

...you might like to give a talk about how priors are useful for modelling spatial data but we certainly would not hold you to that

Håvard Rue
King Abdullah University of Science and Technology
Saudi Arabia

December 4, 2017

Plan

Priors

- Background on priors
- Penalised complexity priors

Examples

- The easier ones
- Area models (more)
- Gaussian fields (less)



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Joint and ongoing work with many, including...



Daniel Simpson



Haakon Bakka



Anna Sterrantino



Andrea Riebler



Geir-A Fuglstad



Finn Lindgren



Massimo Ventrucci



Sigrunn Sørbye

and others

Our background: R-INLA (www.r-inla.org)

- INLA do Bayesian inference on Latent Gaussian models
- Accurate, fast, scale well wrt size, great spatial models support, quite general with an easy R-interface (www.r-inla.org).
- Build models adding model component

$$\eta = \mathbf{X}\beta + f_1(\dots; \theta_1) + f_2(\dots; \theta_2) + \dots$$

for Gaussians $\{f_i(\cdot)\}$ conditioned on some hyperparameters θ

- Likelihood(s) have hyper-parameters as well
- Of course, the model include **prior** specification for θ , which is the topic of this talk

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What do **we** know about priors for θ in this framework?

I cannot say what **you** know, here is my story...

- Not much. And I am not proud of it!
- I knew reference priors, which I, except in simple cases, cannot compute, and I do not want to use. Conjugate priors does not apply here, and is more “math, not priors”.
- I could dig up similar studies/models/examples, and copy and refer to their prior choice. (Risk averse)
- I ran into problems when a student presented his/her hierarchical model and ask about advice for how to set priors for the f.ex 5-dimensional hyperparameter θ ; I did not believed my own advises.
- I do not think that I am *that* unique

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Why is the additive model situation different?

Classical:

- I want to estimate the precision from data \mathbf{y} , without any context
- In this case I just want to get it right!

Additive model:

- From data \mathbf{y} I add an additional iid random effect
$$\text{formula} = \mathbf{y} \sim \dots + f(\text{idx}, \text{model}=\text{"iid"})$$
with the “hope” it is not there.
- In this case I have a preference for “no random effect” doing inference

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How to proceed from here?



- How to think about priors in hierarchical models?
- Is it possible to *understand/have good intuition about them?*

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Parameters!



- I have an issue with parameters.
- $\sigma, \sigma^2, \tau, \rho, p, \dots$
- I want to understand their *impact* on something I understand, not their *numerical values!*
- Invariance

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KISS (Keep it simple, stupid!)



- ...most systems work best if they are kept simple rather than made complicated
- ...there is no value in a solution being “clever” but in one being easily understandable

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Our take on the “prior”-problem

which is

- a principled and practical approach to constructing priors
- KISS-friendly
- a unified way to think about priors
- useful
- is widely applicable
- is transparent
- invariant for reparameterisations
- something I can understand
- better than not knowing what to do

It is **not** “optimal” or “unique” in any sense. If you prefer something else, please do...

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Principle I: Occam's razor

CORE PRINCIPLES IN RESEARCH



OCCAM'S RAZOR

"WHEN FACED WITH TWO POSSIBLE EXPLANATIONS, THE SIMPLER OF THE TWO IS THE ONE MOST LIKELY TO BE TRUE."



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JOSIE CUHM © 2009

- Prefer simplicity over complexity. *Simplicity* defines the **base model**
- $x \sim \mathcal{N}(\mathbf{0}, \tau I)$, base model $\tau = \infty$
- Student-t, base model Gaussian
- Spline model, base model linear/constant effect
- AR(1), base model $\rho = 0$ or $\rho = 1^-$

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Principle I: Occam's razor

Consider the more complex model

$$\pi(x|\xi), \quad \xi \geq 0$$

with base model $\pi(x|\xi = 0)$.

