Physics and Mathematics of Quantum Field Theory

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1 Overview of the Field

The origins of quantum field theory (QFT) date back to the early days of quantum physics. Having developed quantum mechanics, describing non-relativistic systems of a finite number of degrees of freedom, physicists tried to apply similar ideas to relativistic classical field theories such as electrodynamics following early attempts already made by the founding fathers of quantum mechanics. Since quantum mechanics is usually "derived" from classical mechanics by a procedure called quantization, the first attempts tried to mimick this procedure for classical field theories. But it was clear almost from the beginning that this endeavor would not work without qualitatively new ideas, owing both to the new difficulties presented by relativistic kinematics, as well as to the fact that field theories, while possessing a Lagrangian/Hamiltonian formulation just as classical mechanics, effectively describe infinitely many degrees of freedom. In particular, QFT completely dissolved the classical distinction between "particles" (like the electron) and "fields" (like the electromagnetic field). Instead, the dichotomy in nomenclature acquired a very different meaning: Fields are the fundamental entities for all sorts of matter, while particles are manifestations when the fields arise in special states. Thus, from the outset, one is dealing with systems with an unlimited number of particles.

Early successes of quantum field theory consolidated by the early 50's, such as precise computations of the Lamb shifts of the Hydrogen and of the anomalous magnetic moment of the electron, were accompanied around the same time by the first attempts to formalize this theory in a more mathematical fashion. Owing to progress and the emergence of new perspectives on quantum field theory, several answers to this question have subsequently been formulated in the 80 years since its birth, illuminating the many facets of this theory. The quest for the ultimate mathematical framework of quantum field theory is still underway.

One of the main motivations for quantum field theory is the description of particle physics. This endeavor culminated in the early 80's in the formulation of the so-called standard model of particle physics, describing quarks, leptons, vector bosons mediating the various forces, as well as the recently discovered Higgs boson. These particles are believed to encompass the visible matter content of the universe, with the exception of dark matter (probably another invisible sector in the standard model) and dark energy. Although the various forces have qualitatively different features – becoming strong effectively at long, respectively short ranges – by the early 70's physicists developed reliable perturbative schemes to calculate quantities of direct physical interest in experiments. These computational techniques typically proceed via Feynman integrals and a systematic control of perturbation theory and renormalization. Open questions, like the convergence of the perturbation theory and infrared problems, motivated axiomatic approaches. It seems that the axiomatic approach that has proven the most vital is the operator algebraic approach. Its scope has widened in the last decades so as to cover more general situations such as thermal equilibrium, non-equilibrium states, or quantum field theory in curved spacetime.

Thermal equilibrium and non-equilibrium states were previously studied mainly in non-relativistic quantum many-body systems and condensed matter physics, where field-theoretical methods and renormalization also play a major role. Modern issues like the thermodynamics of black holes or of the early universe demand the extension of statistical physics to relativistic systems.

The mathematics involved in quantum field theory is very diverse, to the extent that for mathematicians QFT may appear as a collection of many different and completely unrelated formalisms. The underlying principles of quantum mechanics involve spectral theory and the theory of operator algebras, with modular theory playing a major role in modern applications. Perturbation theory naturally uses analysis (for the computation of Feynman integrals) and functional analysis (for questions of convergence, scattering theory, and adiabatic limits). Gauge theories involve tools from differential geometry, and their anomalies are related to cohomological issues. The theory of distributions and microlocal analysis are at the core of the study of states of quantum fields in nontrivial gravitational backgrounds. Modern constructive approaches to path integrals employ stochastic partial differential equations and regularity structures. Numerical high-performance simulations are employed for computations in numerous places. Computer-assisted exact methods, such as the "conformal bootstrap", have gained an increasing importance. (The list is certainly not complete.)

Quantum field theory triggered the development of various mathematical concepts of independent interest, such as non-commutative geometry, vertex operator algebras, tensor categories, random matrices, or topological invariants, to name but a few.

The aim of the workshop was to bring together researchers from different communities interested in various aspects of quantum field theory, so as to explore and vitalize the common roots of their respective approaches and to foster an exchange and comparison of ideas.

