05w5011 Geometric and Asymptotic Methods in Group Theory June 11 - June 16, 2005

Organizers: Rostislav Grigorchuk (Texas A& M University), Alexander Olshanskiy (Vanderbilt University), Akbar Rhemtulla (University of Alberta), Mark Sapir (Vanderbilt University), Dani Wise (McGill University).

The goal of the conference was to bring together specialists in geometric, probabilistic and asymptotic methods in group theory. Special attention was paid to the following topics:

(1) Amenability and randomness in groups
- Random walks and generic properties of groups
- Poisson boundaries of groups
- Amenable actions of groups
- Minimal volume entropy problem for graphs

(2) Actions on rooted trees, growth and self-similarity
- Growth and diameters of Schreir graphs of groups generated by finite automata
- R. Thompson group
- Subgroup growth of groups

(3) Groups, boundaries and geometries
- Cubulation of groups and right angled Artin groups
- Quasi-isometric rigidity of groups
- Asymptotic cones of groups
- Algebraic geometry over groups and Tarski problems

(4) Lattices in Lie groups
- Bounded generation property
- Property τ
- Expanders

Many informal discussions lead to creating new ideas of collaboration between specialists in different areas of group theory. In particular, it may be possible to make groups of intermediate growth and torsion groups act on cubings using the end structure of their Schreir graphs; several properties of 1-relator groups which are believed to be generic may be connected to important properies of random walks on lattices, etc.

Here is the list of participants of the conference and the abstracts of their talks.

(1) **Miklos Abert** (University of Chicago)
On chains of subgroups in residually finite groups
ABSTRACT: We analyze descending chains of finite index subgroups and the corresponding permutation representations for residually finite groups. As a result we show that we can obtain an arbitrary countable set of residually finite groups as the intersection set of a conjugacy class of a suitable chain. Although the representations do not approximate the group in general, we show that in certain cases, e.g. for higher rank real lattices, a weaker form of approximation holds.

(2) **Roger Alperin** (San Jose State University)
Subgroup Separability in Linear Groups
ABSTRACT: We'll survey some examples and non-examples for separability of subgroups in f.g. linear groups.

(3) **Jason Behrstock** (Columbia University)
Relative hyperbolicity and the mapping class group
ABSTRACT: We will describe some recent work on the asymptotic geometry of the mapping class group. In particular, we will give a geometric proof that the mapping class group is not hyperbolic relative to any finite collection of finitely generated subgroups. We will also contrast this with a new description of a way in which the mapping class group is non-positively curved. Parts of this talk are joint work with C. Drutu and L. Mosher.

(4) **Ievgen Bondarenko** (Texas A&M University)
Growth of Schreier graphs associated to groups generated by bounded automata
ABSTRACT: We describe an algorithm for calculating the growth of diameters of Schreier graphs and the orbital contracting coefficient associated with actions of groups generated by bounded automata on levels and on the boundary of the tree. As a corollary we get estimates for the polynomial growth of Schreier graphs associated with action on the boundary. This is joint work with V. Nekrashevych.

(5) **Jim Cannon** (Brigham Young University)
Dead-end elements in Thompson's group
ABSTRACT: We report on work by student Ben Woodruff (BYU PhD, 2005) who uses Blake Fordham's results to sharpen the Cleary-Taback description of dead-end elements in Thompson's group and to give a characterization of the same, notes the existence of regular strings of dead-end elements, and notes that each has a "tail" (varying lengths) of elements pointing back toward the identity consisting of elements that are almost dead-end elements. Such structures create obvious difficulties in the most obvious proposed methods to prove the nonamenability of Thompson's group. It remains, of course, to study the asymptotic density of such elements.

(6) **Yair Glasner** (University Of Illinois at Chicago)
Finitely generated vs' Normal subgroups in 3-manifold groups
ABSTRACT: Let G be the fundamental group of a 3 dimensional manifold of finite volume. We define an invariant topology on G, by taking the normal subgroups and their cosets as a basis for the topology. This is a refinement of the profinite topology. We prove that every finitely generated subgroup of G is closed in this topology. As a corollary we deduce that a maximal subgroup of infinite index in G cannot be finitely generated. The main tool used in the proof is the Marden conjecture that was recently established by Ian Agol, and by Calegari-Gabai.

