem [BLW17]

Any minor-free graph G has an $(1+\varepsilon)$-spanner S s.t:

$$w(S) \leqslant \tilde{O}\left(\frac{1}{\varepsilon^3}\right) w(\mathrm{MST})$$

MST = Minimum Spanning Tree

Corrolary [BLW17]

TSP in minor-free graphs has a PTAS of running time:

$$O(2^{\frac{1}{\varepsilon^4}} \mathrm{poly}(n))$$

Subset Spanners

Subset TSP Spanners

A subgraph S is a subset spanner of G for a set of terminals $T \subseteq V(G)$ iff:

$$d_G(u,v) \leqslant d_S(u,v) \leqslant (1+\varepsilon)d_G(u,v) \quad \forall\ u \neq v \in T$$

Theorem [Le18]

Any minor-free graph G and any set T of k terminals has a subset spanner S s.t:

$$w(S) \leqslant O\left(\frac{1}{\text{poly}(\varepsilon)}\log(k)\right) w(\text{ST})$$

ST = Minimum Steiner Tree for T

Remarks

Theorem [Le18]

Any edge-weighted minor-free graph G and any set T of k terminals has a subset spanner S s.t:

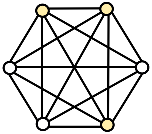
$$w(S) \leqslant O\left(\frac{1}{\mathrm{poly}(\varepsilon)} \log(k)\right) w(\mathrm{ST})$$

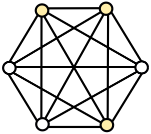
- Non-trivial even in unweighted graphs
- $c(\varepsilon)w(\mathrm{ST})$ subset spanners for unweighted graphs.

→ $c(\varepsilon)w(\mathrm{MST})$ spanners for weighted graphs.

by subdividing edges

Construction

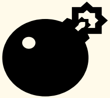




SubsetSpanner(G, w, ε)

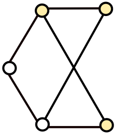
Not enough space

SteinerTree(G, w)



NP-hard

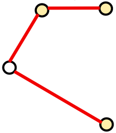
S



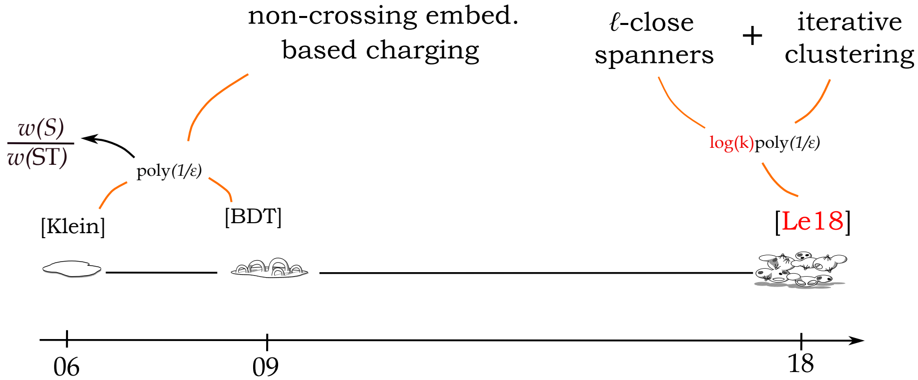
$w(S) > w(ST)$

How much bigger?

ST



History


non-crossing embed.
based charging
ℓ-close
spanners
+
iterative
clustering
$\frac{w(S)}{w(ST)}$
poly(1/ε)
log(k)poly(1/ε)
[Klein]
[BDT]
[Le18]
06
09
18

ℓ-close spanners

ℓ-close spanners

A subgraph S is an ℓ-close spanner of G for a set of terminals $T \subseteq V(G)$ iff:

$$d_G(u,v) \leqslant d_S(u,v) \leqslant (1+\varepsilon)d_G(u,v) \quad \forall\ u \neq v \in T$$

$$\text{and } d_G(u,v) \leqslant \ell$$

ℓ-close terminals

SubsetSpanner(G, w, ε)

works for general graphs

1. SubsetSpanner $\preccurlyeq_{\text{Lightness}}$ EllCloseSpanner

$c(\varepsilon)w(\mathrm{ST})$ $\Longleftarrow$ $c(\varepsilon)k\ell$

Steiner tree

$d_G(u,v) \leqslant \ell$

# of terminals

need minor-freeness

2. $w(S_{\text{close}}) \leqslant \mathrm{poly}(1/\varepsilon)\cdot\log(k)\cdot k\cdot\ell$

an ℓ-close spanner

choose $c(\varepsilon) = \mathrm{poly}(1/\varepsilon)\cdot\log(k)$

$$w(S) \leqslant O(\mathrm{poly}(1/\varepsilon)\log(k))\, w(\mathrm{ST})$$

ℓ-close spanners

Theorem

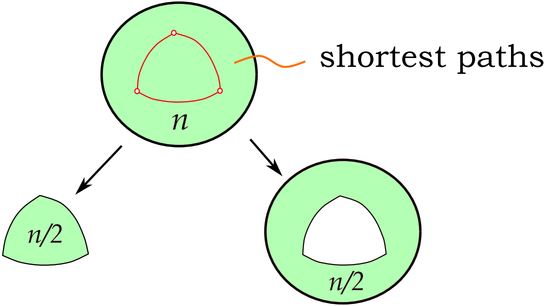
$$w(S_{close}) \leqslant \text{poly}(1/\varepsilon)\log(k).k.\ell$$

