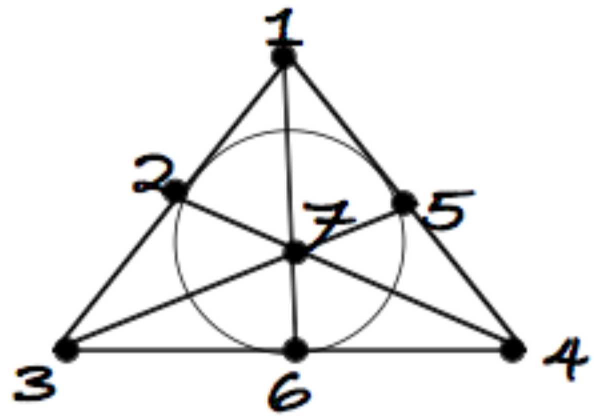
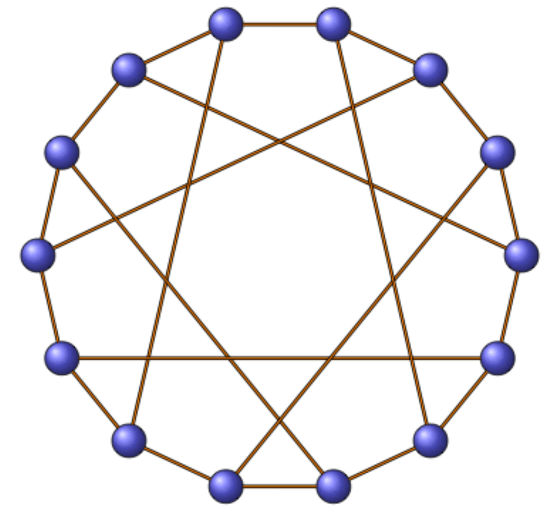
Instituto de Matemáticas





"The Cage problem"

Gabriela Araujo-Pardo

Modern Techniques in Discrete Optimization: Mathematics, Algorithms and Applications

Oaxaca, November 2nd, 2015

Regular cages

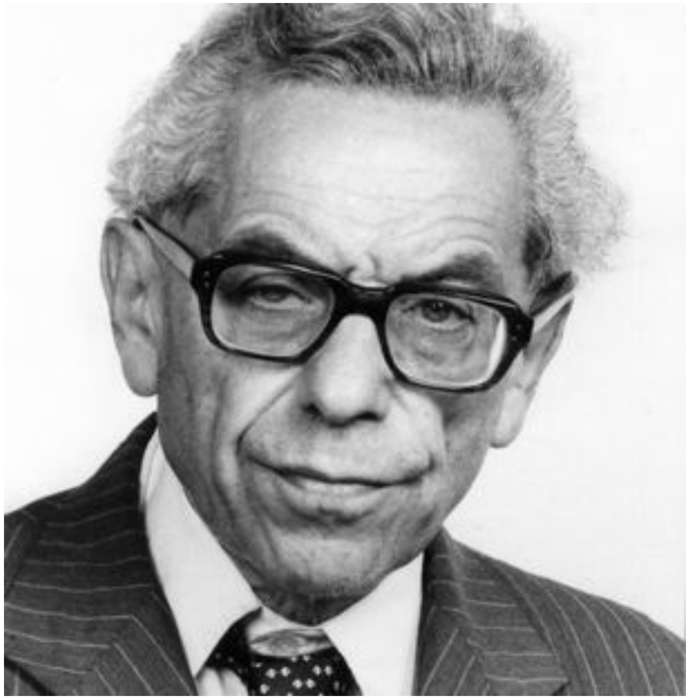
- Let $k \geq 2$ and $g \geq 3$ be two integers; a **(k;g)-graph** is a k-regular graph G with girth $g(G)=g$.

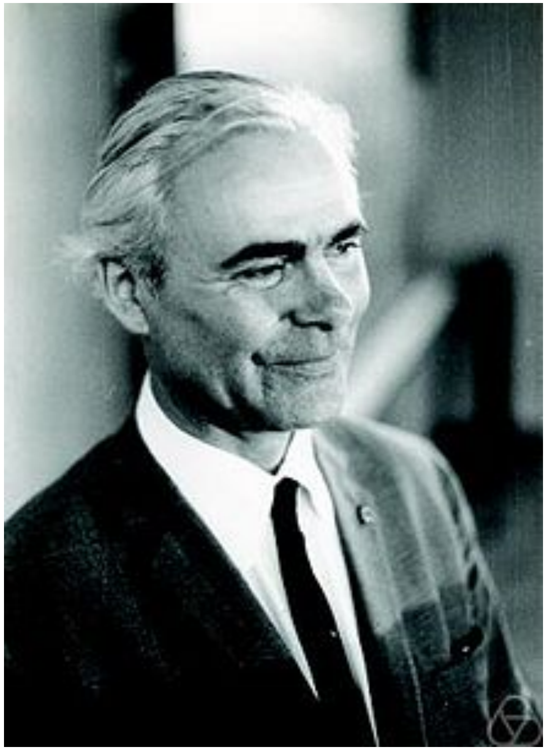
- A *(k;g)*-graph of minimum order is called a ***(k;g)-cage.***

- Denote by ***n(k;g)*** *the order* of a *(k;g)*-cage.

- *Was introduced by Tutte in 1947.*
- In fact as 3-regular or cubical graphs with given girth and minimal order.

The existence of (k;g)-cages was proved by Erdös and Sachs in 1963.

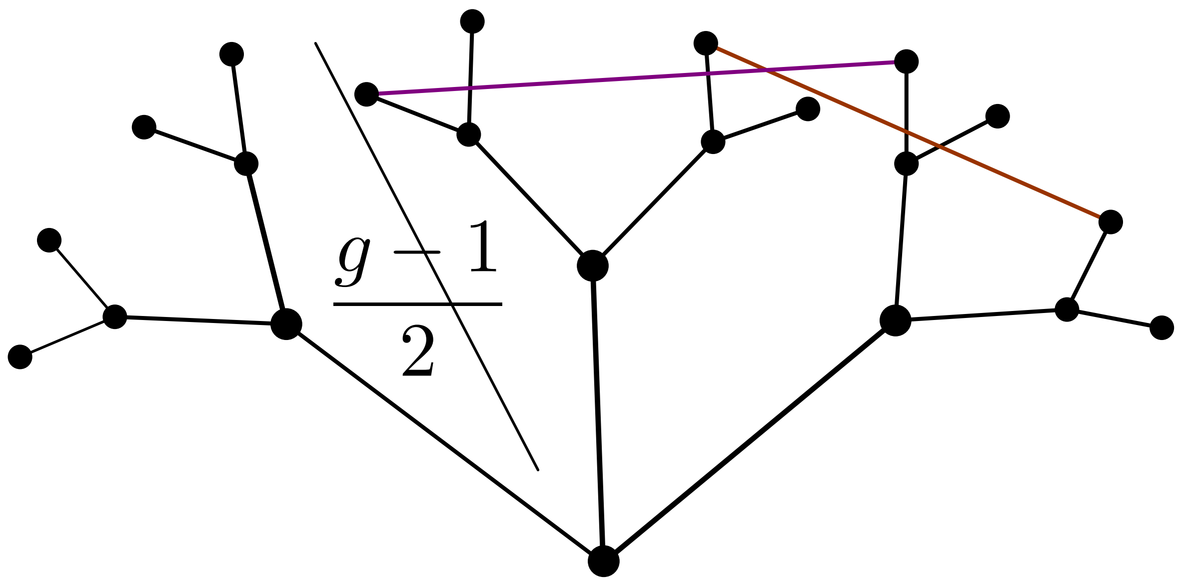




- Counting the number of vertices with respect to their distance from a vertex or an edge depending on whether g is even or odd we obtain the following lower bound.

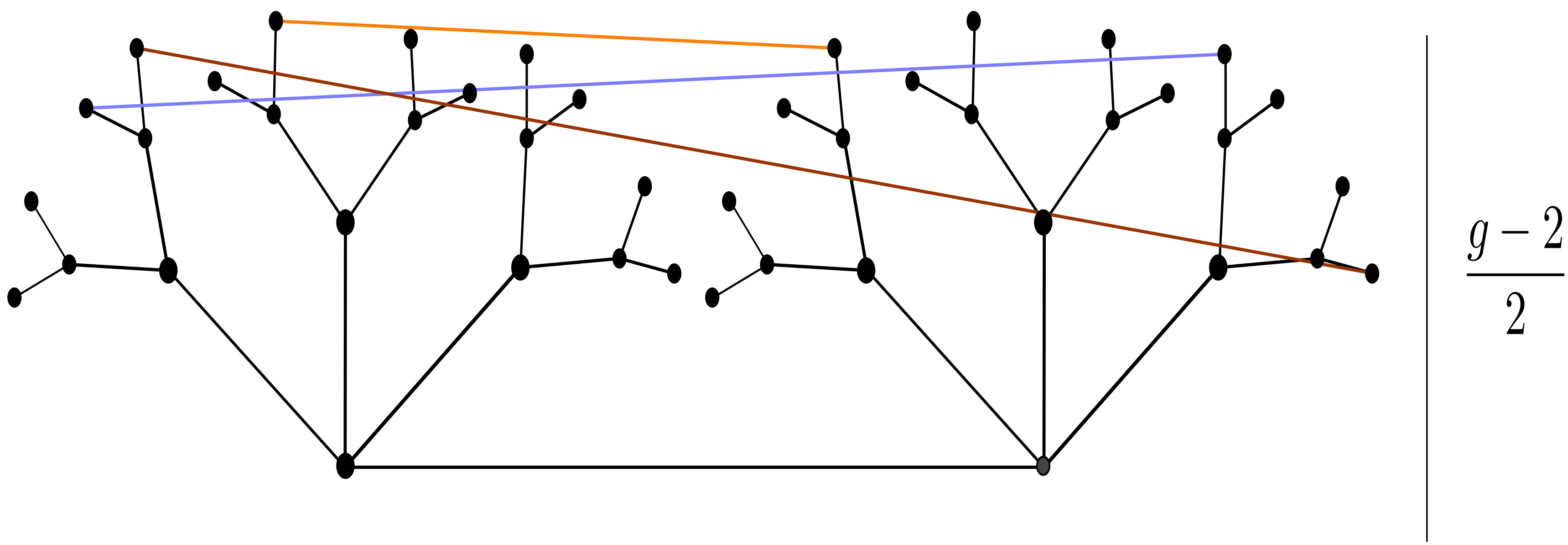
The lower bound of a (k,g)-cage, denoted by $n_0(k,g)$.

If g is odd and k=3



$$1 + \sum_{i=1}^{\frac{g-1}{2}} k(k-1)^{i-1} = \frac{k(k-1)^{(g-1)/2} - 2}{k-2} if\ g\ is\ odd$$

If g in even and k=4



$$2\sum_{i=0}^{\frac{g-2}{2}}(k-1)^i = \frac{2(k-1)^{(g/2)}-2}{k-2} \quad if\ g\ is\ even$$

Biggs (1996) calls excess of a k-regular graph the difference

$$|V(G)|-n_0(k;g)$$

We are interesting in construct graphs with small excess.

Actually, there exist several results focussed on constructing (k;g)-graphs with small excess.