2 Recent Developments and Open Problems

In recent years the research about mathematical structure of quantum field theory made progress in various directions. When organizing this workshop, we decided to focus on the developments that to our minds appeared the most interesting in terms of potential physical applications, as well as from the purely mathematical point of view. Broadly speaking, the areas that we identified as the most interesting by these criteria can be classified as follows:

1. Developments relating **algebraic approaches** to quantum field theory with **perturbative methods**.
2. New ideas in **conformal quantum field theories**, which describe systems in statistical mechanics at the critical point or scale invariant models of particle theory. Among these, we identified the **conformal bootstrap** approach, which is applicable in principle in any dimension, and algebraic methods relating **vertex operator algebras** and non-commutative geometry/operator algebras, which are suitable in two dimensions.
3. **Algorithmic problems** in computing **Feynman integrals**, such as the Laporta algorithm and related ideas.
4. Recent developments in **stochastic quantization/stochastic partial differential equations** (which strictly speaking belong to pure mathematics, however they have strong analogies with quantum field theory).
5. Quantum field theory in **curved spacetime backgrounds**, and microlocal characterization of classes of "states of physical interest".
6. **Mean field approaches** to systems of electrons interacting with electromagnetic fields.
7. Interesting aspects of the **infrared problem** in models with massless particles.
8. **Non-equilibrium** situations in quantum field theory, such as heat waves, non-equilibrium steady states, entropy production, and return to equilibrium.

Our goal was to invite leading researchers representing each circle of ideas as speakers. In our view, this goal has been accomplished. A more detailed overview of the contributions is given in the next section; here we try to explain the context and motivation for our choices.

One major direction of the research over the past decade was an attempt to achieve a synthesis between algebraic and perturbative quantum field theory. This direction has several motivations. One is to put on a firm conceptual foundation the rules for perturbative calculations, especially in the cases where these rules are not clear-cut, or where naive interpretations of the existing rules lead to problems. Examples of this are: Dealing with the infrared problem, which plagues theories with massless particles; finding correct renormalization for quantum field theories on a non-trivial spacetime or e.g. electromagnetic backgrounds without symmetries; renormalization schemes for correlation functions in states other than the vacuum, such as thermal states, non-equilibrium steady states etc. The Epstein-Glaser method turned out to be a convenient tool to carry out such a programme. This method, invented in the 70's, is experiencing in recent years its revival. It works well in quantum field theory on curved spacetime. It is based on the theory of distributions, allowing for renormalization in position space, which is a setting without translation symmetry, where the Fourier transformation is not available. In recent years, complicated analytical questions about the existence and construction of special quantum states in curved spacetimes have been attacked using powerful methods from microlocal analysis instead of the more global technique of Fourier analysis.

One of the most important quantum field theories is quantum electrodynamics (QED). It is often useful to consider its approximate versions, which are easier to study by nonperturbative methods. One such version is the so-called mean-field QED, where charged particles are treated quantum-mechanically and the electromagnetic field is treated classically. The electromagnetic field is divided into a given external part and fluctuations, which are the object of a variational problem. Mean-field QED has been a topic of interesting research involving deep methods from analysis, convex optimization, calculus of variations. In practice, of particular interest are stationary states, but the method is in principle capable of treating non-equilibrium situations as well.

With a similar mix of conceptual and practical motivations, the algebraic viewpoint is exploited to address the notoriously difficult infrared problems in theories with massless particles, thus opening the door to more conceptional treatments of scattering of electrically charged particles and photons. The particularly transparent separation of infra-red and ultra-violet features of perturbative quantum field theories in such a treatment allows for conceptually clear and rigorous proofs of non-renormalization theorems, covering cases of current interest, such as supersymmetric gauge theories and, potentially, supergravity theories.

Algebraic approaches have also received a considerable new impetus from the emergent field of quantum information (QI). Key concepts within this field such a entanglement, capacities, local operations, etc. are naturally formulated in terms of algebraic notions such as entanglement measures, relative entropy functionals, conditional expectations or completely positive maps. While QI historically has been concerned mainly with quantum mechanical systems, it has triggered the interest in the above concepts also in relativistic systems, thus linking QI, for example, with non-equilibrium thermodynamics. Tomita-Takesaki modular theory (a celebrated theory in operator algebras) is used as an important tool when states cannot be described by density matrices.

