Nonlinear Dynamics of Thin Films and Fluid Interfaces

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Abstract

This five-day multidisciplinary workshop focussed on the mathematics of free-surface fluid flow. Building on theoretical and experimental developments of the last few years, the workshop brought together mathematicians and physicists at the forefront of research in experiments, analysis, computation, and modelling. The workshop yielded a rich exchange of ideas in the areas of thin liquid films, dynamic contact lines, slender jets, Hele-Shaw flow, and fluid interfaces.

1 Overview

The mathematics of thin liquid films and fluid interfaces has developed substantially in the last ten years, and is now being pursued by different groups of mathematicians, physicists, and engineers across Canada, Europe, and the United States. The workshop brought together experimentalists, analysts, modelers, and computational fluid dynamics experts for a meeting that yielded a vigorous assessment of the field from many different points of view.

The clear air and an early winter snow blanketing the spectacular surrounding mountains, coupled with the superb facilities of the Banff Institute, provided an ideal setting for the workshop. The meeting was notable for the many informal discussions, lively interaction during talks and generally friendly and cooperative atmosphere. While many of the participants have met before, there were also many new contacts established, and lots of resolve to continue to provide forums for continued dialogue between groups separated by large distances.

The mechanics of fluids with free surfaces or interfaces rests on the largely unexplored area of nonlinear fourth-order partial differential equations. Fourth-order derivatives arise from surface tension, which appears in the equations as gradients of the curvature of the free surface. Fourth-order equations have applications to several active fields of scientific research, including materials science, nanotechnology, and biology. The interaction of physicists and engineers with applied mathematicians and analysts has provided tremendous motivation for the mathematics; the mathematics in turn has contributed substantially to understanding of the wide variety of interesting physical phenomena in this area.

The workshop was linked to and partially supported by an NSF Focused Research Group (FRG) Grant (NSF-DMS #0073841/0074049) on the dynamics of thin films. This grant is directed by four of the workshop organizers: Behringer, Bertozzi, Shearer, and Witelski.

We encourage interested readers to do a literature search on speakers of interest and look at their articles for references. A conference proceedings is planned, to appear as a special volume of the journal Physica D.

2 Free-surface flows: Modelling and Dimensional Reductions

Free-surface flows are a challenging area of research in the physical sciences [1–3]. They are important for industrial and technological processes such as coating flows, film drainage, fluid jetting and droplet formation. Physical experiments with novel and intricate dynamics have pushed classic engineering models to their physical limits. The mathematical modelling and analysis of such dynamics has produced new generations of problems and results in free boundary problems for nonlinear partial differential equations [4, 5].

The mathematical description of a physical free-surface flow is composed of a number of elements. Assume the fluid is located in a time-dependent region $D_t \subset \mathbb{R}^3$. Initial conditions for D_0 and the velocity $\vec{u}_0(\vec{x})$ for all points in the domain are required. Each fluid particle $\vec{x}$ in this region moves with a velocity $\vec{u}(\vec{x},t)$ determined by dynamical conservation laws for the conservation of mass and momentum. For example, if the fluid is an incompressible Newtonian liquid, the governing equations are the Navier-Stokes equations. The system has boundary conditions at ∂D_t. The conditions on ∂D_t can be mixed if portions of the boundary are liquid/gas, liquid/liquid, or liquid/solid interfaces with physical models dictating different conditions on each type of interface. Finally, there is a kinematic boundary condition at ∂D_t which determines the velocity $\vec{u}(\vec{x},t)$ for points on the boundary. The subsequent evolution of the flow will be influenced by any forces that may be acting on the interior or the surface of the domain.

Direct numerical simulation or analysis of the full system of equations is intractable in most cases. And even when possible, large-scale numerical simulations do not necessarily provide insight into the fundamental physical mechanisms at work. Valuable understanding has been gained from studies of asymptotic models that are applicable in limiting cases. These models take more tractable forms, yet retain much of the rich dynamics of the experiments; their study spurs advances in analysis and computation. The use of such models was a major theme of the workshop, with many striking results of analysis and numerical simulation coupled to beautiful and instructive physical experimental findings.