For references see:

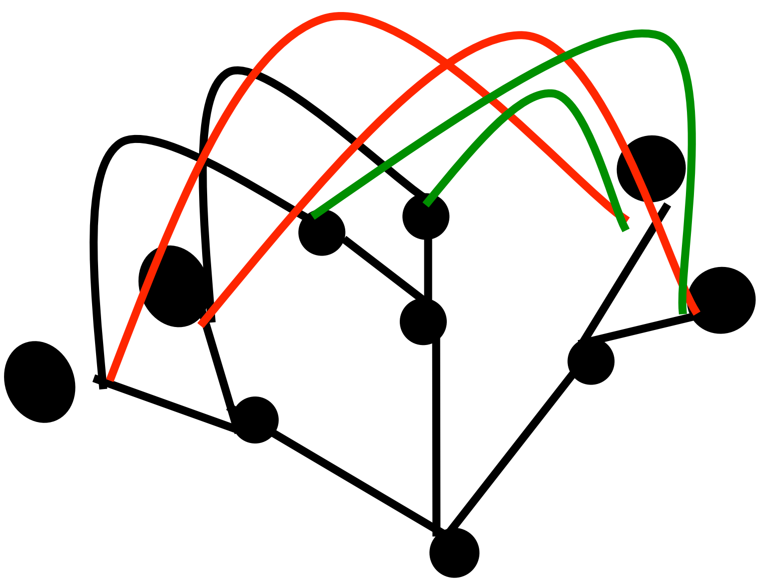
Dynamic Cage Survey

Geoffrey Exoo and Robert Jajcay

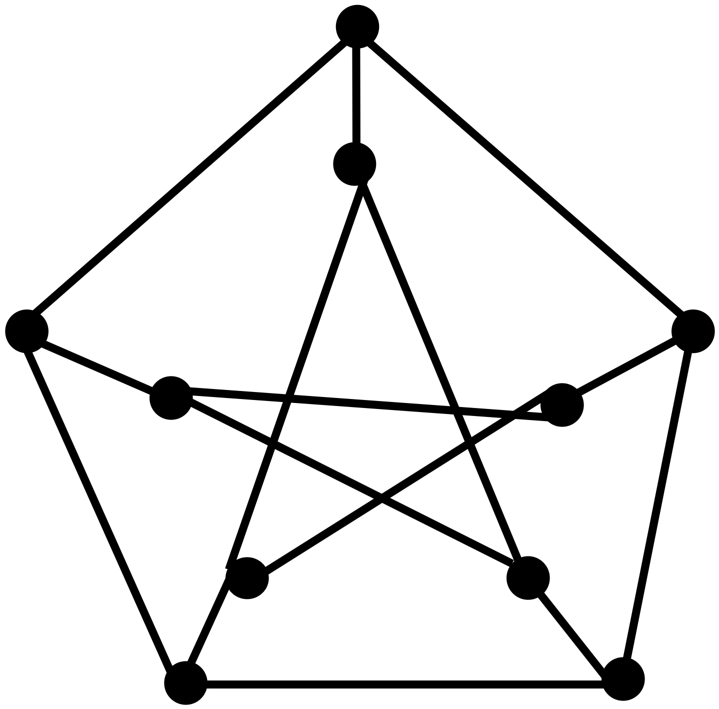
The Electronic Journal of Combinatorics 15 (2008),

DS16

If $n(k,g) = n_0(k,g)$, G is called a Moore (k,g)-cage.



(3,5)-cage



Petersen Graph

Existence of Moore (k,g)-cages:

- (q,3)-cages: Complete graphs
- (2,2d+1)-cages: Odd cycles 2d+1
- (q,5)-cages:
    - o (2,5)-cage: C_5
    - o (3,5)-cage: Petersen Graph
    - o (7,5)-cage: Hoffman-Singleton Graph
    - o (57,5)-cage: May exist

Which are the (q+1,g)-minimal cages for g={6,8,12}?

- As for g=6, $n_0(q+1,6)=2(q^2+q+1)$ then, the (q+1,g)-minimal cage is the incidence graph of a projective plane of order q
- As for g=8, $n_0(q+1,8)=2(q^3+q^2+q+1)$ then, the (q+1,g)-minimal cage is the incidence graph of a generalized quadrangle of order q
- As for g=12, $n_0(q+1,6)=2(q^5+q^4+q^3+q^2+q+1)$ then, the (q+1,g)-minimal cage is the incidence graph of a generalized hexagon of order q

In 1963 Erdös and given the following upper bound:

For all k≥2 and g≥3:

$$n(k;g) \leq 4\sum_{t=1}^{g-2}(k-1)^t$$

Monotonicity (Sachs,1963):

$$n(k;g) \leq n(k;g+1)$$

$$n(k;g) \leq n(k+2;g)$$

In 1997 Sauer give the following upper bounds for $k \geq 2$ and $g \geq 3$:

$$n_0(k,g) = \begin{cases} 2(k-2)^{(g-2)} & \text{if } g \text{ is odd;} \\ 4(k-2)^{(g-3)} & \text{if } g \text{ is even.} \end{cases}$$

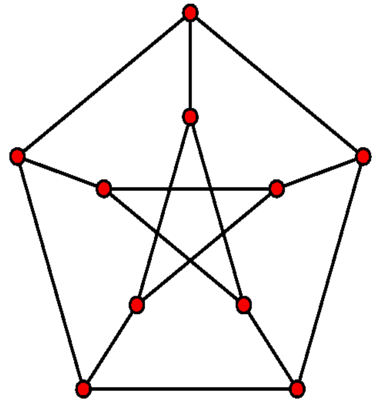
This upper bounds was improvement in the same paper for $k=3$.

Some examples of cages

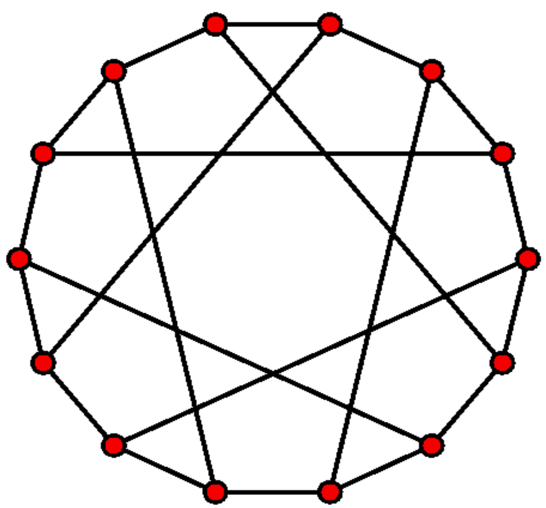
For k=3.

g	5	6	7	8	9	10	11	12
$n_0(3,g)$	10	14	22	30	46	62	94	126
$n(3,g)$	10	14	24	30	58	70	112	126

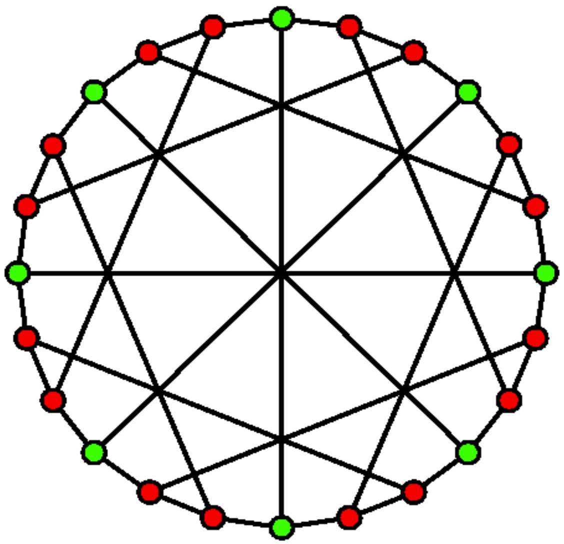
- Petersen Graph



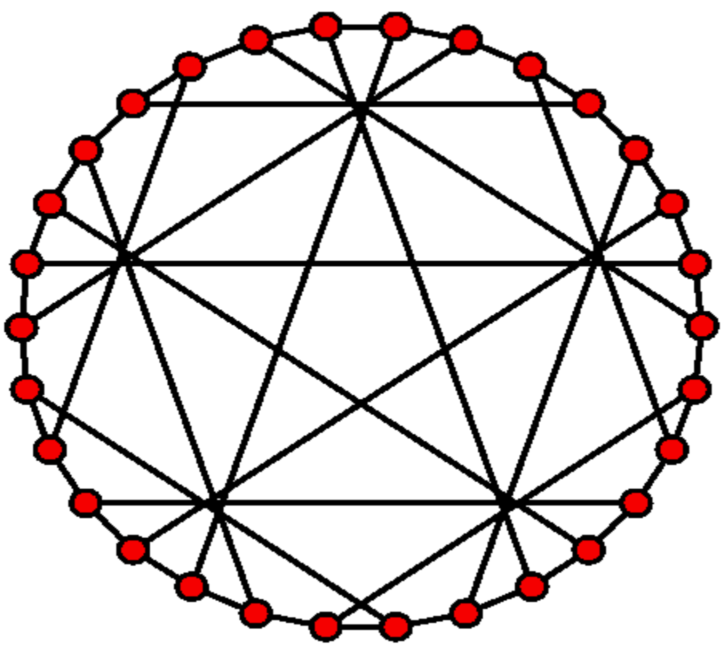
- Heawood Graph



- Mc Gee Graph



Tutte-Coxeter Graph
(3;8)-cage



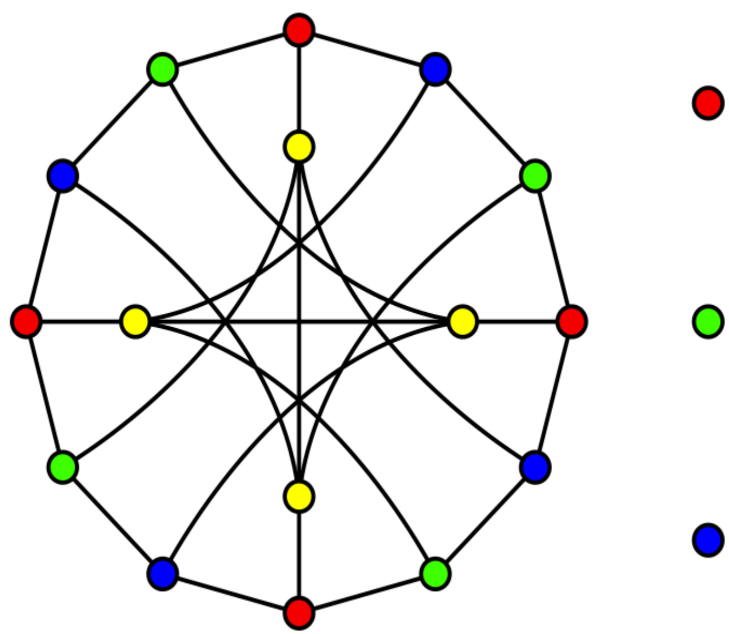
-There exits 18 different (3;9)-cages of order 58

-Three (3;10)-cages of order 70

-There exists only one (3;11)-cage of order 112 constructed by Biggs and the unicity was proved by McKay.

The (3;12)-Moore cage, called Benson´s Graph, is the incidence graph of the generalized hexagon of order 2.

The (4;5)-cage is the Robertson Graph, it has order 19 (three vertices in the excess).



The Geometric Vision

A linear space, denoted by (P,L) consists of two sets, one of points and another of lines (subsets of points) that satisfies:

- Any line has two or more points.
- Two points determine at most one line.

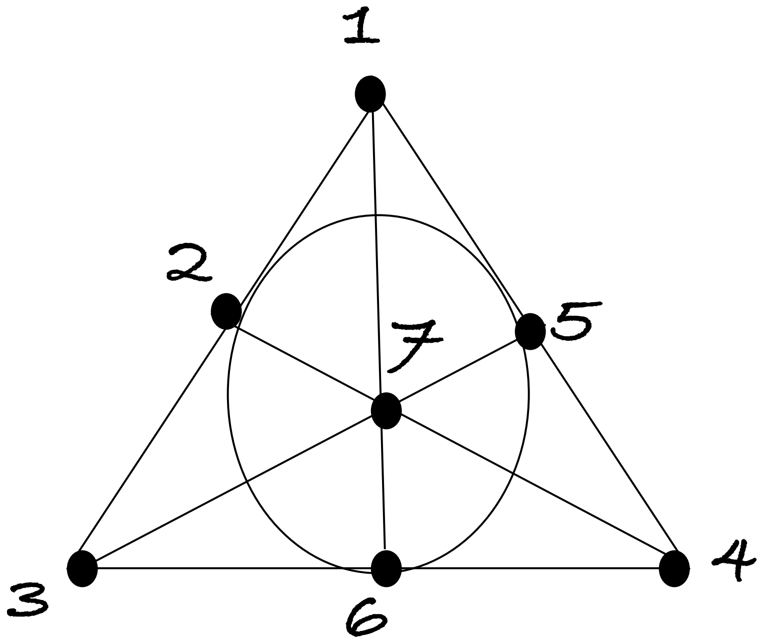
A projective plane of order q is a linear space, denoted by Πq that satisfies also that:
- Any two points determine a line.

- Any pair of lines intersect.

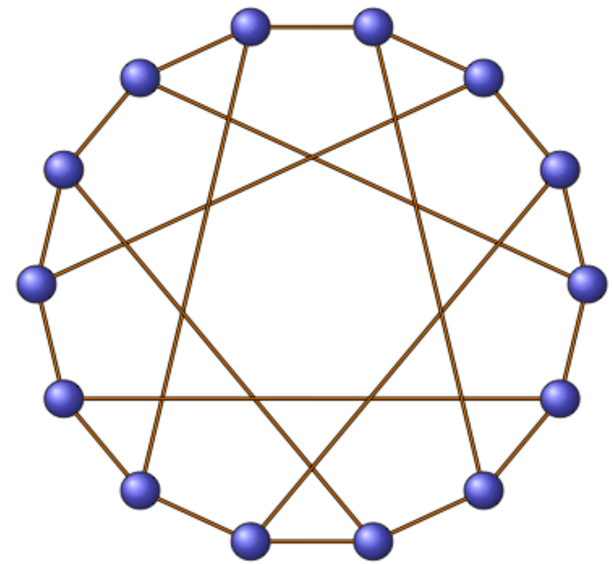
- There exists a set of four points in general position, that is there is not three of them collinear.