$w(S_{close}) \leqslant O(k^2)\ell$ is trivial because we have at most $O(k^2)$ ℓ-close terminal pairs

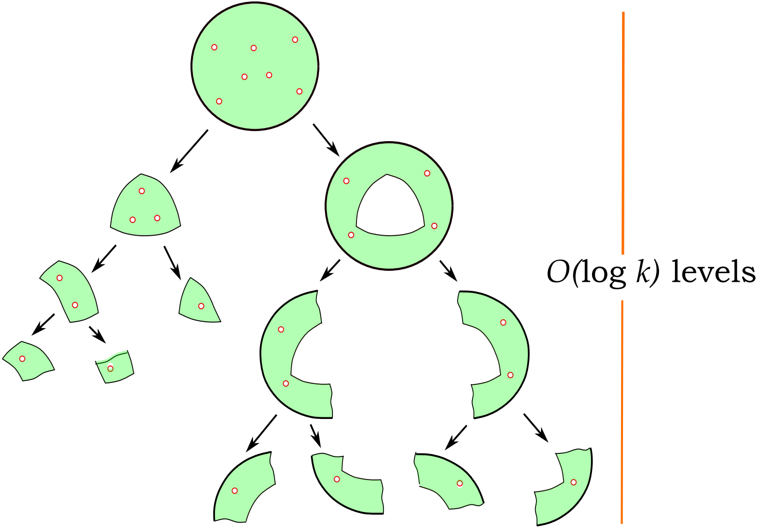
1 st idea: shortest path separator

Theorem [AG06]

Minor-free graphs of size n can be divided into subgraphs of size at most $n/2$ each using $O(1)$ shortest paths.