In practical applications to high-energy physics, once a renormalization scheme is settled upon, calculations of scattering amplitudes typically lead to the problem of evaluating Feynman integrals. Such integrals come with certain parameters, such as masses or the powers of the propagator kernels. Typically, there can exist many relationships between these expressions for different parameters. This naturally leads to the quest for "bases" of independent integrals and algorithms for determining such bases and for computing the corresponding integrals. Such algorithms are thereby not only of practical importance, but also seem to have interesting connections to questions in algebraic geometry and the theory of D-modules.

The idea of stochastic quantization goes back to the early 80's. It is closely related to quantum field theory, especially its Euclidean version. Not surprisingly, stochastic partial differential equations are often affected by divergences of various sorts. A considerable progress how to deal with such divergences has recently been achieved. This progress has triggered renewed interest in various aspects of renormalization.

Conformal quantum field theories form an important subclass of QFT. They are interesting for many reasons: they are relevant for critical phenomena in statistical physics and they have a surprisingly rich structure, which often allows for finding explicit solutions. Besides, they are often good approximations to a realistic description of high-energy physics – note that the Lagrangian of the Standard Model contains only one term that breaks the conformal invariance on the classical level: the Higgs mass term. This approximate conformal invariance is exploited in a recent proposal of Meissner and Nicolai for a quantum field theory that should adequately describe high energy physics over many orders of magnitude beyond the Standard Model.

Conformal invariance is especially rigid in two dimensions. It can be used to describe a large class of models of conformal quantum field theory with rigorous constructive schemes and a complete classification. This classification involves sophisticated mathematical tools such as elements of the theory of categories. Conformal field theory has been developed in various research directions, including the approach based on operator algebras, and a more formal algebraic approach involving the so-called vertex algebras. Heuristically one could expect that the various approaches should be equivalent, however in practice they are not so easy to compare.

In higher dimensions the conformal group is not so large and other tools have to be used in the study of conformally invariant QFT's. One of them is the method of operator product expansions (OPE's). In recent years, there has been a revival of non-perturbative algebraic methods that in essence postulate the OPE of a theory as its defining input. Constraints on the form of this expansion, such as their convergence or associativity, can provide tools to find "exclusion plots" for the conformal data, i.e., the structure constants in the operator product expansion and the conformal dimensions in this expansion. A similar set of ideas, involving constraints on the OPE, seems to be relevant also for QFT's without conformal invariance.

Non-perturbative treatment of quantum field theories (beyond the class of conformal field theories) has a long tradition dating back to works of Glimm and Jaffe, Fröhlich, Nelson, Brydges, Rivasseau, and others. The most traditional subject of this direction of research, usually called Constructive Field Theory, is an attempt to construct interacting models of quantum field theory in various dimensions. It has turned out to be very difficult to construct realistic models in 4 dimensions. In recent years the methods of constructive field theory were applied to construct and investigate some other classes of models, such as hierarchical models and matrix models. They turned out to exhibit many interesting features, which seem to be relevant for realistic theories. Hierarchical models require sophisticated renormalization and lead to interesting phase diagrams. Matrix models are believed to be relevant as approximations of random geometry, and are an important step in our quest for quantum gravity.

3 Presentation Highlights

There were 14 long talks (55min) and 10 shorter talks (30min).

Long talks:

- Vincent Rivasseau presented the theory of random tensors. It is a theory directly inspired by QFT. It leads to a perturbation theory involving sophisticated versions of Feynman diagrams. Random tensor theory can be viewed as an approach to random geometry, and also as a toy model for quantum gravity in $D \geq 3$ dimensions. Vincent reported on the recent progress in the $1/N$ expansion (in the size of the tensors) and on renormalization group methods for these models. They are believed to be approximate AdS-CFT duals of the Sachdev-Ye-Kitaev model. [1]
- Mathieu Lewin reported on recent progress (achieved with Hainzl, Séré and others) in the so-called mean-field QED. This theory can be thought of as a caricature of the true QED, with charged particles treated quantum-mechanically and classical electromagnetic fields. The electromagnetic field is divided into a fixed external field and a "fluctuation", which is subject to a variational treatment. It leads to a well-defined nonlinear functional on density matrices of charged particles. Proving the existence of minimizers amounts to establishing the stability of the approximation. The model needs an ultraviolet regularization – for instance, one can employ the Pauli-Villars method. The Landau pole phenomenon appears when one tries to remove the ultraviolet cutoff. The electrostatic case can be treated non-perturbatively, using methods of convex analysis and the calculus of variations. These methods are closely related to the backreaction equation on curved spacetime for quantum fields coupled to the Einstein Equation. [2]
- Detlev Buchholz presented a construction of a certain new C^*-algebra, which can be used to describe observables of interacting non-relativistic Bose particles. The point of departure of this construction is