This is a joint work with Pete Storm and Juan Souto.

(7) **Frédéric Haglund** (University of Paris-Sud)
Commensurability and separability for uniform lattices in some polygonal complexes
ABSTRACT: We consider a (word hyperbolic) Coxeter group whose Davis-Moussong complex is two-dimensional. We show (in almost every case) that a uniform lattice of this complex is commensurable with the initial Coxeter group if all of its quasi-convex subgroups are separable. The previous "if" is an "if and only if" for example when the Coxeter group is right-angled. As an application there is a single commensurability class of uniform lattices as soon as the link of a vertex in the Davis-Moussong complex is a bipartite graph - for example in Bourdon buildings.

(8) **Chris Hruska** (Chicago)
Cubulating relatively hyperbolic groups
Abstract: We give criteria for proving that a group acts properly discontinuously on a locally finite CAT(0) cube complex. The criteria are inspired by ideas from the theory of relative hyperbolicity but are not limited to relatively hyperbolic groups.

We also give criteria for determining when a relatively hyperbolic group acts on a finite dimensional cube complex and when such an action is cocompact, generalizing a theorem of Sageev from the word hyperbolic setting. More generally, we describe a "cusped cofinite" structure relative to the action of the parabolic subgroups. This structure is analogous to the cusped structure of a finite volume manifold with pinched negative curvature. This is joint work with Dani Wise.

(9) **Tim Hsu** (San Jose State University)
Cubulating graphs of free groups with cyclic edge groups
Abstract: We prove that if G is a finitely generated group that has a decomposition as a graph of free groups with cyclic edge groups, and G is "generic" (essentially, contains no Baumslag-Solitar subgroups), then G is the fundamental group of a compact CAT(0) cube complex. We also discuss generalizations of this result. This is joint work with D. Wise.

(10) **Vadim Kaimanovich** (Bremen)
Amenability of self-similar groups and random walks with internal degrees of freedom
Abstract: The talk is devoted to a discussion of the relationship between random walks with internal degrees of freedom and natural matrix presentations of self-similar groups which allows one to prove amenability of such groups by establishing triviality of the Poisson boundary for appropriate random walks on such groups.

(11) **Martin Kassabov** (Cornell University)
Symmetric groups and Expanders
Abstract: A finite graphs with large spectral gap are called expanders. These graphs have many nice properties and have many applications. It is easy to see that a random graph is an expander but constructing an explicit examples is very difficult. All known explicit constructions are based on the group theory — if an infinite group G has property T (or its variants) then the Cayley graphs of its finite quotients form an expander family.

This leads to the following question: For which infinite families of groups G_i, it is possible to find generating sets S_i which makes the Cayley graphs expanders?

The answer of the question is known only in few case. It seems that if G_i are far enough from being abelian then the answer is YES. However if one takes 'standard' generating sets the resulting Cayley graphs are not expanders (in many cases).

I will describe a recent construction which answers the above question in the case of the family of all symmetric/alternating groups. It is possible to construct explicit generating sets S_n of Alt_n, such that the Cayley graphs $C(Alt_n, S_n)$ are expanders, and the expanding constant can be estimated.

Unlike the usually constructions of expanders, the proof does not use an infinite group with property T (although such group exists) but uses the representation theory of the symmetric groups directly.

(12) **Olga Kharlampovich** (McGill)
Equations in groups with free regular length function.
Abstract: I will discuss the Elimination process for solving equations in groups with free regular length function (in particular, in a free group).

(13) **Avinoam Mann** (Einstein Institute of Mathematics, Hebrew University, Jerusalem)
Positively finitely generated groups and their ζ-functions
ABSTRACT: A profinite group is positively finitely generated (**PFG**) if, for some k, the set of k-tuples generating it has a positive Haar measure. We denote this measure by $P(G,k)$. E.g. finitely generated pronilpotent groups are **PFG**, while free profinite groups of rank at least two are not.