- The prior for $\xi \geq 0$ should penalise the complexity introduced by ξ
- The prior should be decaying with increasing measure by the complexity (the mode should be at the base model)

A prior will cause **overfitting/force complexity** if, loosely speaking,

$$\pi_{\xi}(\xi = 0) = 0$$

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Principle II: Measure of complexity

Use Kullback-Leibler discrepancy to measure the increased complexity introduced by $\xi > 0$,

$$\text{KLD}(f\|g) = \int f(x) \log \left(\frac{f(x)}{g(x)} \right) dx$$

for flexible model f and base model g .

Gives a measure of the information lost when the base model is used to approximate the more flexible models



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Principle III: Constant rate penalisation

Define

$$d(\xi) = \sqrt{2 \text{KLD}(\xi)}$$

as the (uni-directional) “distance” from flexible-model to the base model.
Need the square-root to get the scale right.

Constant rate penalisation:

$$\pi(d) = \lambda \exp(-\lambda d), \quad \lambda > 0$$

with mode at $d = 0$

Invariance: OK

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Principle IV: User-defined scaling

The rate λ is determined from knowledge of the *scale* or some interpretable property or impact, $Q(\xi)$ of ξ :

$$\Pr(Q(\xi) > U) = \alpha$$

- Problem dependent: must be!!!
- Can make the prior more informative or weakly informative this way

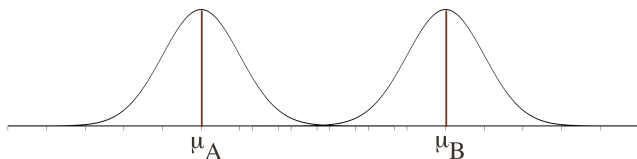
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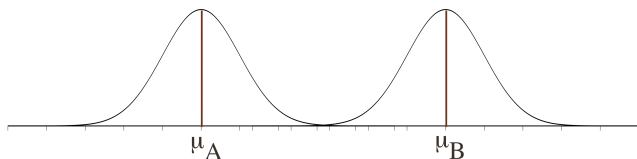
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- Flexible model $\mathcal{N}(\mu, 1)$, $\mu > 0$.
- KLD is $\mu^2/2$ and $d(\mu) = \mu$.
- PC prior:

$$\pi(\mu) = \lambda \exp(-\lambda\mu)$$

- Can determine λ from a question like

$$\text{Prob}(\mu > u) = \alpha$$

Example



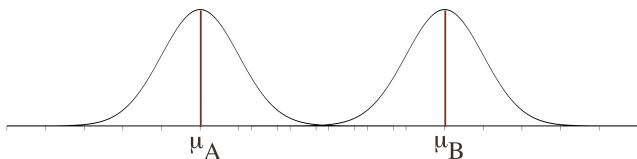
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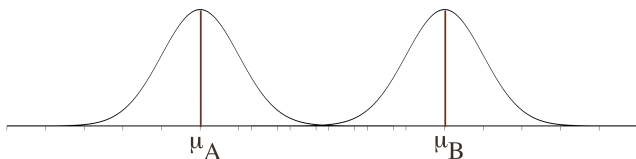
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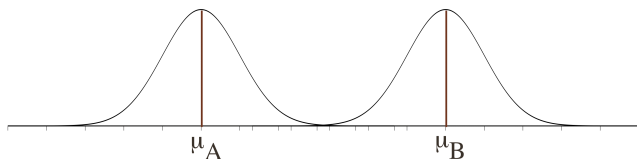
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$$\text{Prob}(\mu > u) = \alpha$$

Example



- Base model $\mathcal{N}(0, 1)$
- Flexible model $\mathcal{N}(\mu, 1)$, $\mu > 0$.
- KLD is $\mu^2/2$ and $d(\mu) = \mu$.
- PC prior:

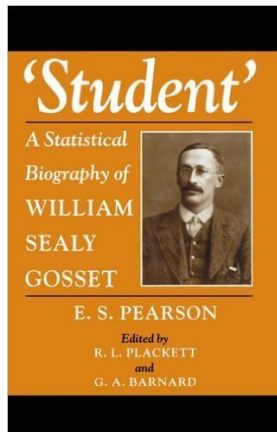
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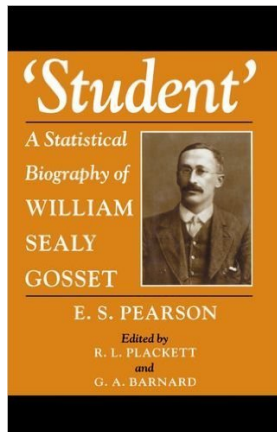
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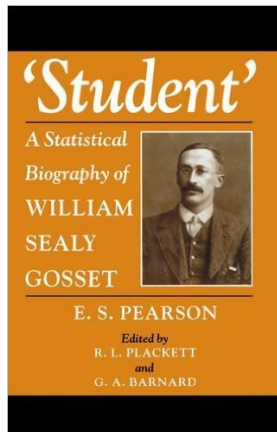
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Result Let $\pi_\nu(\nu)$ be a prior for $\nu > 2$ where $E(\nu) < \infty$, then $\pi_d(0) = 0$ and the prior overfits

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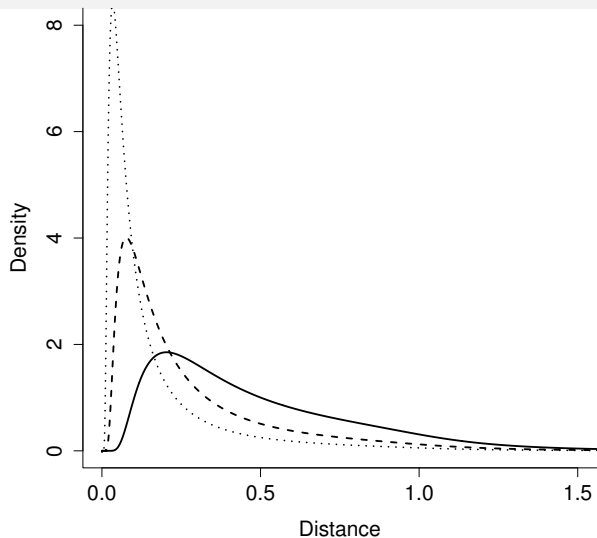
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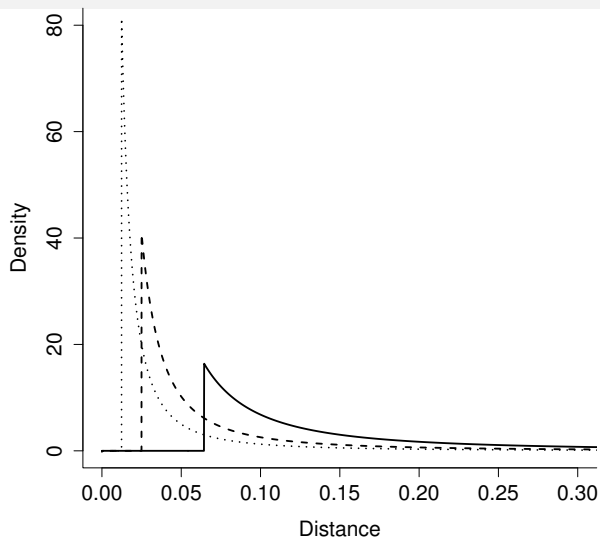
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The exp-prior with mean 5, 10, 20, converted to a prior for the distance



The uniform prior with upper= 20, 50, 100, converted to a prior for the distance



The precision of a Gaussian

PC prior for the precision κ when $\kappa = \infty$ defines the base model

- “random effects” /iid-model
- The smoothing parameter in spline models
- etc...