In deriving an asymptotic model, one often makes an approximation of the region D_t, an approximation of the velocity $\vec{u}(\vec{x},t)$, and approximations of the boundary conditions. This leads to an "asymptotic model" of the full flow. Asymptotic models of free-surface flows yield systems of lower dimension and simpler structure. *Lubrication models* give a single partial differential equation for the motion of the fluid interface, while *long wave models* give coupled evolution equations for the dynamics of the fluid interface and average flow velocity. These modelling techniques have produced striking advances in several areas:

- <u>Thin viscous films on solid surfaces</u>: Lubrication models for low Reynolds number flow of a thin viscous film on a solid surface take the form of a nonlinear fourth-order degenerate PDE for the height of the film's free surface, $h(x,y,t)$,

$$\frac{\partial h}{\partial t} + \nabla\cdot\Big(\,M(h)\,\nabla\big[\nabla^2 h + P(h)\big] \;+\; F(h)\,\vec{e}_1\Big) \;=\; R(h). \tag{1}$$

  Here the liquid/solid interface is assumed to be at $z = 0$ and the air/liquid interface at time t is located at $z = h(x,y,t)$ and so D_t is approximated by $\mathbb{R}^2 \times [0, h(x,y,t)]$. The full velocity $\vec{u}(\vec{x},t)$ is approximated by a laminar velocity $(v_1(x,y,t), v_2(x,y,t), 0)$ which is fully determined in terms of $h(x,y,t)$. Various physical effects including system geometry, gravity, surface tension, thermodynamic effects, chemical kinetics, materials properties and constitutive equations of state are all incorporated into the functions for $F(h)$, $M(h)$, $P(h)$, and $R(h)$. Under different conditions, complicated dynamics including rupture singularities and non-classical shocks occur in this model.

- The Hele-Shaw cell: In a Hele-Shaw cell, a fluid is placed between two closely-spaced parallel plates. The fluid moves in response to pressure gradients arising from surface tension and externally imposed forces. The equations describing this problem are the same as those that describe dendritic solidification in an important limiting case, and are also very similar to those describing flow through a porous medium. In the absence of externally imposed forces, the asymptotic model is

$$\begin{cases} \Delta p = 0 & \text{in } \Omega_t \\ p = \tau\kappa & \text{on } \partial\Omega_t \\ \vec{v} = -\nabla p & \text{on } \partial\Omega_t. \end{cases} \tag{2}$$

  If the parallel plates are a distance 2δ apart, the fluid is in the region $\Omega_t \times [-\delta, \delta]$ at time t. The region Ω_t is the fluid region that one would observe by viewing the Hele-Shaw cell from above (if the two glass plates were on a flat table). The fluid in Ω_t moves with a velocity that is proportional to the gradient of the pressure p. (That is, the full three dimensional domain D_t has been approximated by $\Omega_t \times [-\delta, \delta]$ and the full velocity $\vec{u}(\vec{x}, t)$ has been approximated by a two-dimensional velocity $(-p_x(x, y, t), -p_y(x, y, t), 0)$.) The pressure jump at a point on the air/liquid interface $\partial\Omega_t$ is proportional to the curvature of the interface at that point times the surface tension parameter. The system (2) is elliptic: the velocity at a point $x \in \Omega_t$ is influenced by the entire domain. If the region Ω_t has a thin neck (if Ω_t were shaped like a barbell, for example, with a neck located at $(x, \pm h(x, t))$ for $x \in [-x_0, x_0]$) a further dimensional reduction reduces the nonlocal system (2) to a fourth-order PDE,

$$h_t + (h h_{xxx})_x = 0, \tag{3}$$

  that holds for $x \in [-x_0, x_0]$. This PDE has interesting dynamics and remarkably complex singularity behavior [6].

- Instabilities of slender laminar jets of fluid. Consider a dripping faucet. As a pendant drop begins to form, there is a strand of fluid that connects the drop to the faucet's mouth. This strand of fluid is an example of a jet; it begins to develop instabilities and ultimately breaks and the drop falls. If the full three-dimensional domain D_t is approximately axisymmetric and with the jet's air/liquid interface located at $(h(z, t)\cos(\theta), h(z, t)\sin(\theta), z)$ with gravity is acting in the $-z$ direction, the long-wave model for this system yields a system of partial differential equations for the evolution of the jet radius and of the radially averaged velocity

$$\begin{cases} v_t &= -v v_z - \frac{p_z}{\rho} + 3\nu \frac{(h^2 v_z)_z}{h^2} - g \\ h_t &= -v h_z - \frac{1}{2} v_z h \\ p &= \gamma \left[\frac{1}{h(1+h_z^2)^{1/2}} - \frac{h_{zz}}{h(1+h_z^2)^{3/2}} \right]. \end{cases} \tag{4}$$

  Here the full velocity $\vec{u}(\vec{x}, t)$ has been approximated by an axial velocity $(0, 0, v(z, t))$, p is the pressure, ν is the kinematic viscosity, ρ is the density, γ is the surface tension, and g is the gravitational constant. These equations capture the interaction of viscosity, inertia, and surface tension [1].

