1. A Problem Posed By Vern I. Paulsen

Given an operator $T \in B(\mathcal{H})$ it's n**-th matrix range** is the set

$$W^n(T) = \{\Phi(T) : \Phi : B(\mathcal{H}) \to M_n \text{ is completely positive and unital}\}.$$

This problem is concerned with how well knowledge of matrix ranges for small values determine them for larger values.

Let

$$\mathcal{S}_{n,j}(T) = \{A \in M_n : W^j(A) \subseteq W^j(T)\}.$$

Given $A \in M_n$ and $\mathcal{W} \subseteq M_n$, we let $d(A, \mathcal{W}) = \sup\{\|A - W\| : W \in \mathcal{W}\}$ denote the usual distant between a point and a set and given two sets of matrices $\mathcal{S}, \mathcal{W} \subseteq M_n$ we let $d(\mathcal{S}, \mathcal{W}) = \sup\{d(A, \mathcal{W}) : A \in \mathcal{S}\}$ denote the usual distance between sets.

Problem 1. *Does* $\lim_{j\to\infty} \sup_n d(\mathcal{S}_{n,j}(T), W^n(T)) = 0$ *for every* $T \in B(\mathcal{H})$*?*

This problem turns out to be equivalent to a problem about preserving what are called essential matrix ranges with compact perturbations.

I bring it up at this meeting because I believe that it could also be at the heart of some questions about quantum capacity. It is related to the following vague sort of question. Given a cp map $\Phi : M_n \to M_n$ if we can only have knowledge of $j \times j$ submatrices, with $j << n$, how well can we know the map?

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