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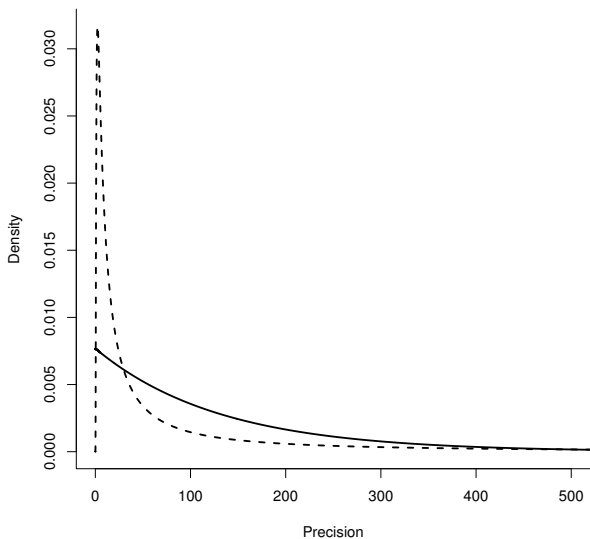
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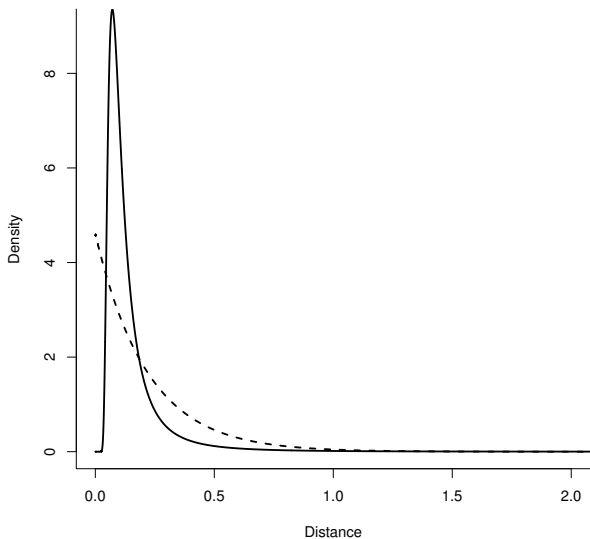
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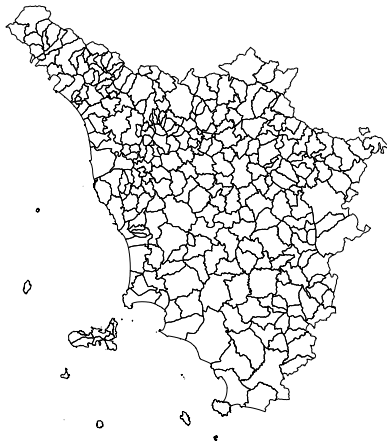
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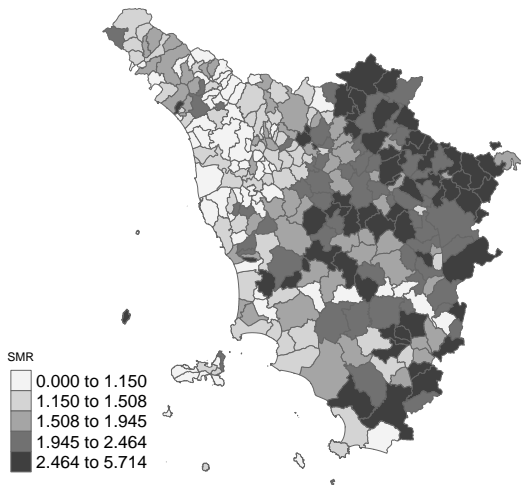
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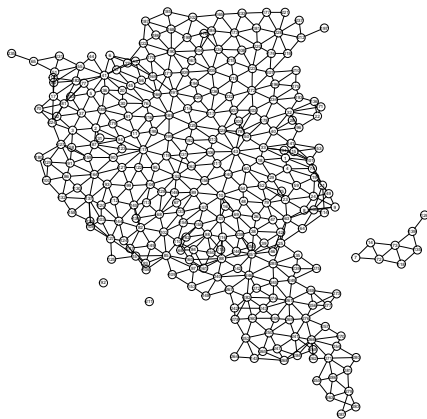