Experiments show that surface tension has important, yet often subtle, effects on interface dynamics. In the modelling equations, surface tension effects are produced by the curvature of the fluid interface, and by gradients of related properties on the interface. Consequently all of these physical problems are modelled by strongly nonlinear, sometimes nonlocal, higher-order nonlinear partial differential equations. These models have provided fundamental and important mathematical challenges. Interesting free boundary problems arise and much is being learned about their solutions.

Important changes in the qualitative structure of the physical systems generally appear as finite-time singularities in the corresponding simplified model: formation of dry spots in a thin film, pinch-off of a thin neck in a Hele-Shaw cell, or pinch-off of a thin jet as in a dripping faucet. A great deal is now known computationally, mathematically and experimentally about the structure of singularities in the model systems. This knowledge is providing insight into the behavior of the full systems.

3 Analysis of thin film equations

One of the fundamental mathematical challenges involved in the study of equation (1) is that the the mobility coefficient, $M(h)$, vanishes at $h = 0$: $M(h) = O(h^n), n > 0$. Assume that the mass is conserved ($R(h) = 0$) and there is no external driving ($F(h) = 0$). Further, consider the case of a film which is uniform in one direction. Equation (1) can then be written as

$$h_t + (hU)_x = 0 \tag{5}$$

where $U(x,t)$ is the average velocity across the cross-section of height $h(x,t)$ located at x at time t. It follows that in order for the thin film to form a dry spot in finite time,

$$h_{\min}(t) := h(x_{\min}(t), t) \to 0 \tag{6}$$

as $t \to T$, there must be a divergence of $U_x(x_{\min}(t), t)$ as $t \to T$. This loss of regularity prevents nonnegative h from going negative at a point (and becoming "unphysical"). Also, if there is a contact line at $x = a(t)$ (i.e. $h(x,t) > 0$ for $x \lesssim a(t)$ and $\lim_{x\to a(t)} h(x,t) = 0$) then this contact line cannot move unless $\lim_{x\to a(t)} U(x,t) = \pm\infty$.

These are heuristic explanations for how the interplay between the mobility $M(h)$ and the regularity of the solution h is crucial for the model to reflect the correct physics: the film thickness should be nonnegative at all times and the contact line should move with finite speed. Proving such behaviors analytically is nontrivial because equation (1) is fourth-order and the comparison-method-based techniques used for second-order degenerate equations do not hold. The development of new analytical techniques is an active area of research; six of the speakers presented analytical work on such thin film equations.

The study of exact solutions such as steady states, travelling waves, and self-similar solutions has proven to be very helpful in understanding general solution properties. In this direction, Mark Bowen presented a collection of exact solutions to the thin film equation

$$h_t + (h^n h_{xxx})_x = 0. \tag{7}$$

These included "dam break" self-similar solutions, separable solutions, and self-similar solutions that had a combination of fixed and free boundary conditions. His studies included how the value of n affected the solutions and their properties [7–9].

Continuing in the theme of this particular equation, Lorenzo Giacomelli presented work in which he rigorously derived the lubrication equation [10]

$$h_t + (hh_{xxx})_x = 0 \tag{8}$$

directly from the related Hele-Shaw problem. The derivation was done by finding the limiting behavior of solutions of the Hele-Shaw problem and the equation they obey, rather than performing asymptotics on the evolution equation. This work is especially exciting because it provides a new bridge between PDE analysts and experts in asymptotics.

Günther Grün presented new interpolation inequalities for

$$h_t + \nabla \cdot (h^n \nabla \Delta h) = 0 \tag{9}$$

and its analogues. He considered a higher-dimensional case than Bowen and Giacomelli: $\vec{x} \in \mathbb{R}^d$ with $d \geq 2$. The interpolation inequalities allow him to prove the existence of weak nonnegative solutions, finite speed of propagation for compactly supported weak solutions, as well as waiting time results [11, 12]. And as a bonus, he presented numerical simulations of complex structures being formed in thin films on silicon wafers that are being driven by molecular forces at the liquid/solid interface [13].

