Linear Algebraic Groups and Related Structures, 09w5026
September 13-18, 2009

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
Lecture rooms available after 16:00 (if desired)
17:30–19:30 Buffet Dinner, Sally Borden Building
20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and small assortment of snacks available on a cash honour-system.

Monday

7:00–8:45 Breakfast
8:45–9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:45 Raman Parimala (Emory University)
Title: *Degree three Galois cohomology of function fields of surfaces*
9:50–10:35 Jason Starr (University of New York at Stony Brook)
Title: Rational simple connectedness and Serre's "Conjecture II"
10:35–11:00 Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45 Daniel Krashen (University of Georgia)
Title: *Patching topologies and local global principles*
11:45–13:00 Lunch
13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00 Group Photo; meet on the front steps of Corbett Hall
14:20–15:05 Arturo Pianzola (University of Alberta)
Title: *Applications of Galois cohomology to infinite dimensional Lie theory*
15:05–15:30 Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:15 Aurel Meyer (University of British Columbia)
Title: *A bound on the essential dimension of central simple algebras*
16:20–17:05 Roland Lötscher (University of Basel)
Title: *A multihomogenization technique for the study of essential dimension of algebraic groups*
17:15–18:00 Mark Macdonald (University of British Columbia)
Title: *Essential p-dimension of algebraic tori*
18:00–19:30 Dinner

Tuesday
7:00–9:00 Breakfast
9:00–9:45 Eric Friedlander (University of Southern California)
Title: *Restrictions to $G(\mathbb{F}_p)$ and $G_{(r)}$ of Rational G-modules*
9:50–10:35 Philippe Gille (Ecole Normale Superieure Paris)
Title: *Algebraic groups with few subgroups*
10:35–11:00 Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45 Alex Ondrus (University of Alberta)
Title: *Minimal Anisotropic Groups of Higher Real Rank*
11:45–13:30 Lunch
13:30–14:15 Sanghoon Baek (University of California, Los Angeles)
Title: *Cohomological invariants of simple algebras*
14:20–15:05 Nikita Karpenko (Universite Paris 6)
Title: *Incompressibility of quadratic Weil transfer of Severi-Brauer varieties*
15:05–15:30 Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:15 Baptiste Calmés (University of Cambridge)
Title: *Invariants, torsion indices and oriented cohomlogies of flag varieties*
(joint work with Viktor Petrov and Kirill Zainoulline)
16:25–17:10 Alexander Vishik (University of Nottingham)
Title: *Rationality of integral cycles*
17:30–19:30 Dinner
Wednesday
7:00–9:00 Breakfast
9:00–9:45 Mikhail Borovoi (Tel Aviv University)
Title: *Extended Picard complexes and homogeneous spaces*
9:50–10:35 Vladimir L. Popov (Steklov Mathematical Institute, Moscow)
Title: *Cross-sections, quotients, and representation rings of semisimple algebraic groups*
10:35–11:00 Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45 David Saltman (Center for Communications Research - Princeton)
Title: *To be announced*
11:45–13:30 Lunch
Free Afternoon
17:30–19:30 Dinner