The order of a projective plane is q if all the lines has q+1 points and in each point incide q+1 lines.

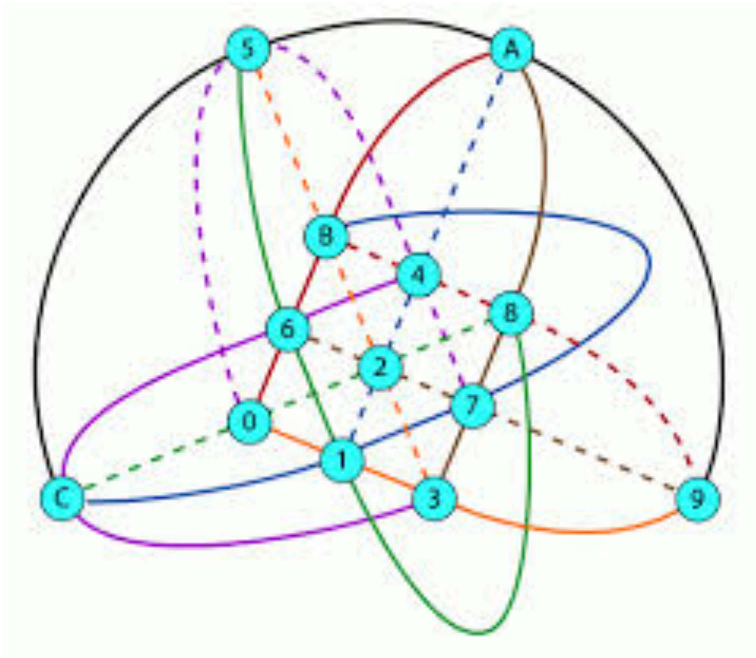
The total of points and lines is the same as it is equal to $q^2 + q + 1$



Fano Plane



Heawood Graph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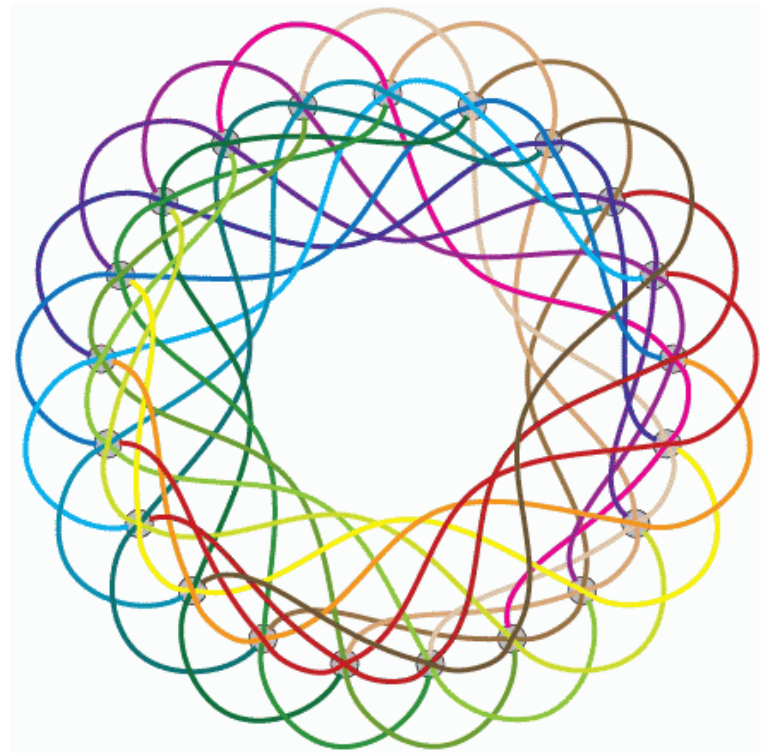
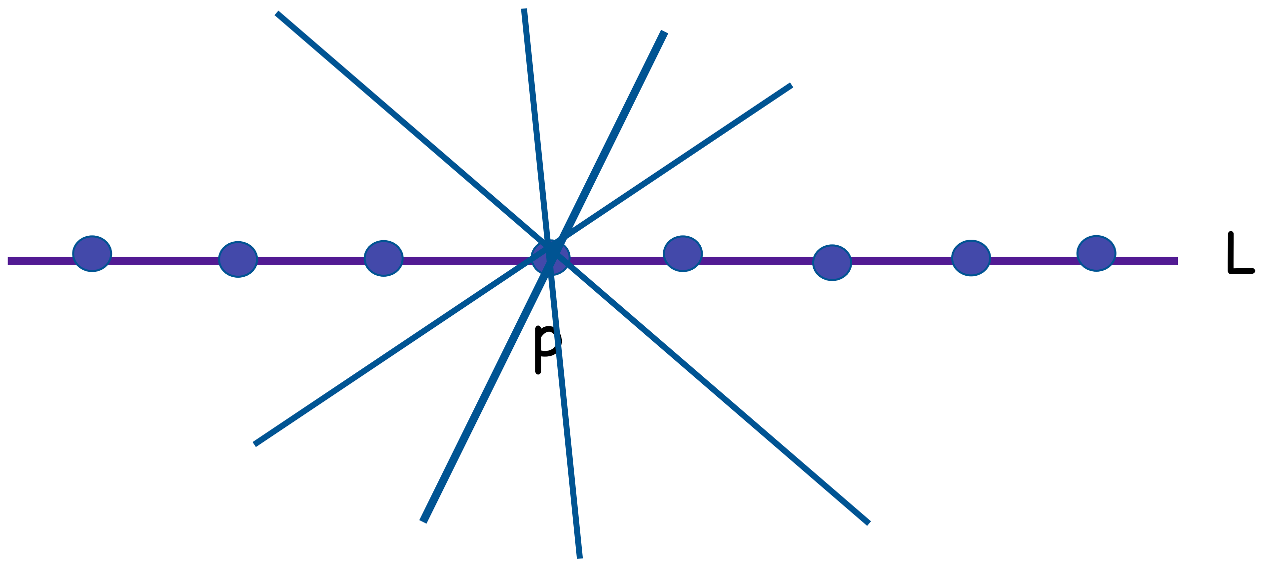


Our work

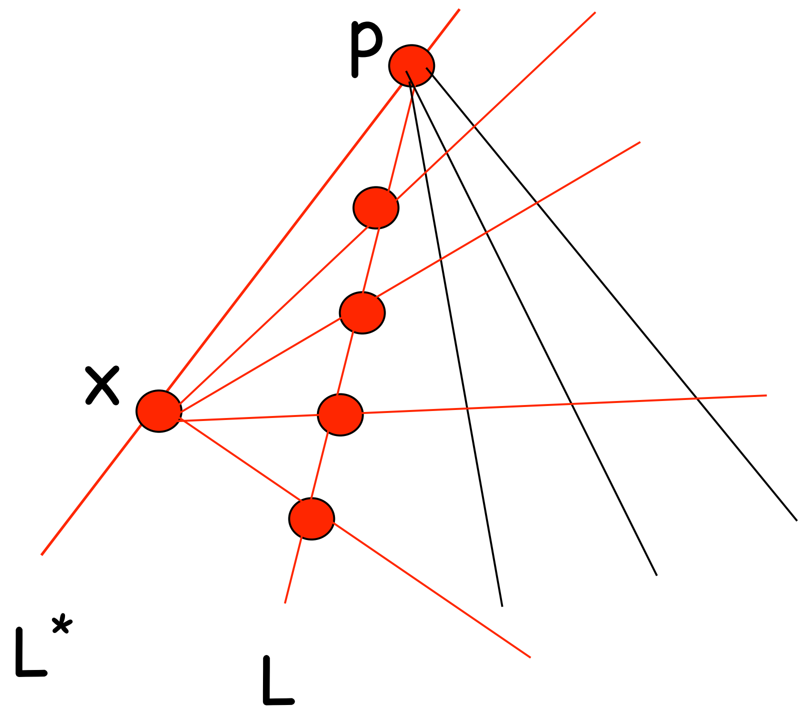
for even

girth

A flag in a partial space is a partial subspace that consist of a point p and an incident line l to p, all the lines incidents to p and all the points in l.

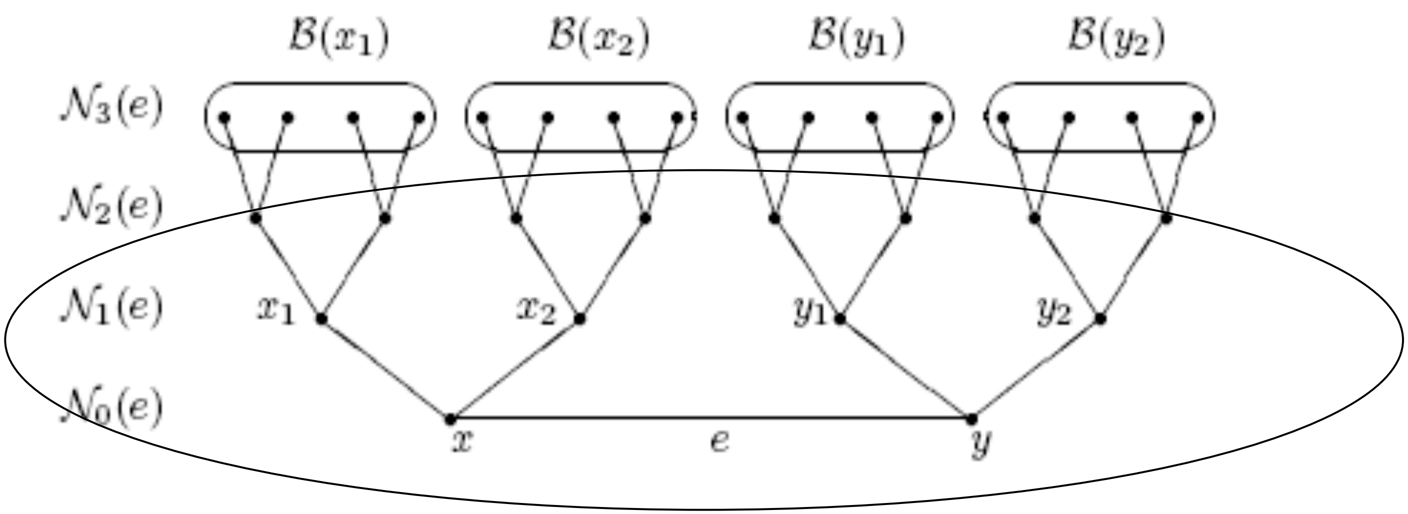


An anti-flag is a linear subespace that consist of a line with their points and a point (not in the line) with all their incident lines.

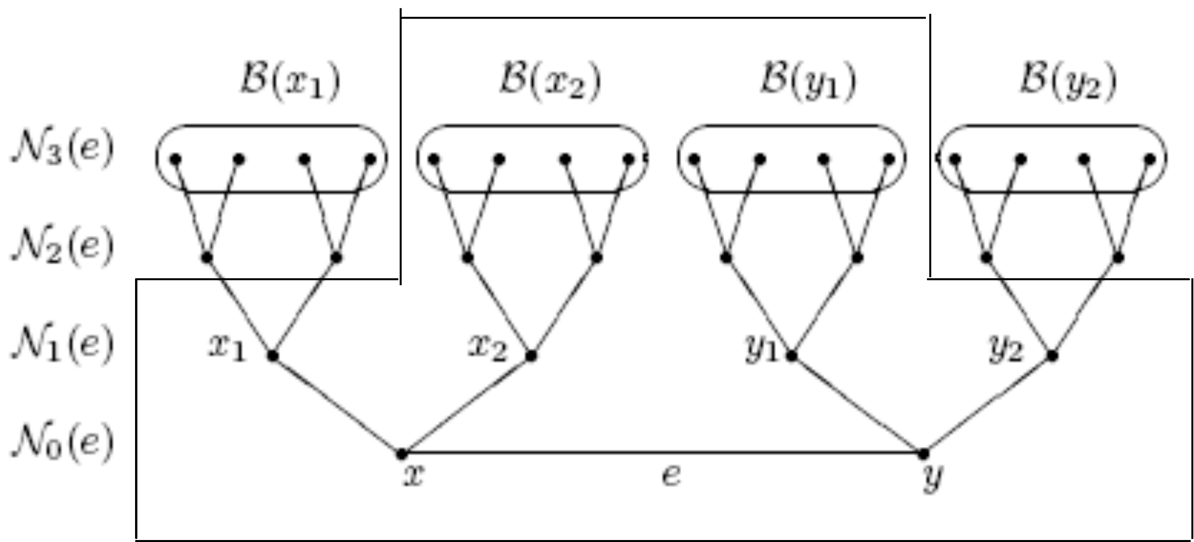


An elliptic semiplane is obtained “deleting” a flag or an anti-flag of a projective plane.