Let $m_n(G)$ be the number of maximal subgroups of G of index n. A theorem of Mann-Shalev characterizes PFG groups by the property that $m_n(G)$ grows polynomially, i.e. $m_n(G) \leq n^s$, for some constant s. It follows that f.g. prosoluble groups are **PFG**, and more generally, any f.g. profinite group that does not generate the variety of all profinite groups is **PFG** (Borovik-Pyber-Shalev). This includes the profinite completions of arithmetic groups with the congruence subgroup property **CSP**.

In many cases we can interpolate the values of $P(G,k)$ to an analytic function defined in a right half-plane of the complex plane. The reciprocal of this function is termed the probabilistic ζ-function of G. It exists, e.g., when G is prosoluble, or when it is an arithmetic group with the **CSP**.

(14) **Dave Witte Morris** (University of Lethbridge)
Bounded generation of special linear groups
ABSTRACT: We present the main ideas of a nice proof (due to D.Carter, G.Keller, and E.Paige) that every matrix in $\mathrm{SL}(3,\mathbf{Z})$ is a product of a bounded number of elementary matrices. The two main ingredients are the Compactness Theorem of first-order logic and calculations of Mennicke symbols. (These symbols were developed in the 1960s in order to prove the Congruence Subgroup Property.) Similar methods apply to $\mathrm{SL}(2,A)$ if $A = \mathbf{Z}[\sqrt{2}]$ (or any other ring of integers with infinitely many units).

(15) **Roman Muchnik** (Chicago)
Amenability of free Grigorchuk group
ABSTRACT: I will describe how the methods developed by V. Kaimanovich can be used to prove that the Free Grigorchuk group is amenable. The main tool used by V. Kaimanovich is to obtain a Random walk with 0 entropy. I will also describe some modifications to simplify computations.

(16) **Graham Niblo** (Southampton)
An eccentric characterisation of hyperbolicity
ABSTRACT: We give a new characterisation of hyperbolicity for geodesic metric spaces in terms of the geometry of balls. It is related to Papasoglu's "thin bigons" characterisation of hyperbolic graphs. This is joint work with Indira Chatterji

(17) **Stephen Pride** (Invariant Ideals for Groups)
University of Glasgow
ABSTRACT: Given a group of type FP_n, David Cruickshank and I defined a table of ideals $E(i,j)$ $(0 \leq i \leq n, j$any integer) in the abelianized group ring. This table is an invariant of the group. The first column $E(1,-)$ is the chain of classical Alexander ideals. I will give the definition of these tables, describe some of their properties, give examples of calculations, and raise some open questions.

(18) **Michah Sageev** (Utah & Technion)
Quasi-isometries and right angled Artin groups.
ABSTRACT: We discuss some results regarding the quasi-isometric rigidity and classification of right angled Artin groups. This is joint work with Bestvina and Kleiner.

(19) **Dmytro Savchuk** (Texas A&M University)
Schreier graphs related to the Thompson's group
Abstract: We will explicitly describe the Schreier graphs of the Thompson group F with respect to the stabilizer of $\frac{1}{2}$ and generators x_0 and x_1 and of its unitary representation in $L_2([0,1])$ induced by standard action on the interval $[0,1]$.

The main result is that these two graphs coincides modulo finite subsets.

(20) **Dan Segal** (All Souls College, Oxford)
Subgroups of finite index in profinite groups
Abstract: We answer a 30-year old question of Serre by proving that *all subgroups of finite index in a finitely generated profinite group are open.* This is deduced from the main theorem: *If w is a d-locally finite word and G is a d-generator finite group then every element of the verbal subgroup $w(G)$ is a product of $f(w,d)$ w-values.* (w is called *d -locally finite* if $F_d/w(F_d)$ is finite, where F_d is the free group of rank d, and $f(w,d)$ denotes a number that depends only on w and d.) The proof is complicated and depends on CSFG. (joint work with Nikolay Nikolov)