Thursday
7:00–9:00 Breakfast
9:00–9:45 Detlev Hoffmann (University of Nottingham)
Title: *Differential forms and bilinear forms under field extensions*
9:50–10:35 Max-Albert Knus (ETH Zürich)
Title: *Severi-Brauer varieties over the field with one element*
10:35–11:00 Coffee Break, 2nd floor lounge, Corbett Hall
11:00–11:45 Eva Bayer-Fluckiger (Ecole Polytechnique Federale de Lausanne)
Title: *Hasse principle for automorphisms of lattices*
11:45–13:30 Lunch
13:30–14:15 John Swallow (Davidson College)
Title: *Galois cohomology groups as Galois modules, and applications*
14:20–15:05 Andrew Schultz (University of Illinois at Urbana-Champaign)
Title: *The first Galois cohomology group as a Gal(E/F)-module, and applications*
15:05–15:30 Coffee Break, 2nd floor lounge, Corbett Hall
15:30–16:15 Sunil Chebolu (Illinois State University)
Title: *Freyd's Generating Hypothesis and The Bloch-Kato conjecture*
16:20–17:05 Mark Blunk (University of British Columbia)
Title: *del Pezzo Surfaces of degree 6 and Derived Categories*
17:15–18:00 Alexander Duncan (University of British Columbia)
Title: *Groups of Essential Dimension 2*
18:00–19:30 Dinner
Friday
7:00–9:00 Breakfast
9:00–9:45 Asher Auel (University of Pennsylvania and Emory University)
Title: *A Clifford invariant for line bundle-valued quadratic forms*
9:50–10:35 Skip Garibaldi (Emory University)
Title: *Applications of the degree 5 invariant of* E_8
10:35–11:00 Coffee Break, 2nd floor lounge, Corbett Hall
11:30–13:30 Lunch
Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Linear Algebraic Groups and Related Structures, 09w5026
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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Asher Auel** (University of Pennsylvania and Emory University)
Title: *A Clifford invariant for line bundle-valued quadratic forms*
Abstract: We construct a generalization of the Clifford invariant for line bundle-valued quadratic forms over a scheme (where 2 is invertible). This similarity class invariant resides in the second étale cohomology group with μ_4 coefficients and interpolates between the classical Clifford invariant and the 1st Chern class modulo 2 of the value line bundle. In this talk, we will discuss the construction of this invariant, its relationship to other cohomological invariants of Azumaya algebras with orthogonal involution, and its place in a possible global Milnor conjecture.

Speaker: **Sanghoon Baek** (University of California, Los Angeles)
Title: *Cohomological invariants of simple algebras*
Abstract: We determine the group of invariants with values in Galois cohomology with coefficients $\mathbb{Z}/2\mathbb{Z}$ of central simple algebras of degree at most 8 and exponent dividing 2 over a base field F such that $\mathrm{char}(F) \neq 2$ and $-1 \in F^{\times 2}$, i.e., determine invariants of $\mathbf{Tors}_G$ for $G = \mathbf{GL}_n/\boldsymbol{\mu}_2$ with n dividing 8. As a consequence, we obtain a close bounds of essential dimension for G. All results in this talk are based on joint work with A.Merkurjev.

Speaker: **Eva Bayer-Fluckiger** (Ecole Polytechnique Federale de Lausanne)
Title: *Hasse principle for automorphisms of lattices*
Abstract: The talk is concerned with the following question: Let f be a monic, integral polynomial. Does there exist a unimodular lattice having an automorphism with characteristic polynomial f?

The foudations for this sort of problems were given by a paper of Milnor in 1969. Partial answers to the above question were given by Gross and McMullen, who also raised an interesting conjecture. The aim of this talk is to present some new results, in particular the following

Theorem: Let f be as above, and suppose moreover that f is symmetric. There exists a unimodular lattice having an automorphism with characteristic polynomial f if and only if there exists such a lattice over $\mathbb{Z}_p$ for all prime numbers p.

Speaker: **Mark Blunk** (University of British Columbia)
Title: *del Pezzo Surfaces of degree 6 and Derived Categories*

Abstract: Let S be a degree six del Pezzo surface over an arbitrary field F. Motivated by the classification of all such S up to isomorphism in terms of a separable F-algebra $B \times Q \times F$, and by the K-theory isomorphism $K_n(S) \cong K_n(B \times Q \times F)$ for $n \geq 0$, we prove an equivalence of derived categories

$$\mathsf{D}^b(\mathrm{coh}\, S) \equiv \mathsf{D}^b(\mathrm{mod}\, A)$$

where A is an explicitly given finite dimensional F-algebra whose semisimple part is $B \times Q \times F$.