Continuing in the direction of liquid film equations with non-advective instabilities, Mary Pugh presented results concerning a critical-exponent long-wave unstable thin film equation [14],

$$h_t + (h^n h_{xxx})_x + (h^{n+2} h_x)_x = 0. \tag{10}$$

This included self-similar solutions that that blow up in finite time in a focussing manner and a study of how these solutions are related to the steady states of the evolution. Dejan Slepcev studied a special case of this critical-exponent equation ($n = 1$) and presented linear stability results for steady droplets, self-similar source-type solutions, and self-similar blow-up solutions [15].

Another system in which there can be surprising instabilities was studied by Sandra Wieland. She presented her thesis work on thin liquid films which have surfactants on the air/liquid surface. This leads to a coupled system involving a lubrication equation for the film thickness and an advection diffusion equation for the surfactant concentration. The system has hyperbolic aspects to it and its analysis is nontrivial and illuminating. A related talk by Barry Edmonstone focussed on new phenomena and their realization in numerical experiments.

4 Filament Instabilities and Moving Contact Lines

Physical experiments are uncovering a broad array of dynamics in thin liquid films and fluid interfaces. The connections to industrial processes, such as spin coating in the manufacture of microprocessors, are also at an early stage of development. Some examples of recent progress in this area include scaling laws for jets and drops, a problem of relevance to inkjet printers, to electrospinning of fibers, and to nanotechnology printing devices.

Two of the speakers presented research on filaments and jets:

Linda Smolka presented her work on liquid filaments in extensional flows [16]. She considered exact, cylindrical solutions in which the radius is time-dependent and used these in a study of various viscoelastic constitutive models. This work is of definite industrial and scientific interest as it suggests ways to use extensional flows to measure rheological properties of nonstandard materials.

Marco Fontelos also considered axisymmetric structures in non-Newtonian flows, specifically in a model where the rheology's constitutive law is Oldroyd-B. He used asymptotics and numerics in an elegant study of the dynamics of droplets in a "beads on a string" configuration in which one has a collection of droplets connected by thin filaments; the filaments allow the droplets to interact and fuse [17].

Another area of intense experimental activity is that of moving contact lines, specifically how the microscopic physics at the contact line can influence large-scale properties of the flow. To date, most of the analysis of driven films has addressed the question of linear instability and wavelength selection. Understanding nonlinear dynamics requires cooperation between the experimentalists, modelers, and analysts in the following ways; (i) from experiments we need to understand basic nonlinear structures that emerge in this problem, including fingers, teeth-like patterns, and capillary ridge instabilities, (ii) once these patterns are well identified, modelers should work to isolate the essential physical mechanisms present and provide simplified models that are tractable for analysts, (iii) a combination of analysis and computation of the models will lead to a general understanding of the problem that can then be compared with experiments and guide further experiments. Ten of the speakers contributed to this program by reporting on their studies involving driven films and contact line instabilities of various types:

Stephen Wilson presented his work on steady rivulet flows down inclined planes. This is an extensive combination of analytical and computational work that considers both constant and temperature-dependent surface tension [18, 19].

Michael Schatz presented his experimental work in which he optically heats regions near the contact line, inducing a surface-tension driven flow [20]. This allowed a direct study of transient amplification near the contact line.

Roman Grigoriev presented careful computational studies of the transient amplification observed by Schatz and others, with the goal of understanding the widely differing predictions made by theorists. In addition, he presented striking experiments in which he used optical heating in a feedback control system to suppress contact line instabilities [20, 21].

Len Schwartz presented a collection of numerical studies of thin film flows with different types of driving forces and substrate geometries [22]. His studies included models with effects such as drying, the effects of surfactants, and the response to local contaminants on the air/liquid surface.

At the end of his talk, he engaged the audience in a lively discussion of the "correct" boundary condition to place at the free boundary represented by the triple junction of fluid, surface and air, when the contact line is moving. His point of view is that a disjoining pressure should be introduced to avoid the contact line and thus avoid the well-known stress singularity that seemingly prevents the contact line from moving if the usual no-slip boundary condition is assumed between fluid and solid substrate [23]. This intriguing issue is a famous outstanding problem in the field of dynamic contact lines; it was alluded to repeatedly during lectures and in discussion.

Barbara Wagner presented her work on dewetting films. Here, one has a thin film which is rupturing in regions. In near a rupture region, the bulk of the film is receding from an area where the film has either ruptured or become very thin. Experiments show intricate transient dynamics and long-time patterns. Wagner presented numerical studies in which she finds that the receding front has a ridge that is linearly unstable in the spanwise direction, thus allowing additional structures to form.