Then, the (q,6)-graphs are the incidence graphs of the elliptic semiplanes, i “deleting” flags:



We obtain (q;6)-graphs with $2q^2$ vertices, called Bq.



We obtain (q;6)-graphs with $2(q^2-1)$ vertices.

Gács and Héger, 2008 use this ideas but, not only in projective planes (also in generalized quadrangles and hexagons) to constuct (k,g)-graphs with few vertices.

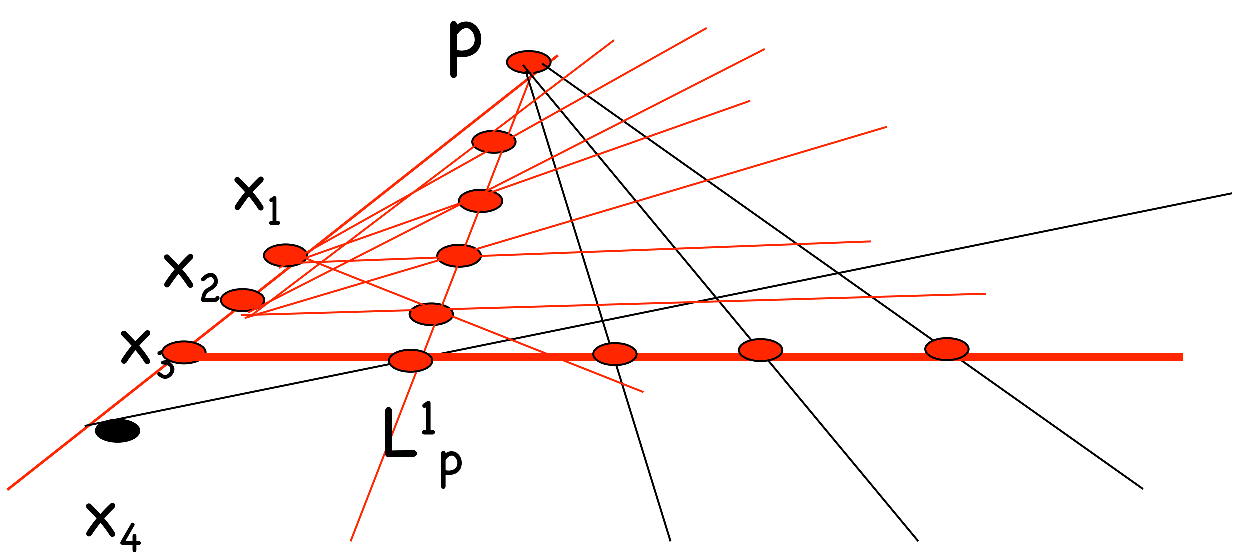
They used the concept of t-good structure.

A pair (P',L') in the generalized n-gon (P,L) is a t-good structure if there are t lines in L' throught any point not in P' and there are t points in P' on any line not in L'.

If you delete a t-good structure in a generalized n-gon, the incidence graph of this partial space is a (q+1-t)-regular with few vertices.

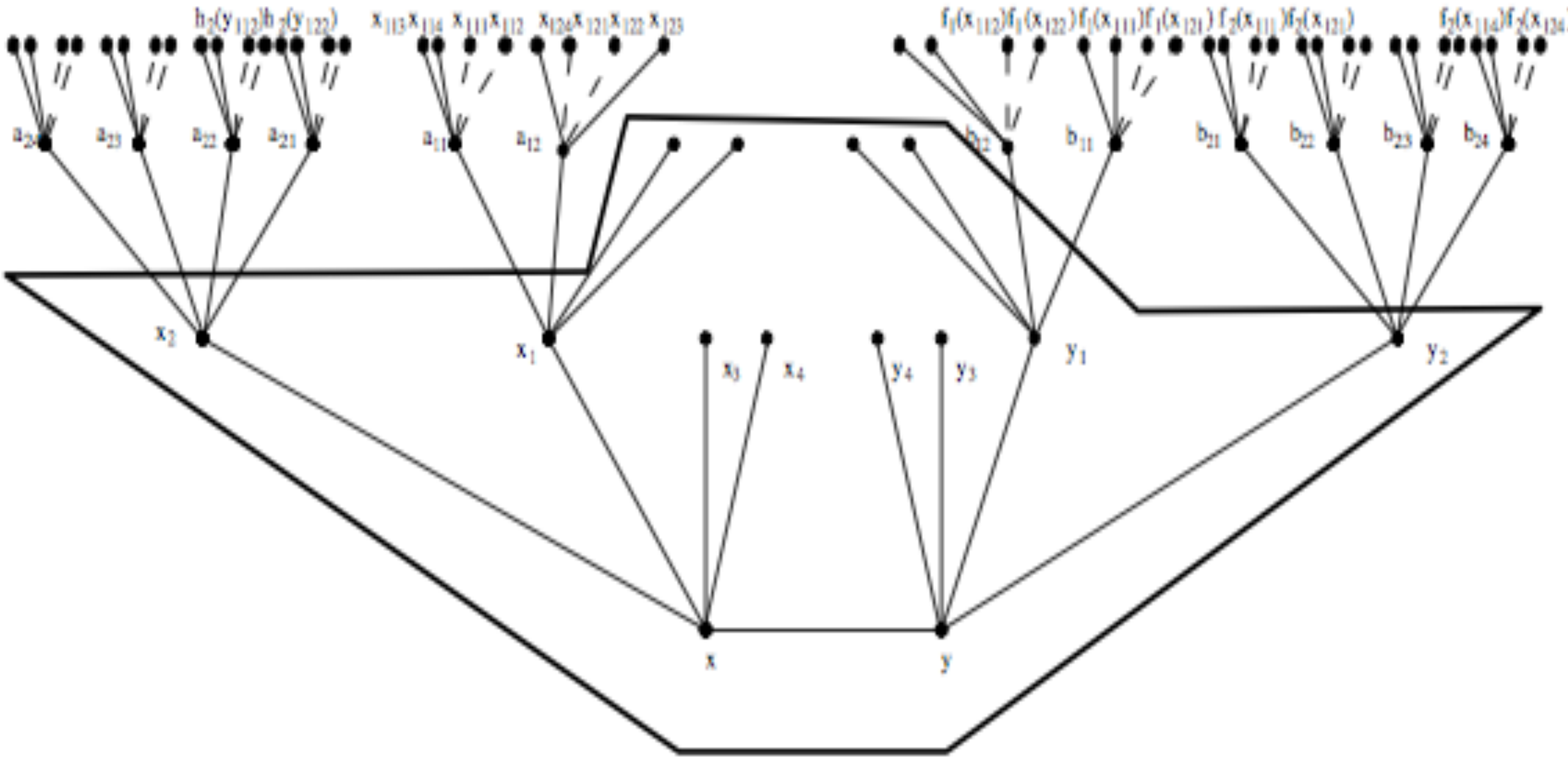
A, Balbuena (2008) generalize the concept of t-good structure a t-nice and in this case delete:

- Points
- Lines
- Incidences



We use this geometric techniques only in projective planes...

... In generalized quadrangles and hexagons we work directly in incidence graphs where the idea of delete incidences is the same that delete edges.



$f_1(x_{111})=y_{111}$, $f_1(x_{121})=y_{112}$, $f_1(x_{112})= y_{121}$, $f_1(x_{122})= y_{122}$

A, Balbuena, Héger (2009)

Para $k \geq q \geq 3$, q una potencia de primo y k un entero positivo tenemos que:

1. $n(k;6) \leq 2(kq-1)$
2. $n(k;8) \leq 2k(q^2-1)$
3. $n(k;12) \leq 2kq^2(q^2-1)$.

A-Balbuena 2010

Let q a power of prime. Then there exists a k-regular graph of girth 6 and order:

- $2(q^2-1)$ si $k=q$.
- $2(q^2-q-2)$ si $k=q-1$.
- $2(qk-2)$ si $k<q-1$.

(A, Abreu, Balbuena, Labbate, 2014)

Using the concept of perfect dominant set in graphs:

A set of vertices U in V(G) is perfect dominant if all vertice non in U has exactly a neighbour in U

2009: If k=q-1, then we construct graphs with order:

$n(q,8) \leq 2q(q^2-1)$

$n(q-1;8) \leq 2(q-1)(q^2-1)=2(q^3-q^2-q-1)$.

2013:

$n(q;8) \leq 2q(q^2-2)$ $q=p^\beta$, β **≥1.**

$n(q;8) \leq 2q(q^2-3q-2)$ for $q \geq 2^\beta$ β **≥2.**

$(q-1;8) \leq 2(q)(q-1)^2=2(q^3-2q^2+q)$, for $q=p^\beta$ and β ≥1.

Work for

odd girth

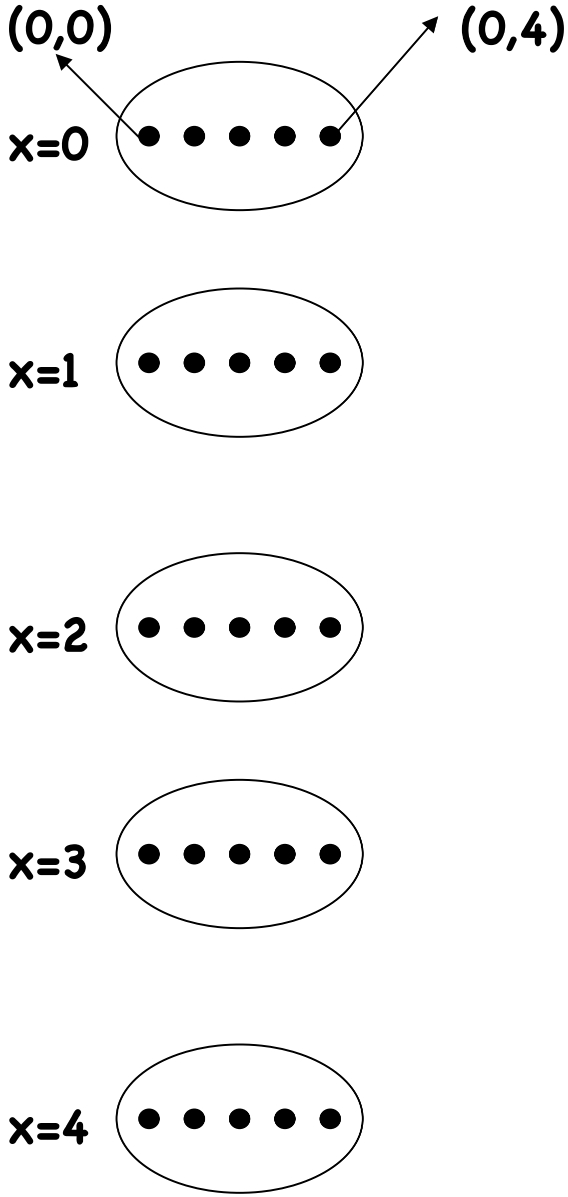
Abreu, A., Balbuena, Labbate (2011)

Also, using the algebraic properties of the projective planes and the Bq graph we obtain some results for girth 5.