Speaker: **Mikhail Borovoi** (Tel Aviv University)

Title: *Extended Picard complexes and homogeneous spaces*

Abstract: Inspired by a work of Kottwitz, for a smooth algebraic variety X over a field k of characteristic 0 we introduce a certain complex of Galois modules UPic(X), related to the Picard group Pic(X). We compute this complex UPic(X) up to isomorphism in the derived category, when X is a linear algebraic group or a homogeneous space of such a group. This is a joint work with the late Joost van Hamel.

Speaker: **Baptiste Calmés** (University of Cambridge)
Title: *Invariants, torsion indices and oriented cohomlogies of flag varieties* (joint work with Viktor Petrov and Kirill Zainoulline)
Abstract: This talk is about the results of the preprint arXiv:0905.1341. We consider the variety of complete flags G/B of a split semi-simple simply connected linear algebraic group G over a field, and we want to give a general description of the ring structure of $h^*(G/B)$ for any oriented cohomology h, in the sense of Levine and Morel. Some examples of oriented cohomology theories are Chow groups, the Grothendieck group K_0, connective K-theory and algebraic cobordism. In the 70's, Demazure showed that the structure of the Chow ring of G/B could be computed in terms of invariants of the Weyl group of G in the symmetric algebra over the characters of a maximal split torus T in G. We generalize this construction to arbitrary oriented cohomology theories by replacing the symmetric algebra by an ad-hoc combinatorial object depending on the formal group law of h. As an application, we prove a generalization to all oriented cohomology theories of a well-known theorem of Borel describing the (singular) cohomology of complete flags of type A_n in terms of symmetric polynomials. We also give an algorithm to compute the ring structure of algebraic cobordism of G/B and examples. This algorithm has been implemented using Macaulay 2.

Speaker: **Sunil Chebolu** (Illinois State University)
Title: *Freyd's Generating Hypothesis and The Bloch-Kato conjecture*
Abstract: I will present an overview of some recent work which surrounds two important conjectures on the cohomology of groups: Freyd's generating hypothesis for finite groups and the Bloch-Kato conjecture for absolute Galois groups.

Freyd's generating hypothesis for finite groups is a fundamental problem on the Tate cohomology. It claims that the Tate cohomology functor is faithful on the subcategory of representations generated by k. In joint work with Minac and Carlson we have shown that this hypothesis holds for a finite group G iff the Sylow p-subgroup ($p = \text{char } k$) of G is C_2 or C_3. This has led to some further basic questions including the problem of finite generation for Tate cohomology and some new invariants for finite groups which measure the degree to which the Generating hypothesis fails.

The Bloch-Kato conjecture is about the Galois cohomology of absolute Galois groups. Very roughly speaking, it claims that the Galois cohomology ring of an absolute Galois group is generated by degree one elements. In joint work with Efrat and Minac, we have shown (using this conjecture) that a certain quotient of an absolute Galois group determines the entire Galois cohomology. We use this result to produce some new examples of profinite groups that are not realisable as absolute Galois groups of fields.

Speaker: **Alexander Duncan** (University of British Columbia)

Title: *Groups of Essential Dimension 2*

Abstract: I will discuss essential dimension and the related concept of versal varieties. The classification of all finite groups of essential dimension 2 over algebraically closed fields of characteristic 0 is made possible by studying Manin and Iskovskikh's classification of minimal rational G-surfaces, and Dolgachev and Iskovskikh's classification of finite subgroups of the Cremona group.