Lou Kondic presented computational studies of gravitationally driven thin films on heterogeneous surfaces [24–26]. His studies focussed especially on the dynamics of the contact line and how it responds to nearby perturbations. Of particular interest was the influence of the spatial structure of the perturbations.

Javier Diez presented experiments studying thin films flowing down a vertical plate. He studied both the constant volume and the constant flux flows and compared the experimental results to numerical simulations, in a study of how the thickness of the precursor film assumed in the simulations affects the relevance of the simulations to the experiments.

Barry Edmonstone presented numerical simulations studying surfactant-ladened thin films flowing down an inclined plane, as modelled by a coupled system for the film thickness and the surfactant concentration. He considered both fixed volume and fixed flux boundary conditions and studied instabilities at the contact line and within the surfactant monolayer concentration.

Martine Ben Amar presented her work on small droplets sliding down inclined planes [27]. Experiments show that there is a critical angle of inclination (or critical sliding speed) at which the the shape of the trailing contact line develops a corner. She presented modelling and asymptotics concerning this phenomenon, which is especially difficult to study because the droplets are fully three-dimensional.

5 Undercompressive shocks in driven films

The discovery of undercompressive shocks in driven films is an example of the interdisciplinary nature of studies on thin liquid films. This discovery arose from a three-way confluence of analytical and computational work on thin liquid films, of analytical studies of hyperbolic PDEs, and of experiments on thermally driven thin liquid films. Early experiments of Cazabat and Fanton [28] concerning a thin film that is climbing an inclined plate found that very thin films produced the characteristic fingering instability, while slightly thicker films showed a strong tendency not to form fingers. The latter behavior was observed to be associated with a broadening of the capillary ridge behind the contact line.

The experiments prompted Bertozzi, Münch, and Shearer to work on a mathematical model for this problem [29, 30]. The relevant form of equation (1) for the experiment has $M(h) = h^3$ and $P(h) = R(h) = 0$. What was unusual about the experiment is that the thermal gradient acts against the pull of gravity, producing a non-convex flux of the form $F(h) = h^2 - h^3$. In this case, equation (1) is a hyperbolic conservation law with degenerate fourth-order diffusion. Travelling wave solutions, correspond to either classical shocks, satisfying the well-known entropy condition, in which characteristics enter the shock from both sides, or to undercompressive shocks for which characteristics pass through. The undercompressive shocks help to explain the lack of fingering in thicker films, and the associated broadening of the capillary ridge. This comparison between theory and experiment is the first clear evidence of the existence of scalar law undercompressive shocks in a physical experiment. Further work on this problem has drawn on different branches of applied mathematics. For example, dynamical systems methods and Lyapunov functions were used to prove existence of undercompressive waves; Evans functions methods resulted in a connection between the topology of the travelling wave phase space and the stability of the waves.

Bob Behringer presented recent experimental and computational results of Jeanman Sur in this context [31]. The film climbs the plate driven by a temperature gradient that induces variation in the surface tension. Using a version of dip coating, undercompressive shocks were induced at the trailing edge of the film, in contrast to the original experiments in which the undercompressive wave appears at the leading edge. In addition, lasers are used to perturb the film at a chosen wavelength, thus experimentally studying the linear stability of the system. Andreas Münch presented simulations of these so-called reverse undercompressive fronts in thin film flow, including numerical stability studies [32].

Michael Shearer presented his work on Lax shocks and undercompressive shocks in the context of driven thin liquid films [33,34]. He demonstrated how the structure of waves observed in experiments and numerical simulations can be explained using only the hyperbolic conservation law, together with an appropriate kinetic relation (that selects the undercompressive shocks) and nucleation criterion (that distinguishes between classical and nonclassical wave structures).

Peter Howard presented his work on nonlinear stability for viscous shock waves arising in conservation laws with high order viscosity [35–37]. He presented an overview of the pointwise Green's function approach for analyzing stability and discussed what this approach can and cannot do in the case of high order viscosity.

Burt Tilley presented modelling and computational work in which he studied flow of two incompressible immiscible viscous fluids in an inclined channel [38, 39]. In this system, he studied Lax shocks, undercompressive shocks, and rarefaction waves. Numerical simulations help identify parameter regimes for the different phenomena.