(21) **Dan Segal** (All Souls College, Oxford)
Groups with polynomial index growth
Abstract: A group G has *PIG* if there exists α such that $\left|\overline{G}/\overline{G}^n\right| \leq n^\alpha$ for every finite quotient $\overline{G}$ of G and every natural number n. This holds for example if G is an arithmetic group with the congruence subgroup property, or if G is 'boundedly generated' (a product of finitely many cyclic groups), in particular if G is a soluble group of finite rank. But there also exist uncountably many finitely generated residually finite groups with PIG that are neither linear nor virtually soluble. We answer a question posed by me 20 years ago with the Theorem: *Let G be a finitely generated soluble residually finite group. Then G has* PIG *if and only if G has finite rank.* Along the way we prove that *every infinite residually finite boundedly generated group has an infinite linear image.* The proofs use some representation theory of finite soluble groups and a lot of 'quasi-commutative algebra': the study of abelian group rings with operators. (joint work with Laci Pyber)

(22) **Vladimir Shpilrain** (The City College of New York)
Translation equivalence in free groups
Abstract: Motivated by the work on hyperbolic equivalence of homotopy classes of closed curves on surfaces, we investigate a similar phenomenon for free groups. Namely, we study the situation where two elements g, h in a free group F have the property that for every free isometric action of F on an $\mathbb{R}$-tree X the translation lengths of g and h on X are equal or have bounded ratio. This is joint work with I.Kapovich, G.Levitt, P.Schupp.

(23) **Tatiana Smirnova-Nagnibeda** (University of Geneva)
Minimizing entropy over the Outer space
Abstract: We solve the minimal volume entropy problem in the class of universal covers of finite connected metric graphs.

(24) **Benjamin Steinberg** (Carleton University)
The spectra of lamplighters and related groups via automata
ABSTRACT: The speaker and Silva showed that any wreath product $G \wr Z$, with G a finite Abelian group, can be realized as the group generated by a special kind of automaton called a Cayley machine. In this talk we calculate the KNS spectral measure associated to the group of a Cayley machine, and in particular to such generalized lamplighters. KNS spectral measures were introduced by Grigorchuk and Zuk for groups acting spherically transitively on rooted trees.

We also show that the KNS spectral measure associated to an automata group coincides with the spectral measure of the simple random walk on the automata group if and only if the action of the group is free in a Baire category sense sense. This is the case for wreath product groups of the above form, and so we have given an automata-theoretic calculation of their spectral measures. A different approach has been used by Dicks and Schick to calculate these spectral measures.

This is joint work with M. Kambites and P. Silva.

(25) **Zoran Sunik** (Texas A&M University)
Free-by-free right-angled Artin groups
ABSTRACT: A group H is poly-free if it has a subnormal series

$$1 = H_0 \trianglelefteq H_1 \trianglelefteq \cdots \trianglelefteq H_n = H$$

in which all factors are free. Equivalently, H is a finitely iterated semidirect product of free groups. The shortest length of a subnormal series with free factors is called the poly-free length of H.

Our main results are as follows.

All right-angled Artin groups are poly-free. The poly-free length of a right-angled Artin group $A\Gamma$ is bounded between the clique number and the chromatic number of the graph Γ that defines the group $A\Gamma$. An explicit realization of a subnormal (in fact normal) series with free factors and of length equal to the chromatic number of Γ is provided.

A complete characterization of graphs that define right-angled Artin groups of poly-free length 2 is given. Such graphs must have an independent set of vertices D such that every cycle in Γ contains at least two vertices from D.

Finally, considerations involving the Euler characteristic allow us to conclude that a right-angled Artin group $A\Gamma$ has poly-free length 2 with both factors of finite rank, i.e., $A\Gamma$ is a semidirect product of two free groups of finite rank if and only if the defining graph Γ is a tree or a complete bipartite graph.

This work is motivated by a question of Bestvina asking if all Artin groups are virtually poly-free.

(26) **Balint Virag** (Toronto)
Torsion generators and slow random drives in Britain
ABSTRACT: A b-spinner graph is a sum of directed cycles of length at most b. An example is a simplified road map of Britain consisting of roundabouts and short two-way streets. Another example is any directed Cayley graph of the Grigorchuk group.

We give up-to-constant optimal upper bounds on the rate of escape of random walks on spinner graphs in terms of their growth.

For torsion groups of intermediate growth, there is no set of generators for which the corresponding random walk escapes at a positive speed. (This statement is false for any infinite nilpotent group.)

This is joint work with David Revelle.

(27) **Andrzej Zuk** (CNRS, Paris VI)
Automata groups
ABSTRACT: We present recent results concerning groups generated by finite automata.