Speaker: **Eric Friedlander** (University of Southern California)
Title: *Restrictions to $G(\mathbb{F}_p)$ and $G_{(r)}$ of Rational G-modules*
Abstract: We fix a prime p and consider a simple algebraic group G over a perfect field k which is defined

over $\mathbb{F}_p$. Let M be a finite dimensional rational G-module M, a comodule for $k[G]$. We seek to somewhat unravel the relationship between the restriction of M to the finite Chevalley subgroup $G(\mathbb{F}_p) \subset G$ and the family of restrictions of M to Frobenius kernels $G_{(r)} \subset G$. In particular, we confront the conundrum that if M is the Frobenius twist of a rational G-module N, $M = N^{(1)}$, then the restrictions of M and N to $G(\mathbb{F}_p)$ are equal whereas the restriction of M to $G_{(1)}$ is trivial. Our analysis enables us to compare support varieties (and the finer non-maximal support varieties) for $G(\mathbb{F}_p)$ and $G_{(r)}$ of a rational G-module M where the choice of r depends explicitly on M.

Speaker: **Skip Garibaldi** (Emory University)
Title: *Applications of the degree 5 invariant of E_8*
Abstract: We give a formula for Semenov's degree 5 invariant of groups of type E_8 with Rost invariant zero. We apply this formula in three settings: to invariants of $Spin_1 6$, to invariants of isotropic but not split forms of E_8, and to embeddings of finite subgroups in forms of E_8. This talk is on joint work with Nikita Semenov.

Speaker: **Philippe Gille** (Ecole Normale Superieure Paris)
Title: *Algebraic groups with few subgroups*
Abstract: This is joint work with Skip Garibaldi. Every semisimple linear algebraic group over a field F contains nontrivial connected subgroups, namely maximal tori. In the early 1990s, J. Tits proved that some groups of type E_8 have no others. We give a simpler proof of his result, prove that some trialitarian groups of type have no nontrivial connected subgroups, and give partial results for types E_6 and E_7.

Speaker: **Detlev Hoffmann** (University of Nottingham)
Title: *Differential forms and bilinear forms under field extensions*
Abstract: Let F be a field of characteristic $p > 0$. Let $\Omega^n(F)$ be the F-vector space of n-differentials of F over F^p. Let $K = F(g)$ be the function field of an irreducible polynomial g in $m \geq 1$ variables over F. We derive an explicit description of the kernel of the restriction map $\Omega^n(F) \to \Omega^n(K)$. As an application in the case $p = 2$, we determine the kernel of the restriction map when passing from the Witt ring (resp. graded Witt ring) of symmetric bilinear forms over F to that over such a function field extension K.

Speaker: **Nikita Karpenko** (Universite Paris 6)
Title: *Incompressibility of quadratic Weil transfer of Severi-Brauer varieties*
Abstract: We show that the Weil transfer with respect to a separable quadratic field extension of any 2-incompressible generalized Severi-Brauer variety is 2-incompressible provided that the norm of the corresponding central simple algebra is trivial. We discuss possible applications to unitary involutions.

Speaker: **Max-Albert Knus** (ETH Zürich)
Title: *Severi-Brauer varieties over the field with one element*
Abstract: Following Tits's ideas (1957) on the geometry over the field F_1 with one element, Severi-Brauer varieties over F_1 are defined. Many properties (over ordinary fields) of central simple algebras and central simple algebras with involution in relation with classical groups occur over F_1. Constructions like exterior powers, Clifford algebras and discriminant have analogues.

This is a report on joint work in progress with Jean-Pierre Tignol.

Speaker: **Daniel Krashen** (University of Georgia)
Title: *Patching topologies and local global principles*
Abstract: In this talk I will give a preliminary report on joint work with Hartmann and Harbater concerning certain local-global principles for cohomology groups of fields of the form $K(X)$ where K is a complete discretely valued field and X is a curve over K.

Speaker: **Roland Lötscher** (University of Basel)
Title: *A multihomogenization technique for the study of essential dimension of algebraic groups*

Abstract: The essential dimension of an algebraic group G is a numerical invariant measuring the complexity of G-torsors over extensions of a fixed base field k. It can be understood in terms of covariants between generically free representations of G. Informally, the essential dimension of G asks for how much the generically free G-action can be compressed by means of covariants.