the so-called resolvent algebra, generated by resolvents of field operators. The next step involves taking the gauge-invariant (particle number preserving) part of that algebra. The final C^*-algebra is capable of supporting a large class of interesting many-body dynamics as C^* automorphisms, giving a rigorous meaning to the dynamics generated formally by many-body Schrödinger Hamiltonians with bounded potentials. The state space and ideal structure are particularly rich and the specific dynamics select states and ideals. The algebra has a quasi-local structure, it possesses positive energy representations and asymptotic ground states. Scattering theory can be formulated in this setting. Progress towards the existence of thermal states has been achieved. [3]

- Abdelmalek Abdesselam presented a non-perturbative construction of hierarchical ϕ^4-type models of statistical mechanics on the space of p-adic numbers. The hierarchical structure allows for a rather detailed analysis of the model. In particular, the application of space-dependent renormalization group analysis leads to a definition composite operators, whose anomalous dimensions establish the non-Gaussian nature of theory. By interpreting the infinite hierarchical level as a boundary, an analogy with Witten diagrams in AdS-CFT can be set up. (Talk prepared jointly with A. Chandra) [4]

- Wojciech Dybalski gave a comprehensive overview of infrared features in relativistic and non-relativistic models of many body systems with massless and massive particles. He gave an existence proof for scattering states of electrons, photons and "atoms" in the context of the Nelson model using the Haag-Ruelle ansatz. He emphasized the notable difference between neutral and charged particles, and reported on recent progress made by combining the Dollard formalism with sophisticated methods from scattering theory, which are developed further in his joint work with A. Pizzo. In relativistic quantum field theory, he proposed a general framework for the superselection structure of scattering states, building on ideas of Buchholz et al. He gave an outlook on the "infrared triangle" put forward by Strominger et al. [5]

- Stefan Hollands reported on a longer programme towards a perturbative construction of quantum field theories in generic globally hyperbolic curved spacetimes. The method is based on a combination of essentially algebraic techniques with analytical methods such as scaling/curvature expansions, extensions of distributions to complicated subspaces such as stratifolds, and microlocal analysis. A culmination of this line of research, carried out partly together with Robert M. Wald and based also on work by Radzikowski and by Brunetti and Fredenhagen, is the perturbative construction of non-abelian gauge theories in curved spacetime. Here, one has to combine the algebraic and analytic methods with cohomological methods (the so called BV-complex) in order to control the gauge invariance of the renormalized theory (see also the contribution by Rejzner). [6]

- Mikołaj Misiak gave an overview of the methods and algorithms for computing Feynman integrals of interest in high energy phenomenology, especially in the calculations of various processes beyond the Standard Model. Feynman integrals satisfy a number of identities, which can be obtained via integration by parts. With help of these identities one can reduce Feynman integrals to a finite number of master integrals. These can be solved because they satisfy a coupled system of differential equations. The choice of master integrals is non-unique and seems important from the practical of view, because it often affects the computer time needed to solve a given problem. Mikołaj pointed out several directions for future research, which seem to lead to interesting questions formulated in an abstract algebraic language. [7]

- Krzysztof Meissner analysed the conformal anomaly in theories of fields of various spin and reported on the observation that contributions of different fields in maximally supersymmetric supergravity theories (embedded in conformal supergravity) cancel from $N = 5$ upwards. Based on an argument for the necessity of this cancellation in the cosmological context, he proposed a novel extension of the standard model motivated by maximally supersymmetric gravity theories. In this extension, all masses, being tiny compared to the Planck mass, arise through a soft breaking of conformal symmetry. The model involves an infinite-dimensional hidden $K(E_{10})$ symmetry of $N = 8$ supergravity, whose mathematics and physical implementations remain to be explored. [8]

- Scott Smith presented an overview of stochastic quantization á la Parisi-Wu. He explained how the necessity of renormalization arises in this set-up and outlined different approaches to the problem: regularity structures á la Hairer, paracontrolled distributions á la Perkovski-Gubinelli, or an RG-approach á la Kupiainen. He gave a detailed outline of recent progress for semi-linear stochastic PDEs with stochastic noise of varying regularity (joint with F.Otto, J.Sauer and H.Weber). [9]