The techniques are similar to construct The Hoffman-Singelton Graph:

- If you describe take the set of points as B_5 as $P(B_5)=\{(x,y) \ / \ \{x,y\} \in Z_5\}$ and the lines as:

$$L(B_5)=\{([x,y] \ / \ \{x,y\} \in Z_5\}$$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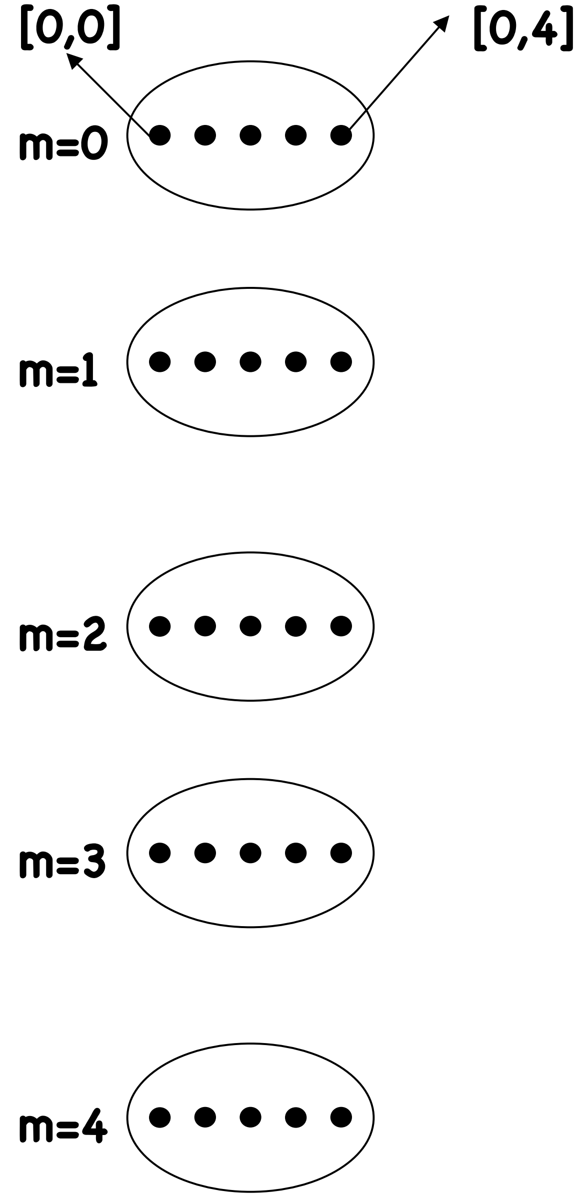
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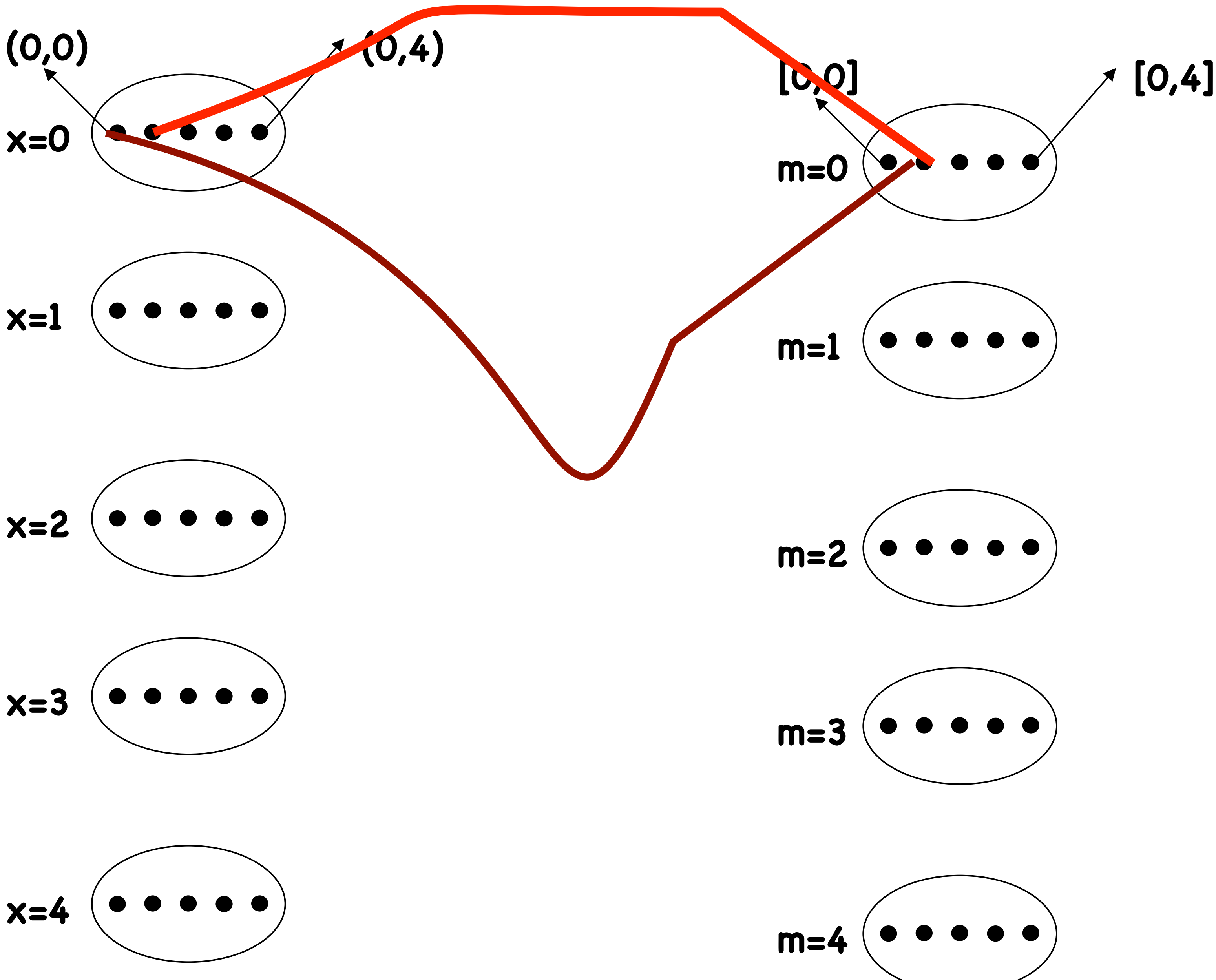
The incidence is given by the rule:

(x,y) be in [m,b] if and only if

$$y=mx+b$$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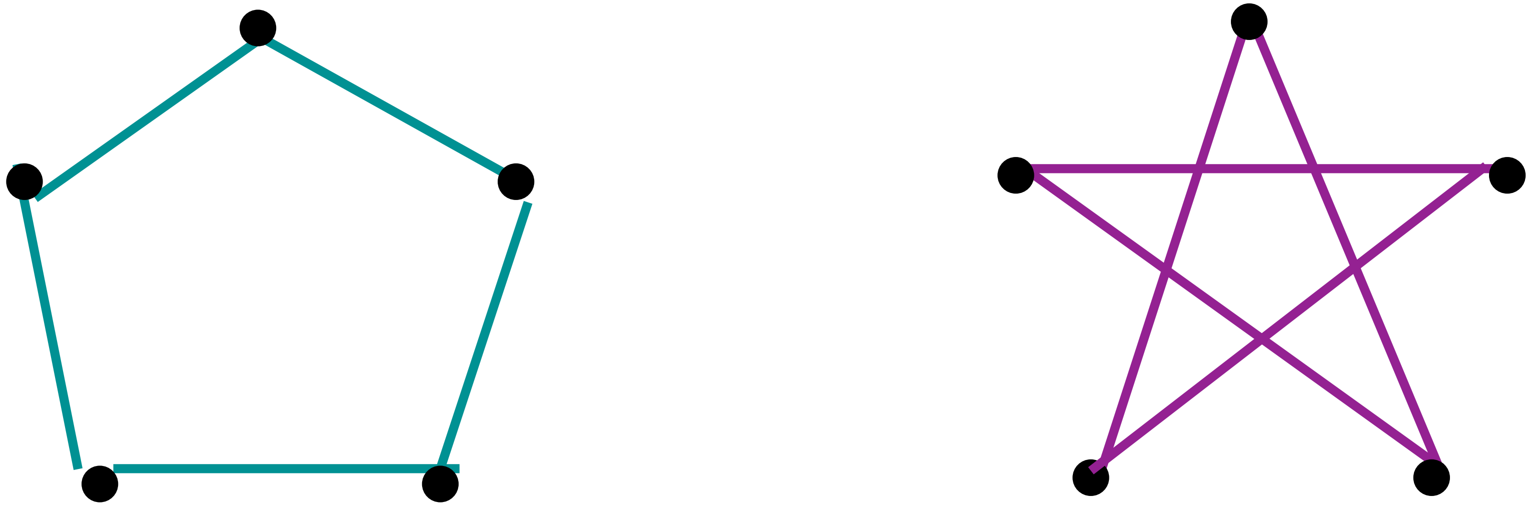

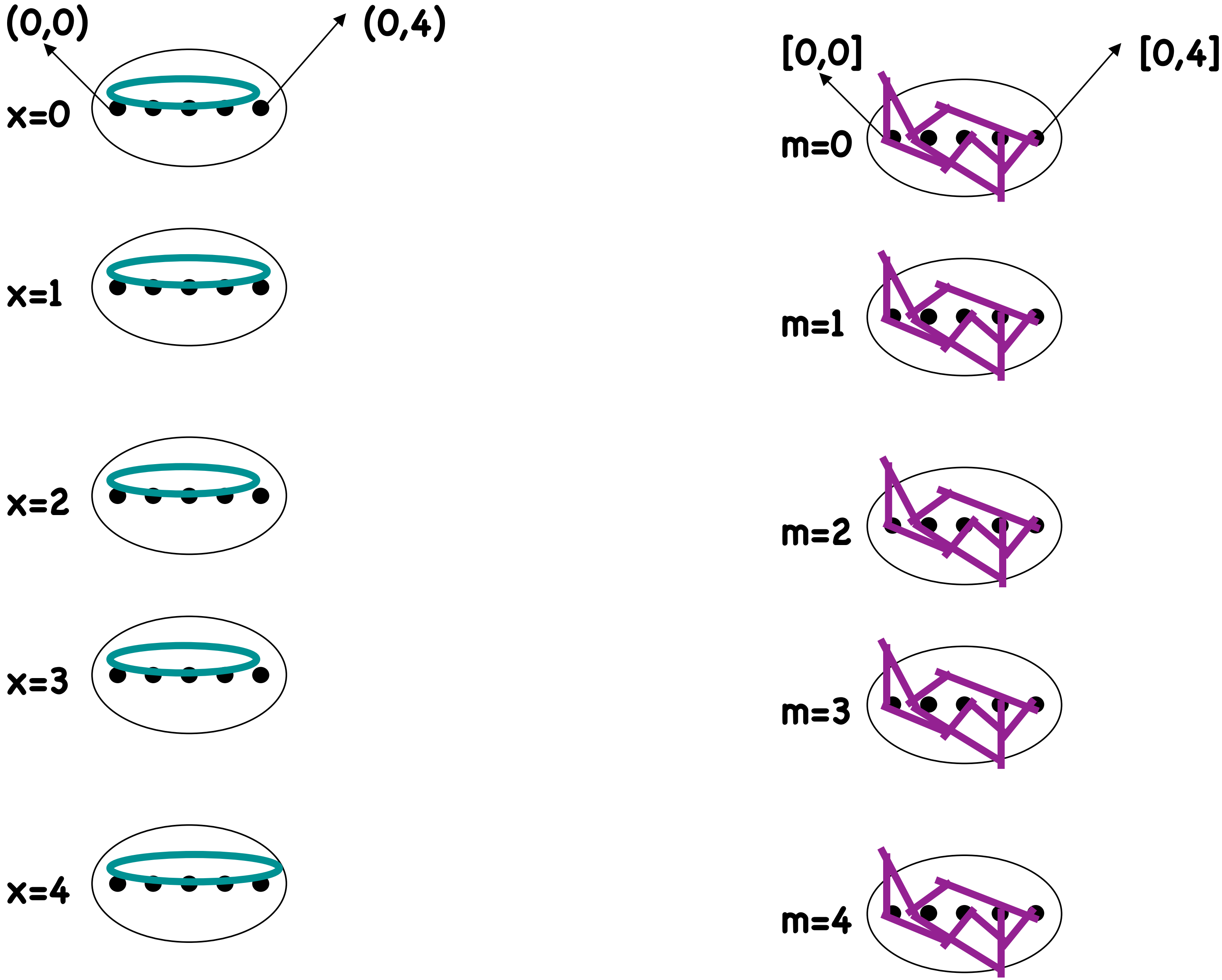
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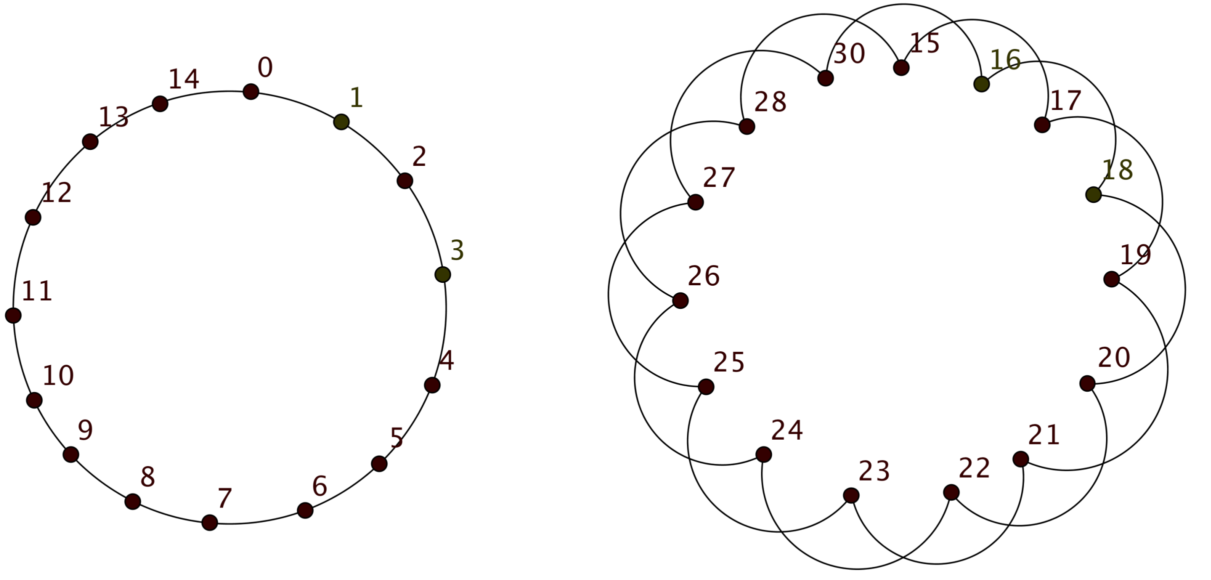
We obtain a general construction and divided the primes in two sets:

$p=6m+1$

$p=6m+5$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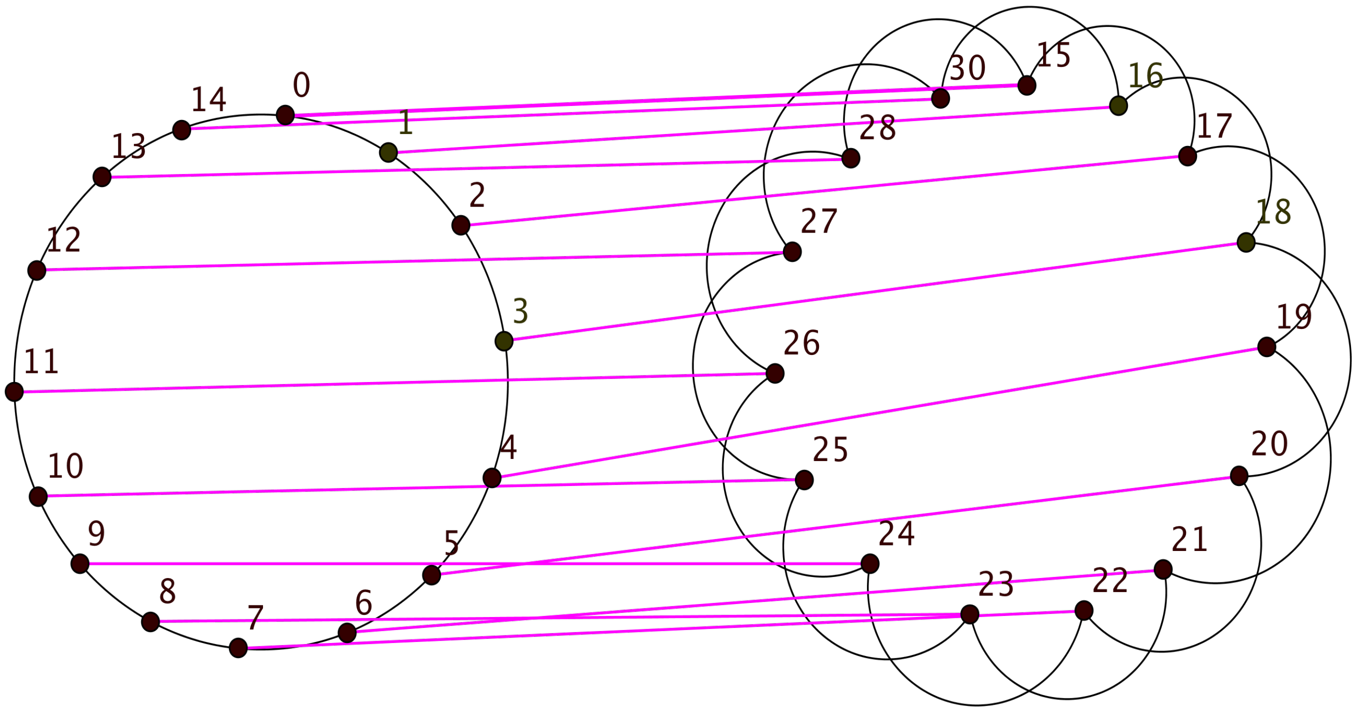

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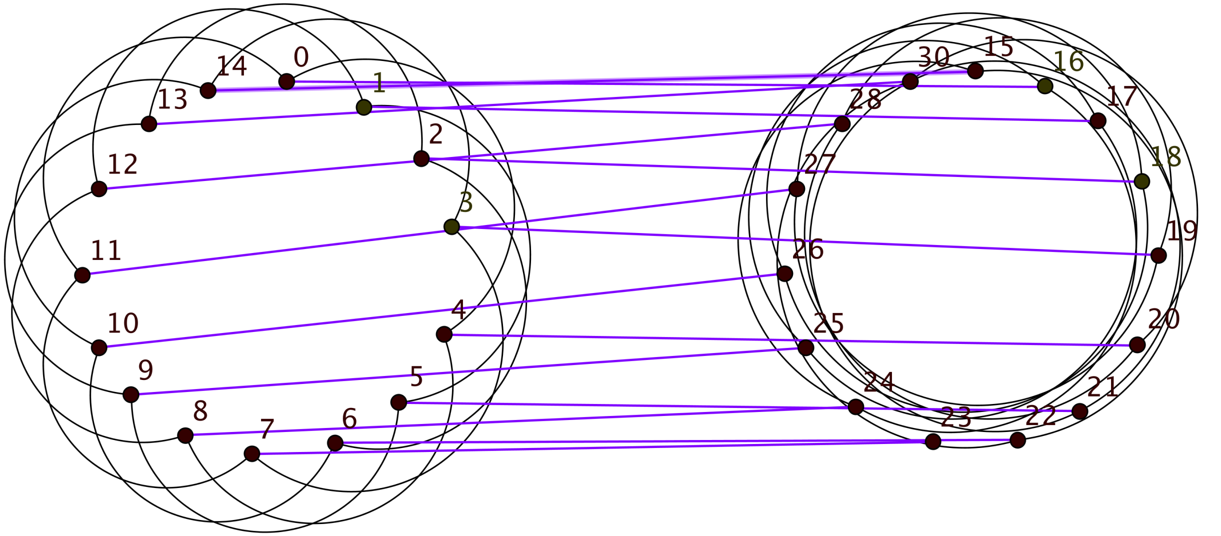
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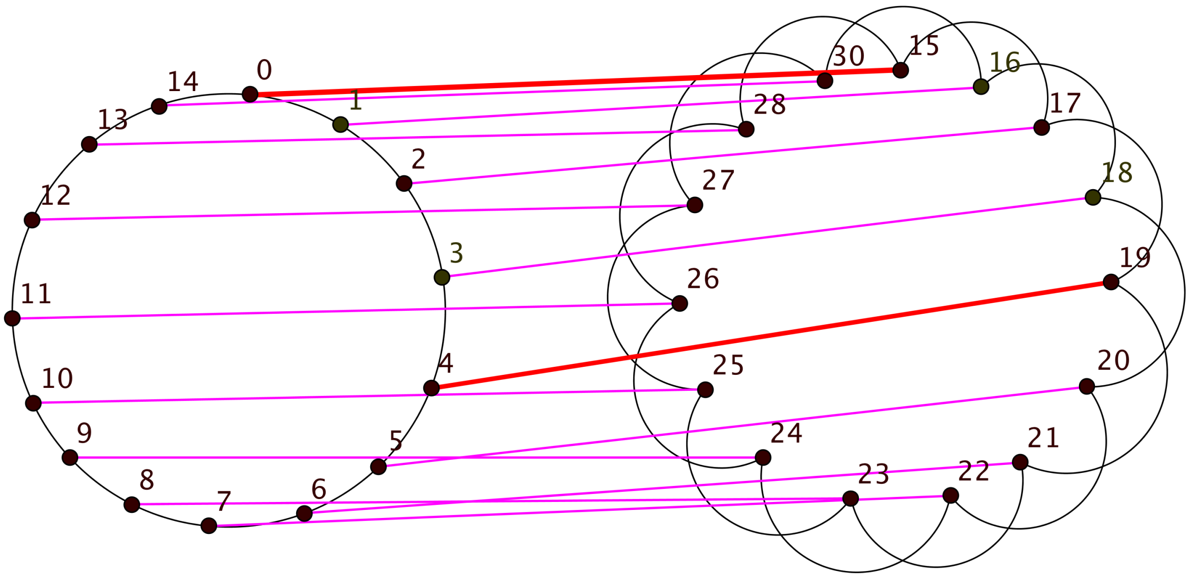
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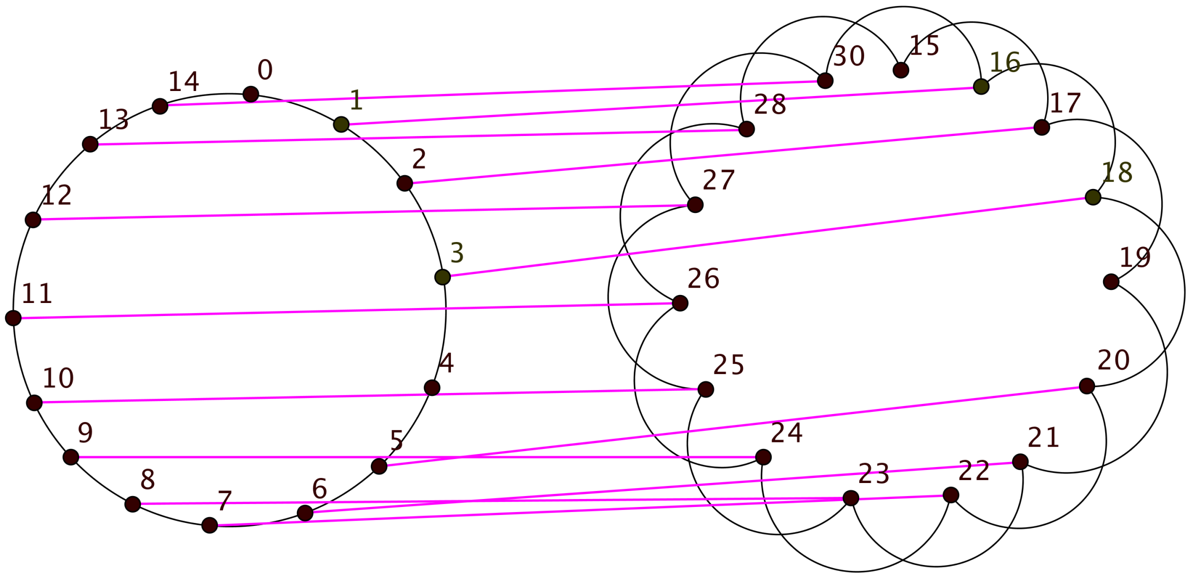