In this talk I will present a multihomogenization technique (introduced in joint work with H. Kraft and G. Schwarz) for covariants and discuss some applications of this technique to the study of the essential dimension of finite algebraic groups. The applications include twisting of multihomogeneous covariants by torsors, which generalizes previous work of M. Florence.

Speaker: **Mark Macdonald** (University of British Columbia)
Title: *Essential p-dimension of algebraic tori*
Abstract: A recent theorem of N. Karpenko and A. Merkurjev gives a simple formula for the essential dimension of a constant finite p-group: the minimal dimension of a faithful representation. We give a similar formula for the essential p-dimension of a broader class of algebraic groups, which includes all algebraic tori. Joint work with R. Lotscher, A. Meyer, and Z. Reichstein.

Speaker: **Aurel Meyer** (University of British Columbia)
Title: *A bound on the essential dimension of central simple algebras*
Abstract: The essential dimension of an algebra describes (in terms of the transcendence degree) minimal fields where the algebra can be defined over. For most central simple algebras this number is unknown. We find upper bounds via certain subfields of the algebra. Combining this with a result by Rowen and Saltman on the universal division algebra, we prove a new upper bound on the essential p-dimension of the projective linear group PGL. (Joint with Z. Reichstein)

Speaker: **Alex Ondrus** (University of Alberta)
Title: *Minimal Anisotropic Groups of Higher Real Rank*
Abstract: Given a simple group G over a number field F, one can ask if G contains a proper subgroup H such that H attains higher rank over all real completions of F such that G does. If not, then we call G minimal. The classification of minimal groups isotropic over F has geometric applications and has been completed by V. Chernousov, L. Lifschitz and D.W. Morris. I will present the classification of anisotropic minimal groups and briefly discuss possible geometric applications of this result.

Speaker: **Raman Parimala** (Emory University)
Title: *Degree three Galois cohomology of function fields of surfaces*
Abstract: In this talk we shall explain certain local-global principle for degree three Galois cohomology of function fields of surfaces over finite fields as well as function fields of curves over p-adic fields. The latter was instrumental to the proof that the u-invariant of function fields of curves over nondyadic p-adic fields is 8. The former has consequences concerning degree three unramified cohomology of conic fibrations of surfaces over finite fields which is studied in the context of Brauer-Manin obstructions.

Speaker: **Arturo Pianzola** (University of Alberta)
Title: *Applications of Galois cohomology to infinite dimensional Lie theory*
Abstract: In this talk I will describe connections between two seemingly unrelated areas: Non-abelian Galois cohomology of Laurent polynomial rings on the one hand, while on the other, a class of infinite dimensional Lie algebras which, as rough approximations, can be thought off as higher nullity analogues of the affine Kac-Moody Lie algebras. The two view points are mostly based on joint work with P. Gille, and with B. Allison and S. Berman. The case of conformal superalgebras (joint work with V. Kac and M. Lau) will also be explored.

Though the algebras in question are in general infinite dimensional over the given base field (say the complex numbers), they can be thought as being finite provided that the base field is now replaced by a ring (in this case the centroid of the algebras, which turns out to be a Laurent polynomial ring). This leads

us to the theory of reductive group schemes as developed by M. Demazure and A. Grothendieck. Once this point of view is taken torsors and their accompanying non-abelian étale cohomology, arise naturally. It is this geometrical approach to infinite dimensional Lie theory that is one of the central themes of the talk.

Speaker: **Vladimir L. Popov** (Steklov Mathematical Institute, Moscow)
Title: *Cross-sections, quotients, and representation rings of semisimple algebraic groups*
Abstract: Let G be a connected semisimple algebraic group over an algebraically closed field k. Let $k[G]^G$ be the algebra of class functions on G and let $\pi_G : G \to G//G$ be the categorical quotient for the conjugating action of G on itself. The following interconnected questions will be addressed and answered:

(1) Is there a section of π_G?

