Banff International Research Station

05w2057 The Kneser-Poulsen Conjecture

November 3 - November 5, 2005

Organizers:

Karoly Bezdek (University of Calgary, Canada)
Robert Connelly(Cornell University, USA).

Confirmed Participants for 05w2057

Name	**Affiliation**
Belk, Maria	Texas A&M University
Bern, Marshall	Palo Alto Research Center
Bezdek, Karoly	University of Calgary
Blekherman, Greg	Microsoft
Brudnyi, Alex	University of Calgary
Connelly, Robert	Cornell University
Csikos, Balazs	Eotvos University, Institute of Mathematics
Kiss, Gyorgy	Eotvos University
Langi, Zsolt	University of Calgary
Ling, Joseph	University of Calgary
Maciejewski, Wes	University of Calgary
Naszódi, Márton	University of Calgary
Papez, Peter	University of Calgary
Rivin, Igor	Temple University

Speaker: Marshall Bern (Palo Alto Research Center, USA)

bern@parc.com

Title: Mo"bius-Invariant Natural Neighbor Interpolation

Abstract:
In interpolation, we are given the value of some function f at a discrete set of sample points, and we wish to determine reasonable values for f at other, unmeasured points. I will describe a 2-d interpolation method using Delaunay triangulation. The method has the property that interpolation followed by a Mo"bius transformation gives the same result as the transformation followed by interpolation. In the limit of closely spaced sample points on a circle, the limit of the interpolated function is harmonic.

Speaker: Karoly Bezdek (University of Calgary, Canada)
bezdek@math.ucalgary.ca

Title: From the Kneser-Poulsen conjecture towards a theory of ball-polytopes

Abstract:
The talk will start with a theorem of Kirszbraun (1934) and then will outline the state of the art of the Kneser-Poulsen conjecture (1954/1955) including a short summary of the main ideas of the recent solution of the (Euclidean) planar case (resp., of the spherical case for congruent balls of radii 90 degrees in any dimension) by Bezdek and Connelly (2002/2004). Then motivated by a conjecture of Alexander (1984) the talk will focus on some of the most striking questions on the geometry of intersection of congruent balls that is of ball-polytopes. (Additional results on this direction will be discussed in more details in the talks of Langi and Papez at the meeting.)

Speaker: Alexander Brudnyi (University of Calgary, Canada)
albru@math.ucalgary.ca

Title: A universal Lipschitz extension property for products of Gromov hyperbolic spaces

Abstract:
In the talk I consider an extension problem for Lipschitz maps defined on subsets of a metric space with images in Banach spaces. I introduce the notion of metric spaces with the absolute Lipschitz extendability property and present some classes of spaces satisfying this property.

Speaker: Robert Connelly (Cornell University, Ithaca NY, USA)
rc46@cornell.edu

Title: Kneser-Poulsen problems-What do we need for further progress?

Abstract:
When a finite collection of circular disks are rearranged so that their centers are contracted, does the area/volume of the union necessarily not increase? In the plane the answer is yes, by a result of Bezdek-Connelly. The proof uses three ideas that just come together for the case of the Euclidean plane. One idea is to use volume formulas comparing the volume of disks in Euclidian d- dimensional space to the (d+1)-dimensional volume of the boundary of the corresponding disks in (d+2)-dimensional space. This idea, going back to Archimedes, generalizes nicely to other spaces. Another idea is to use some formulas of Csikos for the change in the boundary volume for a continuous motion. This works nicely in all dimensions and in other spaces. The third idea is to show a discrete contraction of a configuration of points can be made continuously monotone, if the configuration is allowed to move through higher dimensions. This idea should extend to other natural situations, but there do not seem to be any counterexamples or proofs known. Two natural situations are the following: If one configuration is a discrete contraction of another in the 2-sphere, is there a corresponding extension to a monotone continuous contraction in the 4-sphere? If one configuration is a discrete contraction of another in the hyperbolic plane, is there a corresponding extension to monotone continuous contraction in hyperbolic 4-dimensional space? There is reasonable evidence that the answer to these questions are both yes, but I do not know how to do it.

Speaker: Balazs Csikos (Eotvos University, Budapest, Hungary)
csikos@math.elte.hu

Title: Volume Variation Formulae and Volume Monotonicity

Abstract:
In this talk, we give a survey of variation formulae for the volume of polytopes with curved faces, in particular, for the volume of polytopes bounded by spherical faces. These formulae turn out to be quite useful in the study of the Kneser-Poulsen conjecture. We present some Kneser-Poulsen-type volume monotonicity results obtainable from them.

Speaker: Balazs Csikos (Eotvos University, Budapest, Hungary)

Title: The Kneser-Poulsen Conjecture in the Elliptic Space

Abstract:
First we show that the Kneser-Poulsen conjecture cannot be extended to the elliptic space: we construct 3 smoothly moving congruent balls with centers getting closer to one another in such a way that the volume of the union of the balls strictly increase during the motion. In spite of this counterexample, we shall prove a special case of the conjecture in the second part of the talk. As a corollary, we obtain that $n+1$ balls in the n-dimensional elliptic space cover maximal volume if the distances between the centers are all equal to the diameter $\pi/2$ of the space. This is a joint work with Gabor Moussong.

Speaker: Zsolt Langi (University of Calgary, Canada)
zlangi@math.ucalgary.ca

Title: A Generalization of the Discrete Isoperimetric Inequality for Piecewise Smooth Curves of Constant Geodesic Curvature

Abstract:
The discrete isoperimetric problem is to determine the maximal area polygon with at most k vertices and of a given perimeter. It is a classical fact that the unique optimal polygon on the Euclidean plane is the regular one. The same statement for the hyperbolic plane was proved by K\'aroly Bezdek and on the sphere by L\'aszl\'o Fejes T\'oth. In the present paper we extend the discrete isoperimetric inequality for ``polygons'' on the three planes of constant curvature bounded by arcs of a given constant geodesic curvature.

Speaker: Marton Naszodi (University of Calgary, Canada)
nmarton@math.ucalgary.ca

Title: Touching Homothetic Bodies and Antipodality

Abstract:
An antipodal set in Euclidean n-space is a set of points with the property that through any two of them there is a pair of parallel hyperplanes supporting the set. In this talk, I will present two research topics that are connected by the idea of antipodality.
The first part of the talk will focus on the extension of the above concept to hyperbolic n-space. This is a joint work with Karoly Bezdek and Deborah Oliveros.
In the second part, the maximum number of touching positive homothetic copies of a convex body in Euclidean n-space will be discussed. According to a conjecture of Karoly Bezdek and Janos Pach, this number is 2^n; which bound, if it holds, is sharp as it is attained by cubes. The previously known bound was 3^n, I improved it to $2^(n+1)$.

Speaker: Peter Papez (University of Calgary, Canada)
pdpapez@math.ucalgary.ca

Title: Ball-Polytopes

Abstract:
The study of polytopes is one of the oldest and most well researched areas in all of mathematics. One way of looking at polytopes is to interpret them as the region bounded by intersecting hyperplanes. These hyperplanes are just surfaces of

zero curvature. Suppose that we use surfaces of non-zero curvature, say of curvature one. What do we obtain by doing this? With some care we obtain ball-polytopes. Intuitively, we can think of these as fattened polytopes, but the concept is more delicate than may first appear. The aim of this talk is to survey the results obtained by our research group in the study of ball-polytopes. These results range over many different areas of geometric interest. Most results pass to higher dimensions, but we will focus on the two- and three-dimensional cases to provide insight regarding the techniques used in this field of study. This is a joint work with Karoly Bezdek, Zsolt Langi, Marton Naszodi.