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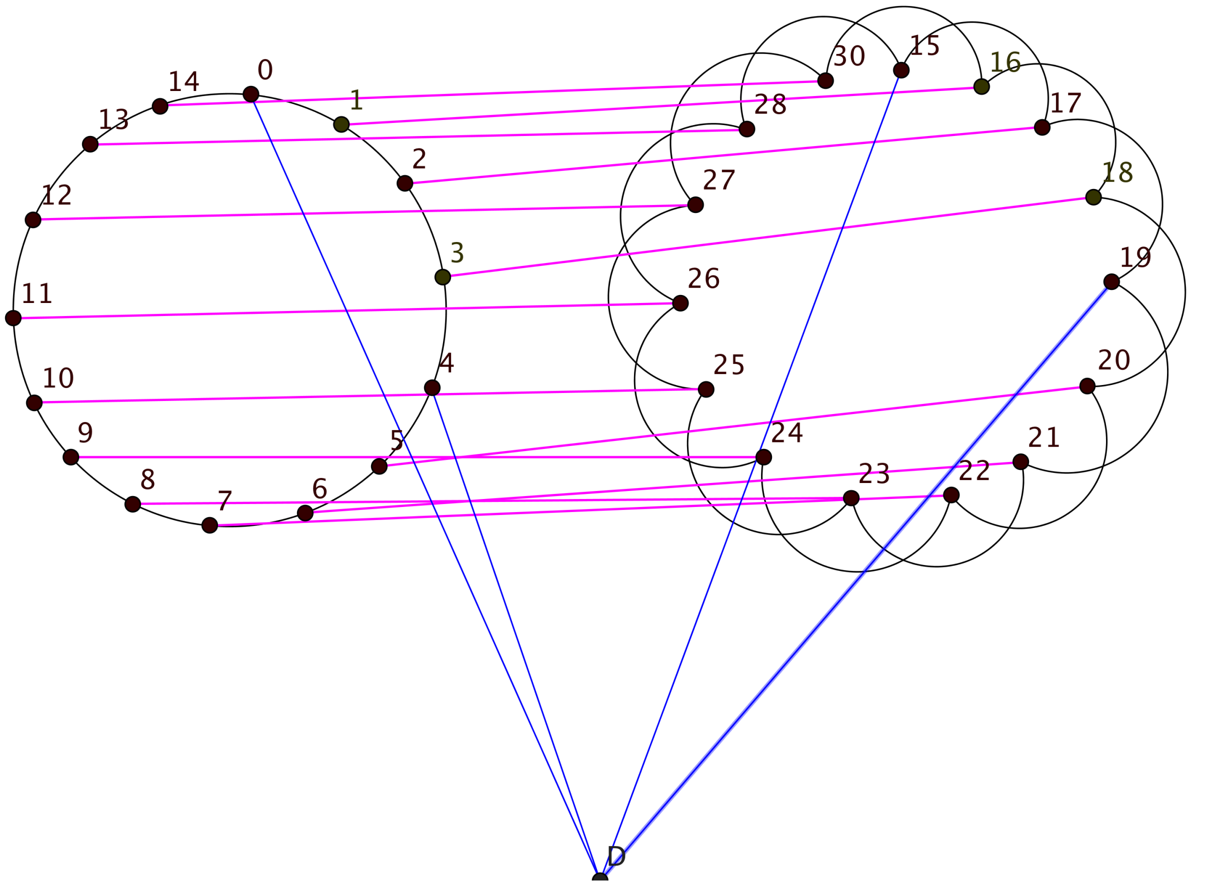

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Abreu, A-P, Balbuena, Labbate (2011):

For q=13,17, 19 and all prime q≥23 we obtain (q+3)-regular graphs of girth 5 with few vertices.

k	New value	Previous value	
13	236	240	Exoo
14	284	288	Jørgensen
15	310	312	Jørgensen
20	572	576	Jørgensen
21	682	684	Jørgensen

New values Abajo, A-P, Balbuena, Bendala (2015):

$q = 16$ → $(17,6) - cage$ → $(19,5) - grafo$

$q = 17$ → $(18,6) - cage$ → $(20,5) - grafo$

$q = 19$ → $(20,6) - cage$ → $(22,5) - grafo$

regularidad	cota superior orden	nueva cota
17	448	436
18	480	468
19	512	500
20	572	564
21	682	666
22	720	704

$g \in \{6,8,12\}$ → *La* $(q+1, g)-$ *jaula es minimal*

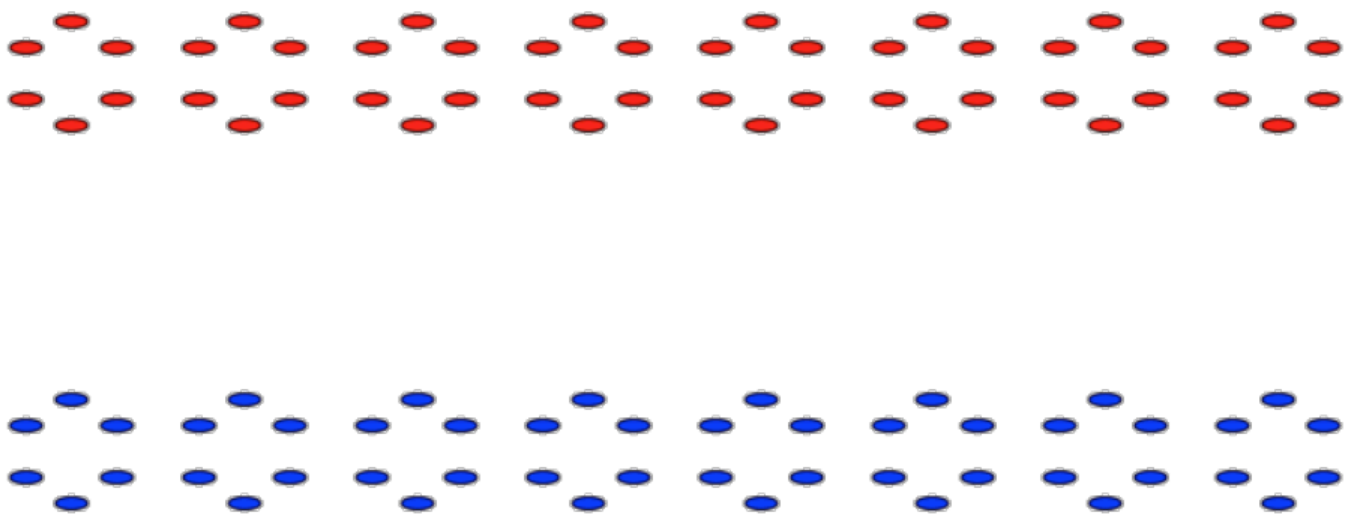
q potencia de primo

Semiplano elíptico tipo L

$$2\,(q^2-1) = 2\,(q+1)(q-1) \;\; \textit{vértices}$$

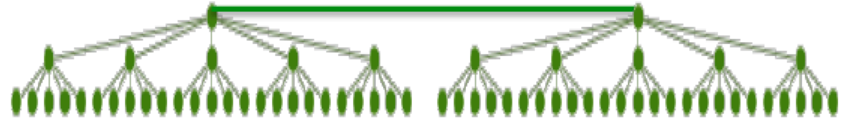
$$q = 7 \longrightarrow 2\ (q+1)(q-1)\ \textit{vértices}$$

$$2 \cdot 8 \cdot 6 = 96\ \textit{vértices}$$



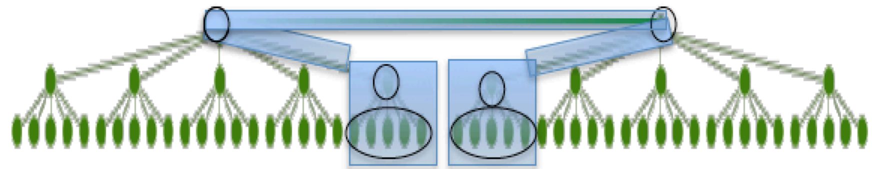
$(q+1,6) - cage$

Plano proyectivo



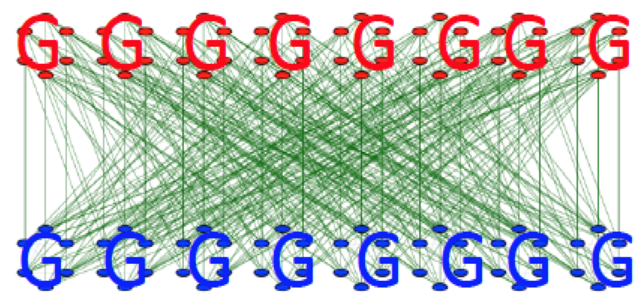
$2\,(q^2-1)$ *vértices*

Semiplano elíptico tipo L



$q - regular$

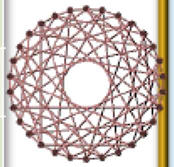
cintura 6



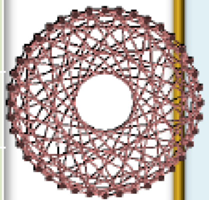
$q+?$ $-regular$

cintura 5

q	r	regularidad	cota superior orden		nueva cota
		32	1680	Jørgensen	1624
29	4	33	1856	Funk	1680
		34	1920	Jørgensen	1800
		35	1984	Funk	1860
31	5	36	2048	Funk	1920
32	5	37	2514	Abreu et al.	2048
		38	2588	Abreu et al.	2448
		39	2662	Abreu et al.	2520
		40	2763	Jørgensen	2592
		41	3114	Abreu et al.	2664
37	5	42	3196	Abreu et al.	2736



q	r	regularidad	cota superior orden		nueva cota
		43	3278	Abreu et al.	3040
		44	3310	Jørgensen	3120
		45	3610	Abreu et al.	3200
		46	3696	Jørgensen	3280
41	6	47	4134	Abreu et al.	3360
43	5	48	4228	Abreu et al.	3696
		49	4332	Abreu et al.	4140
		50	4416	Jørgensen	4232
		51	4704	Jørgensen	4324
47	5	52	4800	Jørgensen	4416



Teorema: *Dado un entero* $k \geq 53$,

sea q *el número primo impar más próximo tal que* $k \leq q + 6$.

Se cumple que

$$n(k,5) \leq 2(q-1)(k-5)$$

Ejemplo:

q	r		regularidad	cota superior orden		nueva cota
49	6		55	5510	Abreu et al.	4800

-710

Teorema: *Dado un entero* $k \geq 68$,

sea $q = 2^m$ *el número primo par más próximo tal que* $k \leq q + 6$.

Se cumple que

$$n(k,5) \leq 2\,q\,(k-6)$$

Ejemplo:

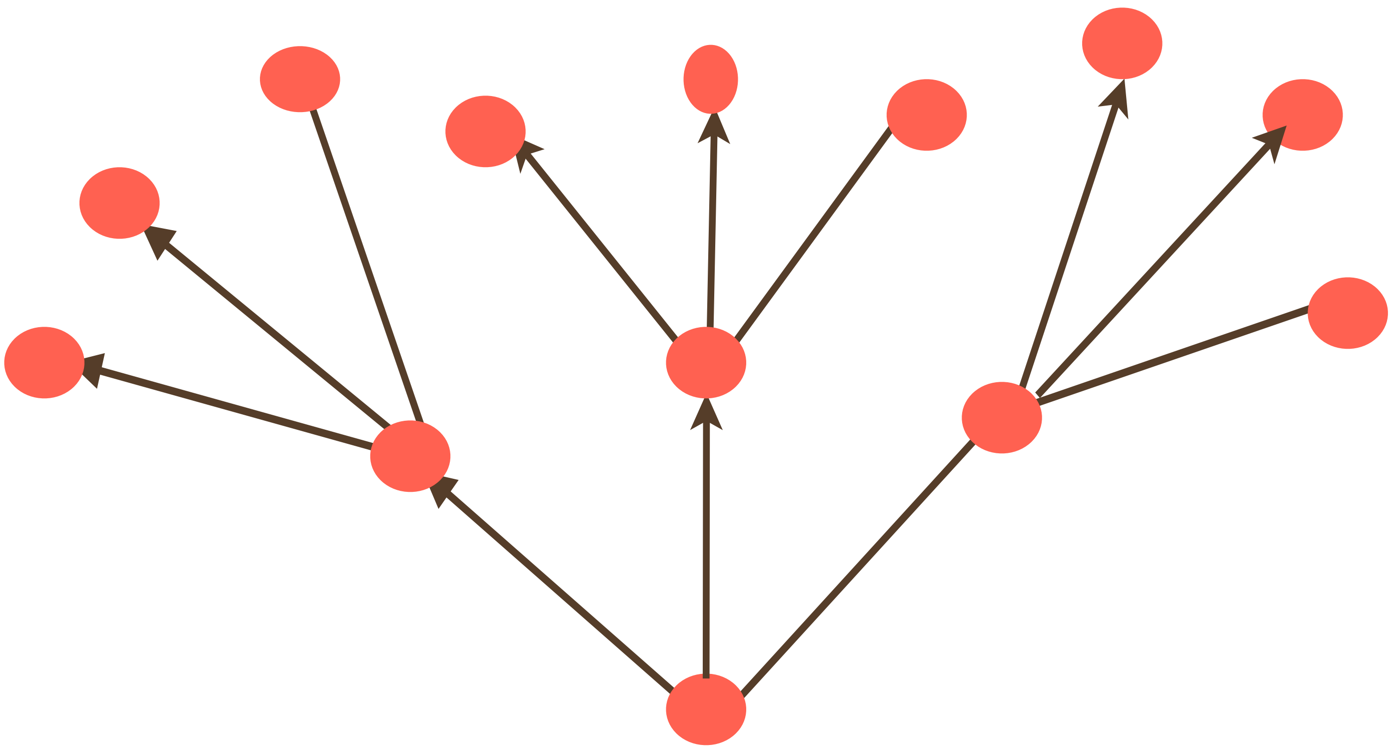
q	r	regularidad	cota superior orden		nueva cota
64	6	70	8976	Abreu et al.	8192

$-\,784$

Thank you for your attention !!!