- Christopher Fewster reported on ongoing work with R. Verch on the problem of measurement schemes and observables in quantum field theory in curved spacetime. Their basic scenario involved a *system* coupled to a *probe*. Both were described in the framework algebraic quantum field theory on curved spacetime. It was assumed that the interaction happened in a compact subset of spacetime and the probe was measured elsewhere. The talk discussed effect-valued measures, their non-selective measurement and joint unsharp measurement of observables. [10]

- Michał Wrochna discussed the importance of Hadamard states in the context of linear quantum field theory in curved spacetime, their applications, microlocal characterization, and methods how to establish them. He gave examples of important Hadamard states in the Schwarzschild black-hole spacetime, recent constructions of Unruh and Hartle-Hawking states, and commented on the non-existence of stationary Hadamard states in the Kerr black-hole spacetime. He presented a microlocal construction of Hadamard states in smooth asymptotically flat spacetimes, and also in analytic spacetimes. The latter construction involves the so-called Calderon projectors, defined after performing a microlocal version of the "Wick rotation". (Talk prepared jointly with C. Gérard) [11]

- Nicola Pinamonti reviewed the construction of thermal equilibrium states of interacting quantum field theories on Minkowski spacetime in the setting of renormalized perturbative algebraic quantum field theory. He compared this modern approach (due to Fredenhagen and Lindner) with other more formal ones á la Landsman and á la Keldysh. He presented results about perturbed dynamics and return to equilibrium. Secular divergences in time are observed in the thermodynamic limit if the initial state is not appropriately chosen. He characterized the asymptotic stationary state as a non-equilibrium steady state (NESS), computed the relative entropy between the NESS and the initial Gibbs state, adapting methods from Araki's perturbation theory. [12]

- David Simmons-Duffin reported on the key ideas and recent progress in the conformal bootstrap approach to conformal field theories. This approach is based on an old idea of Polyakov, Migdal, and others: One tries to exploit the bounds on the so-called conformal data of the theory (the structure constants and dimensions) by writing down the consequences of the associativity law for the operator product expansion ("bootstrap equation"). Recent progress has been achieved thanks to better suited analytical expressions for the so called conformal blocks appearing in those conditions. A concrete algorithm proposed by Rychkov, Rattazzi, Tonni, and others is to test the bootstrap equation via certain well-chosen linear functionals. This algorithm yields exclusion plots in the space of conformal data. Particularly impressive results are available in the 3-dimensional Ising model. [13]

- Sebastiano Carpi reported on the status of the connection between two axiomatic settings: Borcherds' vertex operator algebras (VOA) and the algebraic formulation of chiral conformal QFT á la Haag-Kastler. Both approaches can be used to axiomatize conformal QFT models, even though the original motivation of the first approach was purely mathematical (the "moonshine conjecture" and number theory). Both have complementary advantages that can be transferred to the other, once a suitable "dictionary" has been established. Provided certain energy bounds on their states (the so-called $a_n b$ bounds) are satisfied, unitary VOAs can be turned into Wightman-like fields. One can then introduce nets of local(ized) algebras. Unfortunately, their locality is not automatic. If it holds, then the VOA can be reconstructed from the net via the Fredenhagen-Jörß construction. Many conjectures and questions concerning the representation theory of the two approaches remain open. [14]

Short talks:

- Daniel Siemssen reviewed some properties of the Klein-Gordon equation on curved spacetimes, which are relevant for applications in Quantum Field Theory. In particular, he described a proof of the existence of a distinguished Feynman propagator on a large class of asymptotically static globally hyperbolic manifold. He also discussed the relationship of the distinguished Feynman propagator with the problem of self-adjointness of the Klein-Gordon operator. [15]