6 General problems on interface motion

Alexander Golovin studied the self-organization of quantum dots in thin solid films [40]. This included a weakly nonlinear stability analysis of spatially regular patterns in the presence of both epitaxial stress and anisotropic surface tension.

Amy Novick-Cohen studied a problem in solid materials in which a grain boundary that couples an exterior surface which is evolving under the influence of surface diffusion to an interior material which is antiphase. The problem is fully nonlinear and no assumption has been made about the interface's being the graph of a function. She presented a family of exact travelling wave solutions for the problem.

John Bush performed aquabatics. He presented experiments in which a jet of water impacts on a flat surface, or two jets impact on each other. Both situations produced structures with surprising symmetries and great temporal stability [41]. Analysis of the flows help to explain the observed structures. In addition, he discussed various types of insects that walk on water, including a robotic insect that his research group had built out of a soda can [42]. John's presentation was further distinguished by his use of artistic graphics and colourful photos of experiments.

Dan Lathrop presented experiments involving topological changes of Newtonian thin jets as well as experiments in turbulence and possible connections to singularities in inviscid flow. He presented an impressive collection of power-law behaviors in real fluids, as well as a novel way to "measure" turbulence [43, 44].

John King presented his studies of a thin fluid film in which the air/liquid free interface has been replaced with a rubber sheet, resulting in a sixth–order modelling equation:

$$h_t = (h^n h_{xxxxx})_x. \tag{11}$$

He demonstrated the power of local asymptotics, finding a large collection of special solutions and studying their dependence on the exponent n [45].

Linda Cummings presented thin film models for nematic liquid crystals, including different types of anchoring for the rods at one or both of the surfaces (air/liquid and liquid/solid). Lubrication models were found for the film thickness, allowing for stability studies [46].

Andy Bernoff presented experiments and modelling for polymer monolayers resting on the flat air/liquid interface of a sub fluid, like an oil slick on water. Experiments show a different dynamic than that of oil on water — when mixed, the droplets of polymer do not break apart, instead they return to their droplet shape however extremely they have been sheared. Bernoff proposed a clever model that blends two and three dimensional behavior; he compared his model to the classical (two dimensional) Hele Shaw problem.

Mike Miksis presented extensive level set simulations of air bubbles rising in inclined channels as part of his continuing studies of air bubbles in blood vessels [47, 48]. Varying the Reynolds number and the inclination angle of the channel, he observed steady bubbles, time-periodic bubbles, and bubbles that rupture onto the wall of the channel.

Brian Wetton presented a new numerical method for elliptic problems in which the domain is separated into two regions by a free boundary, on which mixed linear Dirichlet-Neumann conditions are specified. He presented an iterative approach in which a "generalised Stefan velocity" is computed and used to decrease the residual.

7 Speakers

Analysis:	Lorenzo Giacomelli	University of Rome
	Günther Grün	University of Bonn
	Peter Howard	Texas A & M
	Amy Novick-Cohen	Technion
	Mary Pugh	University of Toronto
	Michael Shearer	North Carolina State University
	Dejan Slepcev	University of Toronto
	Sandra Wieland	University of Bonn
Modeling:	Martine Ben Amar	Ecole Normale Superieure
	Andrew Bernoff	Harvey Mudd College
	Mark Bowen	University of Nottingham
	Linda Cummings	University of Nottingham
	Alexander A. Golovin	Northwestern University
	Roman Grigoriev	Georgia Tech.
	John R. King	University of Nottingham
	Andreas Münch	Humboldt-Universität Berlin
	Leonard W. Schwartz	University of Delaware
	Linda B. Smolka	Duke University
	Burt S. Tilley	Franklin W. Olin College of Engineering
	Barbara Wagner	Weierstrass Institute
	Stephen K. Wilson	University of Strathclyde
	Thomas Witelski	Duke University
Computation:	Barry D. Edmonstone	Imperial College London
	Marco Fontelos	Universidad Rey Juan Carlos
	Lou Kondic	New Jersey Institute of Technology
	Michael Miksis	Northwestern University
	Brian Wetton	University of British Columbia
Experiments:	Robert Behringer	Duke University
	John Bush	MIT
	Javier Diez	Universidad Nacional de Buenos Aires
	Daniel Lathrop	University of Maryland
	Michael Schatz	Georgia Tech

References

[1] J. Eggers. Nonlinear dynamics and breakup of free-surface flows. *Reviews of Modern Physics*, 69(3):865–929, 1997.

