The Mathematics of Invasions in Ecology and Epidemiology

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1 Overview of the Field

This summer school was a continuation of the MITACS summer school series on disease modelling. Previous schools have been held at BIRS (2004), York University (2006), Xian Jiaotong University (2006), the Atlanta Center for Disease Control (2007), the University of Edmonton (2008) and the University of Ottawa (2009). In contrast to these schools, which focused on the mathematics of epidemiology and public health, this school focused on the dynamics of invasions and evolution. The school was organized jointly by two MITACS research groups: a group of researchers working on mathematical models of infectious diseases (www.liam.yorku.ca/research/MADI/) and a group of researchers working on mathematical models of biological invasions and dispersal (www.unb.ca/bid).

Historically, the fields of mathematical ecology and the dynamics of evolution have developed separately and it is only recently that work has been done to begin to bridge these two fields. Models for the evolution of populations assumed slowly changing or constant populations, and models for ecological populations assumed evolution took a much longer time scale than population dynamics. Recent theoretical work has begun to bridge these two approaches allowing population traits to change on the same timescale as population size. This advance is necessary for a theoretical framework for pathogen evolution in many systems. The influenza virus provides a pressing example. The timescale of viral evolution is similar to the rate of spread of the virus though the host population. Any control measures, such as vaccines or antiviral medications, must take into account the rapid appearance of drug resistant strains. Other examples presented in lectures include weedy species [7], HIV and vector-borne parasites such as malaria.

The lectures were divided roughly along two lines: the evolution of pathogens; and the spread of an invading pathogen. The mathematical foundations of pathogen evolution were outlined in the lectures of Day and Gomulkiewicz, and applied in the lectures of Gilchrist, Reisberg and Reid. The lectures of Allen and Brauer introduced the basic stochastic and deterministic models for pathogen invasion and spread, while the lectures of Arino and Gourley covered spatial approaches to modelling, first in the context of metapopulations, and the latter in the continuous space. Nelson applied these methods to the spread of a forest insect and introduced additional modelling techniques for structured host populations. The lecture of Zou extended the models introduced by Gourley to include delays.

2 Recent Developments and Open Problems

Several key open problems were introduced in the lectures.

1. A single theoretical framework for pathogen evolution and spread has yet to be developed.
2. New modelling paradigms are necessary for pathogen spread and evolution that incorporate both the scales of within-host and between-host into a single theoretical framework [3].
3. Key public concerns surrounding treatment and vaccination require models for the evolution of resistance, such as a resistance to treatment or vaccine in a human infectious disease.
4. Analytical tools are needed to address the spatial component of resistance evolution. For example, Chloroquine resistance in mosquitoes seems to arise in areas of low transmission [1], which suggests that control measures must take spatial dynamics and evolution of the pathogen and vector into account.
5. Much more analytical work is needed to understand the path of spread of a pathogen through a structured population. Two examples given were the global spread of novel human pathogens [4] and the spread of the mountain pine beetle [5]. In the first case, the host population is spatially structured, and in the second, the population is structured by host defenses.
6. Most diseases involve a delay between infection and onset. This leads to many open mathematical problems in a spatial setting [6]

3 Presentation Highlights

As with previous schools, there were many more applications than could be accepted. In attendance were 35 students from mathematics and ecology of which 8 were from the US and 4 from outside north america.

The format for the school consisted of a series of short courses, case studies and student group projects. Short courses were an important component, providing students with the basic theory on a topic. The participants were able to apply the theory in group projects. Each short course consisted of one 90 minute lecture and one 60 minute tutorial. They covered topics on basic modelling, both deterministic and probabilistic, and specialized modelling topics such as metapopulations, evolution and dispersal. Case studies consisted of 90 minute lectures by leading international researchers. This summer school covered relevant mathematical background and recent progress in the fields of biological invasions in ecology and epidemiology

The highlight of the meeting was the presentations by the students. These students varied from upper level undergraduates to Postdoctoral and young researchers. The projects were assigned on the second day of the workshop, giving a mere five days for development and analysis of an appropriate model. As in the past, the projects were well done, with some exceptional presentations.

The student projects identified six open problems:

- the role of disease in the decline of amphibian species, specifically the role of *Chytridiomycosis* as a cause of species extinction;
- the role of a pathogen in the spread of an invasive forest or agricultural insect;
- the evolution of virulence in the spread of syphilis;
- the reasons for spectacular failures of biological control using exotic species;
- the evolution of a species in a changing habitat;
- the spread of a novel strain of influenza.

4 Scientific Progress Made

The lectures and case studies presented techniques, ideas and problems at the leading edge of mathematics and its applications to ecology and evolution. Most of the case studies covered applications of mathematics from an ecological perspective, encouraging all participants to think about new mathematical approaches to problems. Some of the project reports covered new ground, and we encourage the students to continue their collaboration wih the goal of publishing their results in a peer reviewed journal.

5 Outcome of the Meeting

In summary, the school brought together students and researchers in evolution, ecology and epidemiology using a variety of modelling and analytical techniques, including game theoretic, statistical, pde, ode and dynamical systems. We hope that many students will continue to work in this emerging research area of theoretical epidemiology at the interface of mathematics, ecology and evolution.

References

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