Area models



The “CAR”-model

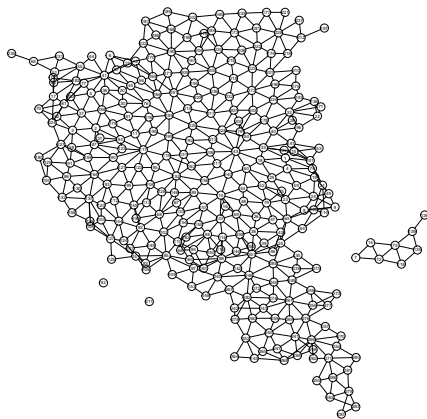
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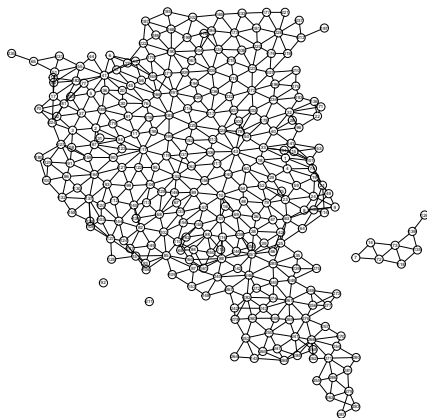
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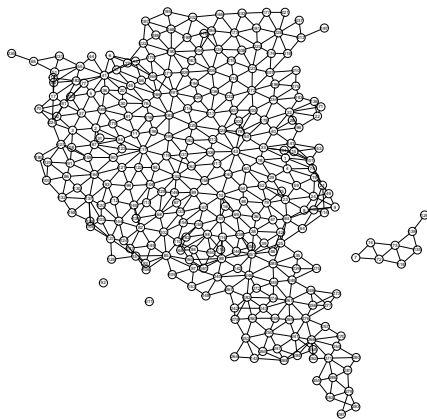
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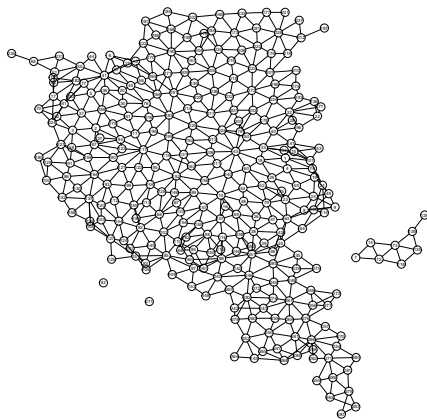
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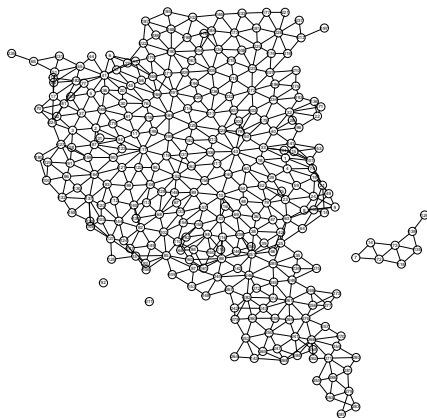
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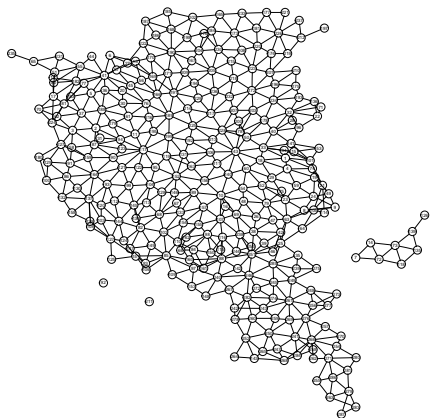