[2] P. G. de Gennes. Wetting: statics and dynamics. *Reviews of Modern Physics*, 57(3):827–863, 1985.

[3] A. Oron, S. H. Davis, and S. G. Bankoff. Long-scale evolution of thin liquid films. *Reviews of Modern Physics*, 69(3):931–980, 1997.

[4] A. L. Bertozzi. The mathematics of moving contact lines in thin liquid films. *Notices Amer. Math. Soc.*, 45(6):689–697, 1998.

[5] T. G. Myers. Thin films with high surface tension. *SIAM Rev.*, 40(3):441–462, 1998.

[6] R. Almgren, A. Bertozzi, and M. P. Brenner. Stable and unstable singularities in the unforced Hele-Shaw cell. *Phys. Fluids*, 8(6):1356–1370, 1996.

[7] J. R. King and M. Bowen. Moving boundary problems and non-uniqueness for the thin film equation. *European J. Appl. Math.*, 12(3):321–356, 2001. The dynamics of thin fluid films.

[8] M. Bowen and J. R. King. Asymptotic behaviour of the thin film equation in bounded domains. *European J. Appl. Math.*, 12(2):135–157, 2001.

[9] M. Bowen, J. Hulshof, and J. R. King. Anomalous exponents and dipole solutions for the thin film equation. *SIAM J. Appl. Math.*, 62(1):149–179 (electronic), 2001.

[10] L. Giacomelli and F. Otto. Variational formulation for the lubrication approximation of the Hele-Shaw flow. *Calc. Var. Partial Differential Equations*, 13(3):377–403, 2001.

[11] G. Grün. Droplet spreading under weak slippage: a basic result on finite speed of propagation. *SIAM J. Math. Anal.*, 34(4):992–1006 (electronic), 2003.

[12] R. Dal Passo, L. Giacomelli, and G Grün. Waiting time phenomena for degenerate parabolic equations—a unifying approach. In *Geometric analysis and nonlinear partial differential equations*, pages 637–648. Springer, Berlin, 2003.

[13] J. Becker, G. Grün, R. Seemann, H. Mantz, K. Jacobs, K. R. Mecke, and R. Blossey. Complex dewetting scenarios captured by thin-film models. *Nature Materials*, 2:59–63, 2003.

[14] A. L. Bertozzi and M. C. Pugh. Finite-time blow-up of solutions of some long-wave unstable thin film equations. *Indiana Univ. Math. J.*, 49(4):1323–1366, 2000.

[15] D. Slepcev and M.C. Pugh. Selfsimilar blowup of unstable thin-film equations. *preprint*, 2003.

[16] L. B. Smolka, A. Belmonte, D. M. Henderson, and T. P. Witelski. Exact solution for the extensional flow of a viscoelastic filament. *submitted to European Journal of Applied Mathematics (in review)*, 2003.

[17] J. Li and M. A. Fontelos. Drop dynamics on the beads-on-string structure for viscoelastic jets: A numerical study. *Physics of Fluids*, 15(4):922–937, 2003.

[18] B. R. Duffy and S. K. Wilson. A rivulet of perfectly wetting fluid with temperature-dependent viscosity draining down a uniformly heated or cooled slowly varying substrate. *Physics of Fluids*, 15(10):3236–3239, 2003.

[19] D. Holland, S. K. Wilson, and B. R. Duffy. Similarity solutions for slender rivulets with thermocapillarity. *Q. J. Mech. Appl. Math.*, 56(3):411–439, 2003.

[20] R. O. Grigoriev N. Garnier and M. F. Schatz. Optical manipulation of microscale fluid flow. *Phys. Rev. Lett.*, 91:054501, 2003.

[21] R. O. Grigoriev. Contact line instability and pattern selection in thermally driven liquid films. *Phys. Fluids*, 15(6):1363–1374, 2003.

[22] S. B. G. O'Brien and L. W. Schwartz. Theory and modeling of thin film flows. In *Encyclopedia of Surface and Colloid Sci.*, pages 5283–5297. Marcel Dekker, 2002.

[23] L.W. Schwartz. Unsteady simulation of viscous thin-layer flows. In P. Tyvand, editor, *Free-Surface Flows with Viscosity*, pages 203–233. Computational Mech. Publ., Southampton, 1997.

[24] L. Kondic. Instabilities in gravity driven flow of thin fluid films. *SIAM Rev.*, 45(1):95–115 (electronic), 2003.