(2) Is there a rational section of π_G?

(3) What is a minimal generating set of $k[G]^G$?

(4) What are the singularities of $G//G$?

(5) What is a minimal generating sets of the representation ring of G?

Speaker: **David Saltman** (Center for Communications Research - Princeton)
Title: *To be announced*
Abstract: To be announced

Speaker: **Andrew Schultz** (University of Illinois at Urbana-Champaign)
Title: *The first Galois cohomology group as a Gal(E/F)-module, and applications*
Abstract: For E/F cyclic of degree p^n, we decompose $H^1(G_E, F_p)$ as an F_p Gal(E/F)-module. The problem goes back to Faddeev, who considered the local case in the 1960s. The decomposition contains at most one indecomposable of dimension not a power of p, and this exceptional dimension must be of the form $p^m + 1$, $m < n$. We show that all $p^m + 1$, $m < n$ can be realized as exceptional dimensions. We then give an application of these results to the automatic realizability of Galois groups. We discuss ongoing work on E/F with other Galois groups, including the Klein 4-group and Z_p-extensions. This work is joint with John Swallow and Jan Minac.

Speaker: **Jason Starr** (University of New York at Stony Brook)
Title: Rational simple connectedness and Serre's "Conjecture II" Abstract: In topology a fibration over an n-dimensional base admits a continuous section provided the fibers are (n-1)-connected. Rational connectedness and rational simple connectedness are algebraic analogues of path connectedness and simple connectedness. Some years back, Graber-Harris-Starr and de Jong-Starr used rational connectedness to prove the n=1 case of an algebraic analogue of the section result. I will report on joint work with de Jong and X. He using rational simple connectedness to prove the n=2 analogue, and verifying that projective homogeneous varieties are rationally simply connected.

Together with an induction argument of Ph. Gille, this implies the split, geometric case of Serre's "Conjecture II": a torsor over a surface for a simply connected, semisimple algebraic group over an algebraically closed field always admits a rational section. Together with work of many people, this settles Serre's "Conjecture II" in the geometric case. Time permitting, I will also explain some connections to weak approximation for projective homogeneous varieties over the function field of a curve (originally proved by Colliot-Thélène and Gille), and to fixed point theorems for algebraic actions of $Z/aZxZ/bZ$ on projective homogeneous spaces (which appears to be new).

Speaker: **John Swallow** (Davidson College)
Title: *Galois cohomology groups as Galois modules, and applications*
Abstract: The proof of the Bloch-Kato Conjecture provides important tools for new results in field theory.

In particular, for E/F cyclic of degree p, we use these tools to decompose $H^i(G_E, F_p)$ as F_pGal(E/F)-modules. We show that not all indecomposable F_pGal(E/F)-modules appear, and we show how this fact has interesting consequences, for instance in showing that certain pro-p-groups are not absolute Galois groups. We also discuss other applications of the proof of the Bloch-Kato Conjecture, to Schreier's formula for Galois cohomology, analogues of theorems of Artin-Schreier and Becker, and to conditions for freeness and triviality of Galois cohomology. This work is joint with Dave Benson, John Labute, Nicole Lemire, and Jan Minac.

Speaker: **Alexander Vishik** (University of Nottingham)
Title: *Rationality of integral cycles*
Abstract: We will extend the Main Tool Lemma to the case of integral Chow groups. We will also prove the following result:

For any field k of characteristic zero, and any $r \in N$, there exists a field extension E/k such that:

1) $K_r^M(E)/2 = 0$;

2) for all smooth quasiprojective Y/k, and any $m < 2^{r-1} - 1$ the map $CH^m(Y) - -- > CH^m(Y_E)$ is surjective.

This result shows that (mod 2) and degree r cohomological invariants of smooth varieties could not affect rationality of cycles of codimension up to $2^{r-1} - 2$.