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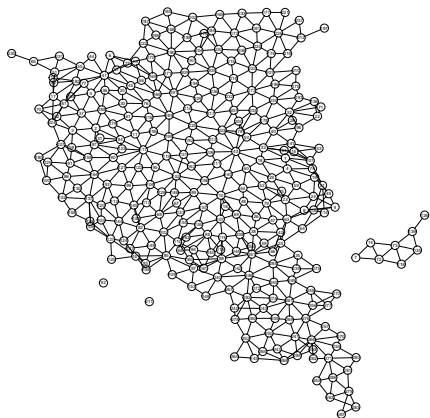


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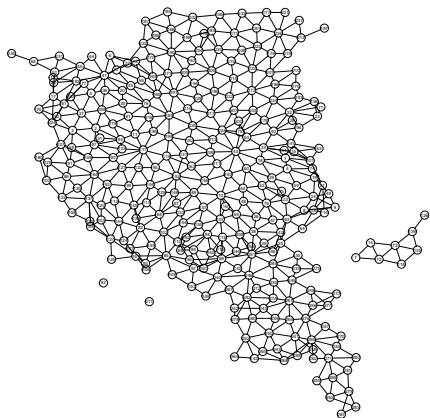


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Scaling (I)

- Assume a connected graph
- κ controls the deviation from the null-space
- The geometric mean of the marginal variances are

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## largest area  
> gmean(diag(INLA:::inla.ginv(Q)))  
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- Scale each connected component to have unit g_{mean}
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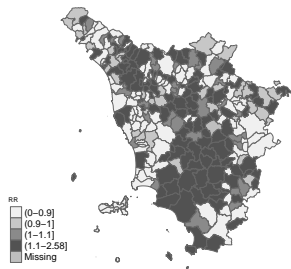
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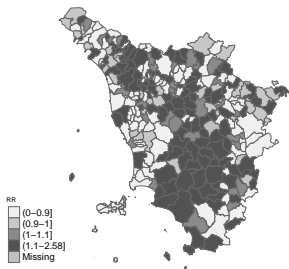
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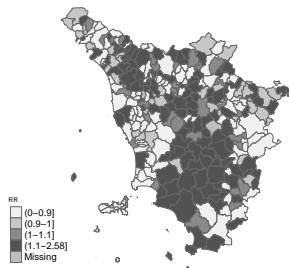
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- Additional random effect: structured and unstructured
- Here, there is a lot of “confusion” in the literature
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- Unstructured precision matrix I
- Structured precision matrix R
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- The basic idea is to have a convex combination of two limiting cases
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- Computations need to make use of the sparse structure of R
- Let $z = x + y$, where x and y are indep normal
- Then z is the marginal from the joint distribution of (x, z)

$$x \sim \mathcal{N}(0, \dots) \quad \text{and} \quad z|x \sim \mathcal{N}(x, \dots)$$

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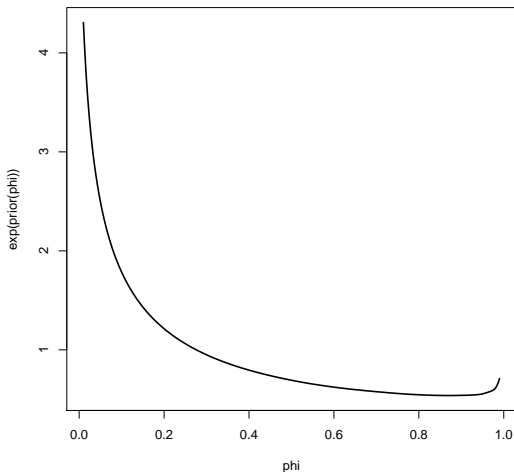
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Prior for ϕ