[25] J. A. Diez and L. Kondic. Computing three-dimensional thin film flows including contact lines. *J. Comput. Phys.*, 183(1):274–306, 2002.

[26] L. Kondic and A. L. Bertozzi. Nonlinear dynamics and transient growth of driven contact lines. *Phys. Fluids*, 11(11):3560–3562, 1999.

[27] M. Ben Amar, L. J. Cummings, and Y. Pomeau. Transition of a moving contact line from smooth to angular. *Phys. Fluids*, 15(10):2949–2960, 2003.

[28] X. Fanton, A. M. Cazabat, and D. Quere. Thickness and shape of films driven by a marangoni flow. *Langmuir*, 12:5875–5880, 1996.

[29] A. L. Bertozzi, A. Münch, and M. Shearer. Undercompressive waves in driven thin film flow: theory, computation, and experiment. In *Trends in mathematical physics (Knoxville, TN, 1998)*, volume 13 of *AMS/IP Stud. Adv. Math.*, pages 43–68. Amer. Math. Soc., Providence, RI, 1999.

[30] A. L. Bertozzi, A. Münch, and M. Shearer. Undercompressive shocks in thin film flows. *Phys. D*, 134(4):431–464, 1999.

[31] J. Sur, A. L. Bertozzi, and R. P. Behringer. Reverse undercompressive shock structures in driven thin film flow. *preprint*, 2002.

[32] A. Münch. Pinch-off transition in marangoni-driven thin films. *Phys. Rev. Lett.*, 91:016105, 2003.

[33] M. Shearer and P. G. LeFloch. Nonclassical riemann solvers with nucleation. *Isaac Newton Institute for Math. Sciences preprint. Submitted to Proc. Roy Soc Edinburgh*, 2003.

[34] M. Shearer and R. Levy. Comparison of dynamic contact line models for driven thin liquid films. *Preprint, submitted to Euro. J. Applied Math.*, 2003.

[35] P. Howard. Local tracking and stability for degenerate viscous shock waves. *J. Differential Equations*, 186(2):440–469, 2002.

[36] P. Howard. Pointwise estimates and stability for degenerate viscous shock waves. *J. Reine Angew. Math.*, 545:19–65, 2002.

[37] P. Howard. Pointwise Green's function approach to stability for scalar conservation laws. *Comm. Pure Appl. Math.*, 52(10):1295–1313, 1999.

[38] B. S. Tilley, S. H. Davis, and S. G. Bankoff. Nonlinear long-wave stability of superposed fluids in an inclined channel. *J. Fluid Mech.*, 277:55–83, 1994.

[39] B. S. Tilley, S. H. Davis, and S. G. Bankoff. Linear stability theory of two-layer fluid flow in an inclined channel. *Physics of Fluids*, 6(12):3906–3922, 1994.

[40] A. A. Golovin, S. H. Davis, and P. W. Voorhees. Self-organization of quantum dots in epitaxially strained solid films. *Phys. Rev. E*, 68:056203–11, 2003.

[41] J. W. M. Bush and Aristoff J. M. The influence of surface tension on the circular hydraulic jump. *J. Fluid Mech.*, 489:229–238, 2003.

[42] D. L. Hu, B. Chan, and J. W. M. Bush. The hydrodynamics of water strider locomotion. *Nature*, 424(6949):663–666, 2003.

[43] D. R. Sisan, W. L. Shew, and D. P. Lathrop. Lorentz force effects in magneto-turbulence. *Physics of the Earth and planetary interiors*, 135(2-3):137–159, 2003.

[44] B. W. Zeff, D. D. Lanterman, R. McAllister, R. Roy, E. J. Kostelich, and D. P. Lathrop. Measuring intense rotation and dissipation in turbulent flows. *Nature*, 421(6919):146–149, 2003.

[45] J. C. Flitton and J. R. King. Moving-boundary and fixed-domain problems for a sixth-order thin-film equation. *preprint*, 2003.

[46] M. Ben Amar and L. J. Cummings. Fingering instabilities in driven thin nematic films. *Phys. Fluids*, 13(5):1160–1166, 2001.

[47] J. Zhang, M. J. Miksis, S. G. Bankoff, and G. Tryggvason. Nonlinear dynamics of an interface in an inclined channel. *Phys. Fluids*, 14(6):1877–1885, 2002.

[48] K. M. DeBisschop, M. J. Miksis, and D. M. Eckmann. Bubble rising in an inclined channel. *Physics of Fluids*, 14(1):93–106, 2002.