- Krzysztof Gawędzki discussed the time evolution of states called "heat waves" in conformal field theory. These states are defined by a density matrix with non-trivial temperature profiles. Methods using Virasoro symmetry allow us to compute the energy density and currents. Conformal welding techniques (related to the Riemann-Hilbert problem) yield a semi-explicit expression for the exact probability distribution. [16]
- Alessandro Pizzo reported on an unexpected emergence of a non-commutative recurrence problem in the context of non-relativistic scattering theory (as discussed in the talk by Dybalski). He presented various closed form solutions of the recurrence in terms of multi-variate polynomials. This methods leads to results about detailed regularity properties of n-particle wave functions. [17]
- Daniela Cadamuro reviewed the construction of interacting quantum field theories in two spacetime dimensions with a factorizing scattering matrix via an "inverse approach": The S-matrix is taken as an input, and is used to define "deformed" creation and annihilation operators. The associated fields fail to be local, but are localized in wedge-like regions, so that local algebras can be defined as intersections of two algebras of wedge-localized observables. In the case of the Bullough-Dodd model, where the poles of the S-matrix indicate the presence of bound states, one has to add a "bound-state operator" in order to achieve weak commutativity of wedge-localized fields. (Talk continued by Yoh Tanimoto)
- Yoh Tanimoto, in continuation of Daniela's talk, focused on the problem of establishing the Haag-Kastler axioms for the local algebras of the Bullough-Dodd model. This problem involves three steps: the self-adjointness of the wedge-localized generators, their strong commutativity, and finally the "size" of the intersection of two opposite wedge algebras. He reported on the progress in this program. [18]
- Markus Fröb spoke about recent results on the operator product expansion in non-conformally invariant theories. He presented convergence results and estimates on the remainder of the expansion in various theories, originally due to Kopper and Hollands. Furthermore, he explained a flow equation for the coefficients in the operator product expansion, which in principle can be used to self-consistently determine these quantities beyond perturbation theory. When solved in perturbation theory, the flow equation gives a recursive definition of the coefficients which agrees with traditional methods, but is conceptually much clearer. The methods can be generalized to non-abelian gauge theories. [6]
- Kasia Rejzner advertized the application of the Batavin-Vilkovisky formalism in perturbative algebraic quantum field theory with local symmetries (Yang-Mills, effective quantum gravity) on curved spacetime. Using homological algebra, she derived a rigorous quantum Master Equation involving potential anomalies, thereby reinterpreting a formula of Hollands. This approach exhibits the cohomological nature of consistency conditions for anomalies. [20]
- Paweł Duch reviewed the problem of the adiabatic limit (infrared problem) in perturbative QFT with massless particles. He covered the weak adiabatic limit (convergence of correlations and Green functions) and the strong adiabatic limit (construction of fields and their scattering matrix). The existence of the weak adiabatic limit was recently established by him in a large class of models. For the construction of the S-matrix, he reported on his work in progress, which involves a version of ideas of Dollard and Faddeev-Kulish. [21]
- Joseph Várilly reconsidered Wigner's "continuous spin representations" of the Poincaré group, by identifying them with coadjoint orbits of its Lie algebra, in the spirit of the Kirillov approach. Massive and massless orbits have rather different properties. Massive orbits do not have a massless limit. Massless continuous spin orbits allow to define a helicity operator and position operators that are conjugate to the momentum operators. The position operators, however, do not commute among themselves, indicating an intrinsic "noncommutative geometry". The resulting wave-function representation of the one-particle Hilbert space can be cast into a form compatible with the recent "string-localized" second-quantized theory (Mund-Schroer-Yngvason). [22]
- Christian Jäkel discussed QFT on 2-dimensional deSitter spaces. He explained how free quantum fields can be constructed using the theory of representations of the deSitter group $SO(1,2)$. Then he discussed how one can construct interacting fields using the modular theory. He also described a natural generalization of Haag-Kastler axioms to the context of deSitter spacetimes. [23]

4 Scientific Progress Made / Outcome of the Meeting

The workshop brought together researchers rooted in different communities who address the same subject "Quantum Field Theory", but use widely different concepts and methods, to the extent that they sometimes do not even understand the language of the others. It was our objective to bridge such gaps and thus trigger communication between various communities. The pay-off of such a meeting may not be immediate, but we are convinced that it will develop over time.

For this purpose we gave priority to overview talks. We have the impression that the talks were sincerely appreciated by participants, who engaged in lively discussions during and after the talks. It was visible that many participants were eager to understand the complementary approaches. Most talks were recorded by the BIRS video system, and pdf's of slides are available for all but a few blackboard talks. These documents are valuable resources for who wants to delve deeper into the subjects.

In spite of a large number of talks, there was sufficient time for informal scientific interaction among the participants. Several small groups of participants benefitted from the workshop and its facilities to pursue or initiate joint research projects.

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