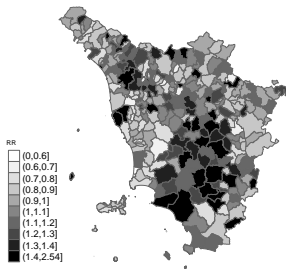
```
Q = INLA:::inla.pc.bym.Q("Toscana.graph")
prior = INLA:::inla.pc.bym.phi(Q,
  u = 0.5, alpha = 0.5)
phi = seq(0.01, 0.99, len=1000)
plot(phi, exp(prior(phi)), lwd=2, type="l")
```



Application

Proper Poisson quantile regression...

```
formula = obs35 ~ 1+f(id,  
                      model='bym',  
                      scale.model=TRUE,  
                      graph='Toscana.graph')  
res = inla(formula, family="poisson",  
           E=exp35, data=counties@data,  
           control.family = list(  
             control.link = list(  
               model = "quantile",  
               quantile = 0.9)))
```



Gaussian fields

- Gaussian field in \mathbb{R}^d ($d \leq 3$) with a Matérn covariance function with fixed smoothness ν .
- PC-prior for range r and variance σ^2 , with base model $\sigma^2 = 0$ and $r = \infty$ (a constant).
- Joint PC-prior is (dim= 2)

$$(1/r, \sigma) \sim \text{Exp}(\lambda_r) \times \text{Exp}(\lambda_\sigma),$$

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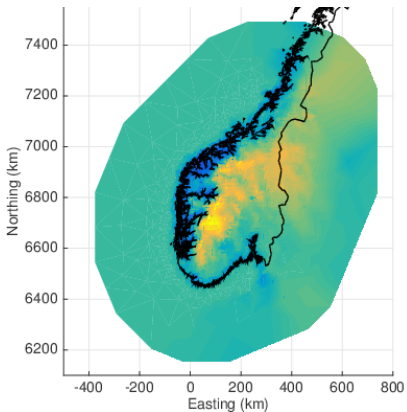
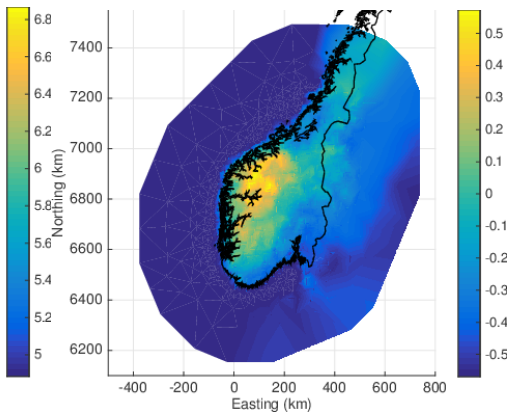
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Non-stationary Gaussian fields

$$\pi(\theta) = \underbrace{\pi(\theta_{\text{stationary}})}_{\text{PC-prior for range \& stdev}} \times \underbrace{\pi(\theta_{\text{non-stationary}} \mid \theta_{\text{stationary}})}_{\text{shrinkage towards stationarity}}$$

 $\text{Log}(\text{range})$  $\text{Log}(\text{stddev})$

Non-separable space-time model

Based on Finn's ideas

$$\begin{aligned}(\gamma_t \frac{\partial}{\partial t} - \Delta)^{\alpha_t} z(s, t) &= \gamma_s^{-1/2} \mathcal{E}(s, t) \\ (1 - \gamma_\varepsilon \Delta)^{\alpha_\varepsilon/2} \mathcal{E}(s, \delta t) &= \mathcal{W}_\varepsilon(s, \delta t)\end{aligned}$$

written up in the forthcoming PhD-thesis of Elias Krainski.

We need to understand the parameters in this model, which we **can** map into

- marginal variance
- spatial range
- temporal range

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New nonsep space-time model

Discussion: Priors

- **Not easy**
- Makes a difference
- Need to '*calibrate*' priors based on intuitive model properties
- PC-priors is a principled constructive approach to construct priors, and seems very promising
- Easy and natural interpretation, as a well defined shrinkage to a base-model: KISS!
- Still work in progress

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References

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- and others...