RESOLUTION OF SINGULARITIES

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The subject of this mini-series is the main ideas in the proofs of resolution of singularities in characteristic zero by Bierstone-Milman and Villamayor, as well as in the more recent work of Włodarczyk.

Each of these approaches involves reduction of the main desingularization problems (embedded resolution of singularities, principalization of an ideal,...) to simplification ("desingularization") of a certain collection of local data (a "marked ideal", in the language of Włodarczyk). Canonical desingularization of a marked ideal is proved by induction on dimension. Włodarczyk's proof is structured to emphasize that the only inductive assumption should be canonical desingularization of marked ideals in lower dimension.

"Canonical" desingularization, in a general sense, means that "equivalent" objects should undergo the same sequences of blowings-up. Although the algorithms for desingularization of a marked ideal by Bierstone-Milman and Włodarczyk are essentially the same, their notions of equivalence are different. Roughly speaking, that of the latter means there is an automorphism taking one object to the other, while that of the former means the objects have the same sequences of "test blowings-up". The notion of equivalence bears in a crucial way on the desingularization theorems that follow from simplification of a marked ideal, as well as on practical computation.

We will explore the ideas involved, using concrete examples to illustrate the relationships among the approaches, and their consequences.

References

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