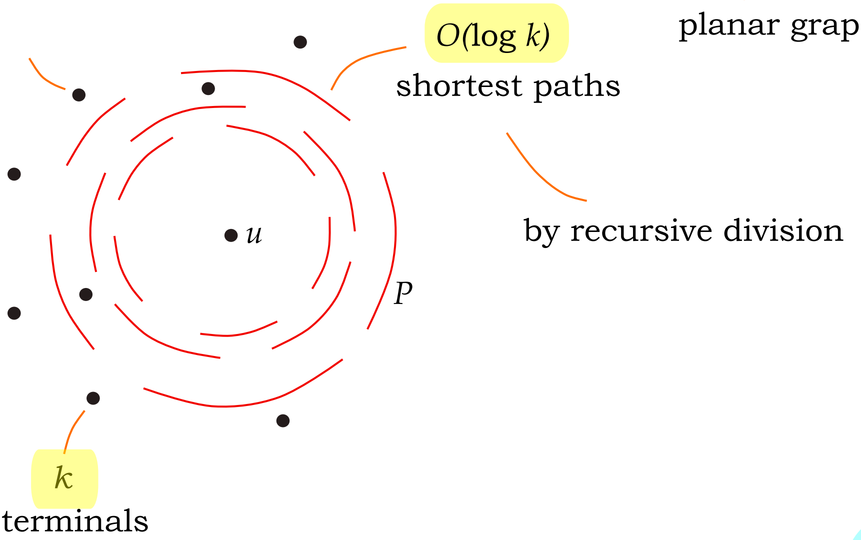


2nd idea: recursive division

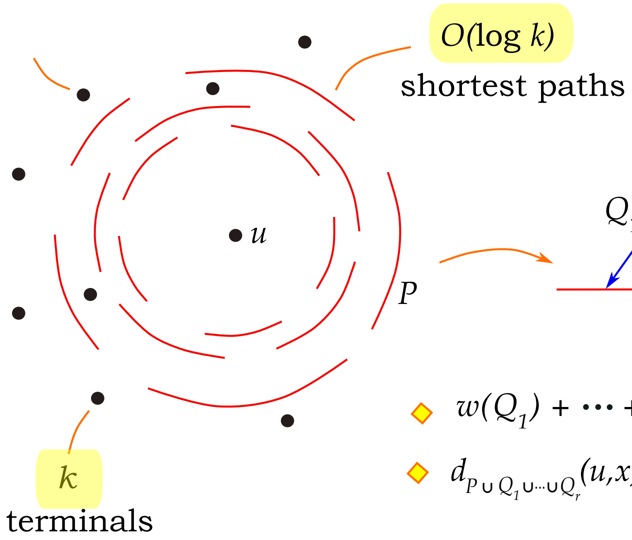


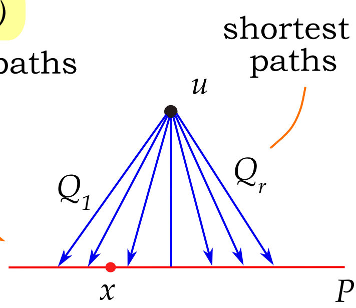
3rd idea: single source spanners [Klein06]

planar graphs



3rd idea: single source spanners [Klein06]

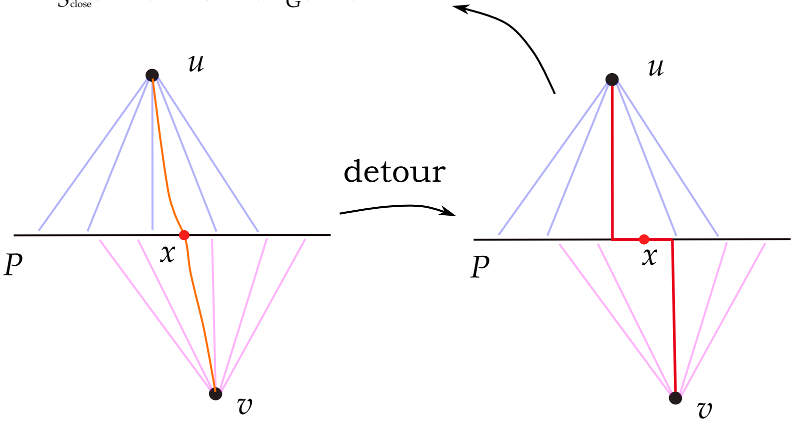




- $w(Q_1) + \cdots + w(Q_r) \leqslant \text{poly}(1/\varepsilon)\ell$
- $d_{P \cup Q_1 \cup \cdots \cup Q_r}(u,x) \leqslant (1+\varepsilon) d_G(u,x)$ $\forall x \in P$

3rd idea: single source spanners [Klein06]

- $w(S_{close}) \leqslant \text{poly}(1/\varepsilon)\log(k).k.\ell$ ✔
- $d_{S_{close}}(u,v) \leqslant (1+\varepsilon)d_G(u,v)$?



Recap

Theorem [Le18]

Any minor-free graph G and any set T of k terminals has a subset spanner S s.t:

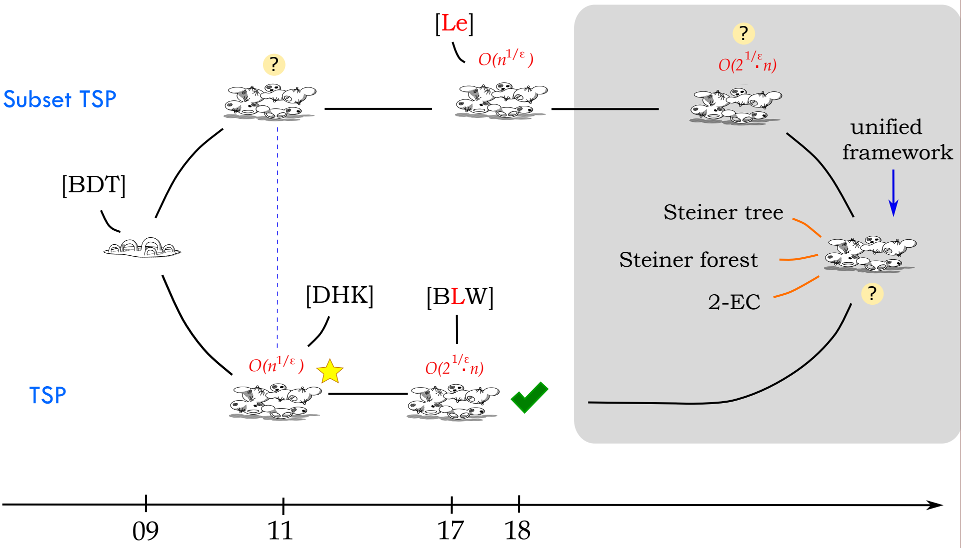
$$w(S) \leqslant \mathrm{O}\left(\frac{1}{\mathrm{poly}(\varepsilon)}\log(k)\right) w(\mathrm{ST})$$

Corrolary [Le18]

Subset TSP in minor-free graphs has a PTAS with running time:

$$O(n^{poly(1/\varepsilon)})$$

Open problems



Thank you

OSU Oregon State University

please catch up

University of Victoria