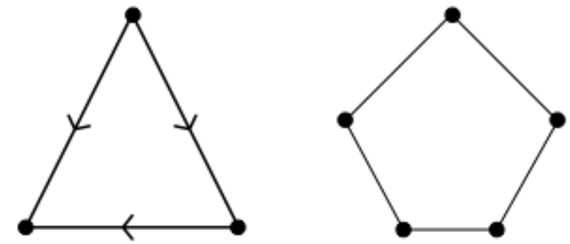
Mixed Moore Graphs

Mixed Moore Bound



The existence is conditioned

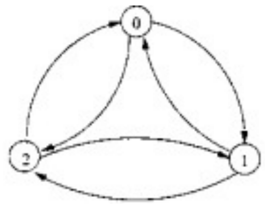
- **Bosák (1979):**
- **The obviuos:**



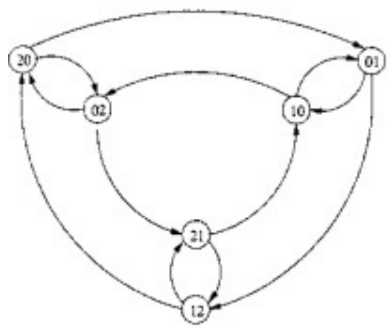
- **If z and r satisfies that there exists c ∈ Z such that:**
- **c divides to (4z-3)(4z+5) and r=1/4(c^2+3).**

n	d	z	r	Existence	Unicity
3	1	1	0	Z_3	YES
5	2	0	2	C_5	YES
6	2	1	1	Ka(2,2)	YES
10	3	0	3	Petersen	YES
12	3	2	1	Ka(3,2)	YES
18	4	1	3	Bozák	YES
20	4	3	1	Ka(4,2)	YES
30	5	4	1	Ka(5,2)	YES
40	6	3	3	Not known	Not known

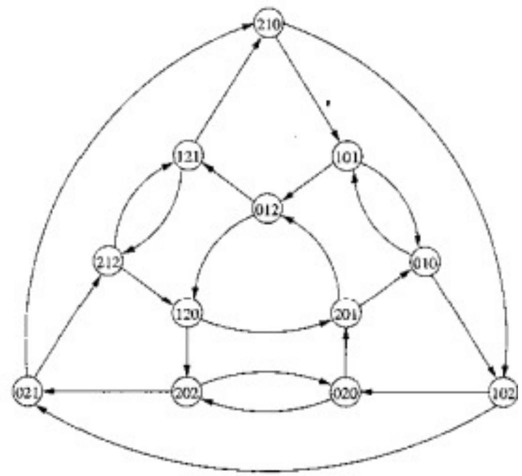
Gráficas de Kautz



$KG(2,1) = K_3^+$

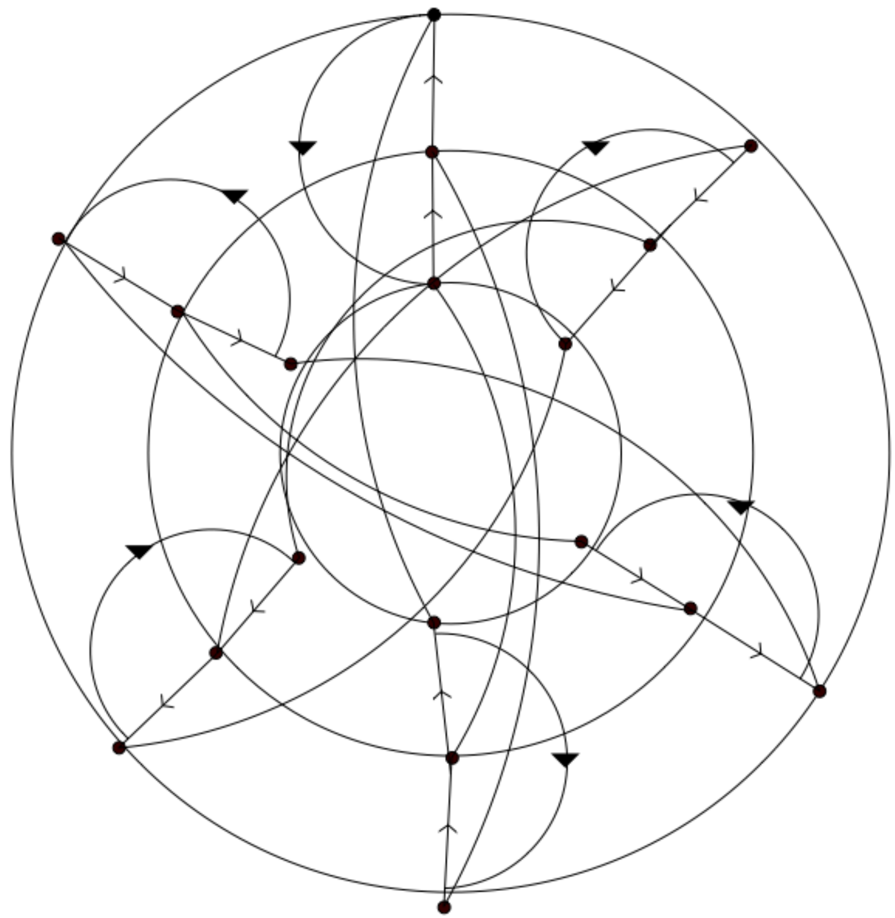


$KG(2,2)$



$KG(2,3)$

Gráfica de Bozák



n	d	z	r	Existencia	Unicidad
56	7	6	1	Ka(7,2)	YES
72	8	7	1	Ka(8,2)	YES
84	9	2	7	Not known	Not known
88	9	6	3	Not known	Not known
90	9	8	1	Ka(9,2)	YES
108	10	7	3	Jorgensen	NO
110	10	9	1	Ka(10,2)	YES
132	11	10	1	Ka(11,2)	YES
150	12	5	7	Not known	Not known
156	12	11	1	Ka(12,2)	YES
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Thank you for your attention !!!

Jajcay y Exoo, 2011: Cuello impar.
A, Jajcay, Exoo ??: Cuello Par.

Jaulas bi-regulares

"contra"

jaulas regulares

FOTOS: Julián Ramírez Araujo

COLOQUIO QUERETANO de Matemáticas

Septiembre 2015

Septiembre

Miércoles 2

"Movimiento rígido a través de líneas rectas"

Luis Montejano

Instituto de Matemáticas, Campus Juriquilla

17:00 horas

diseño: Víctor Alcántara / IM-UNAM

Teatro Auditorio "Dr. Flavio M. Mena Jara"

Centro Académico Cultural, UNAM Campus Juriquilla

Informes:
garaujo@matem.unam.mx /// hernandez@im.unam.mx

UNAM JURIQUILLA · Instituto de Matemáticas · CAC

Chartrand, Gould y Kapoor mostraron que n(D; 3)=1+a_k para D={a_1,a_2,...a_k} (1981)

Downs, Gould, Mitchem y Saba probaron la siguiente cota inferior n(D;g) (1981)

$$f(D;g) \geq \begin{cases} 1 + \sum_{i=1}^{t} a_k (a_1 - 1)^{i-1} & \text{if } g = 2t + 1 \\ 1 + \sum_{i=1}^{t-1} a_k (a_1 - 1)^{i-1} + (a_1 - 1)^{i-1} & \text{if } g = 2t. \end{cases}$$

Cuando D={r,m} entonces (D;g)-jaulas se llaman jaulas bi-regulares

Para jaulas bi-regulares y cuello g=6, Yang and Liang (2003) demostraron que para $2 \leq r < m$ se tiene la siguiente cota inferior:

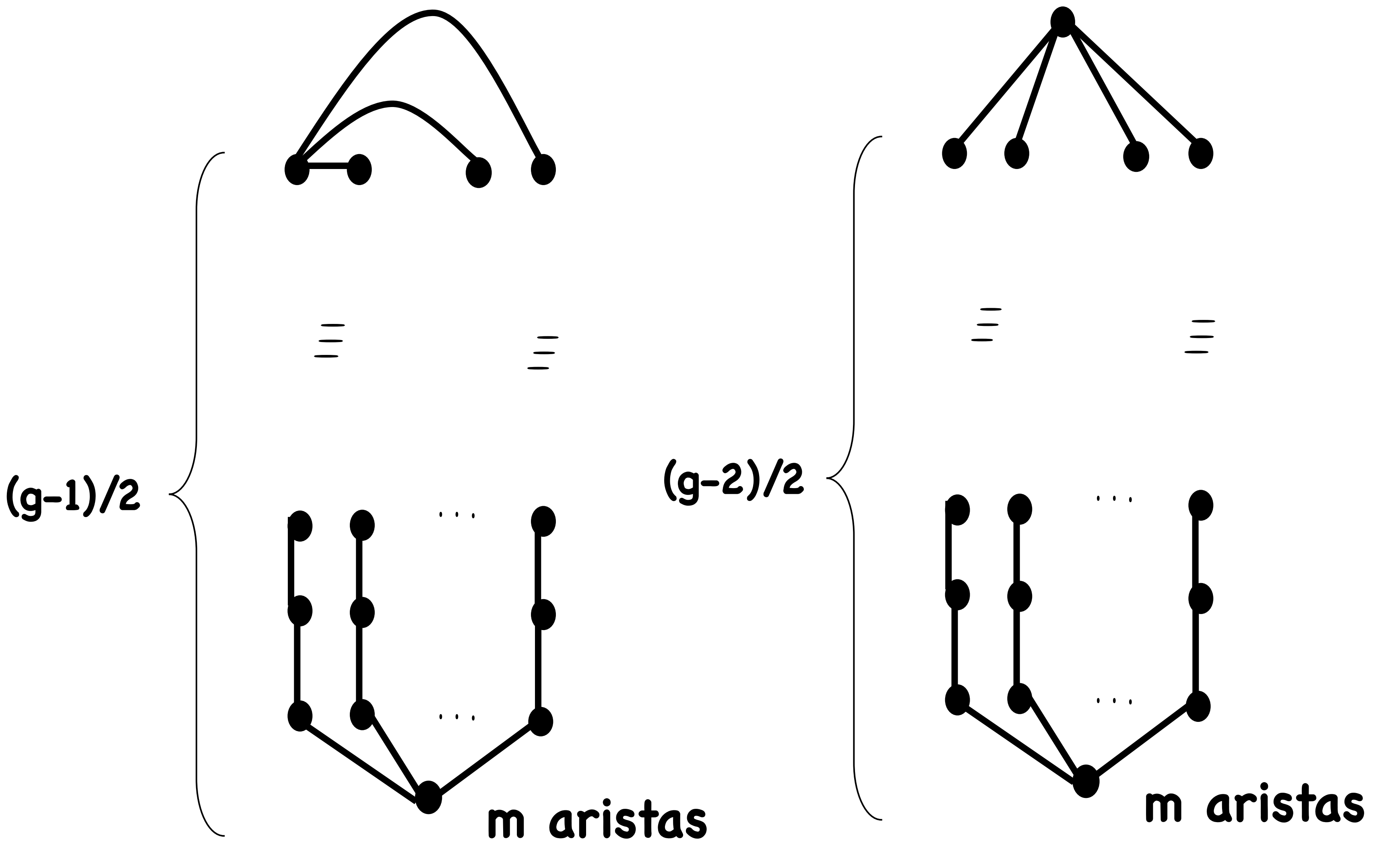
$$f(\{r, m\}; 6) \geq 2(rm - m + 1).$$

Chartrand, Gould y Kapoor (1981) proved that:

$$For\ m \geq 3,\ \ g \geq 3$$

$$f(2, m; g) = \begin{cases} \dfrac{m(g-1)+2}{2} \ if\ \ g\ is\ odd, \\ \\ \dfrac{m(g-2)+4}{2} \ if\ \ g\ is\ even. \end{cases}$$

Las ({2,m};g)-jaulitas, para g impar y para g par



Otros valores exactos:

$f(\{r,m\};4) = r + m$ for any r with $2 \leq r \leq m$*

$f(\{3,m\};g) = 1 + gm$ for $m \geq 4$ and $g = 5, 6$*

$f(\{3,m\};9) = 1 + 15m$ for $m \geq 9$ *

- Chartrand, Gould and Kapoor (1981)
- Downs, Gould and Mitchem (1981)

* Downs, Gould, Mitchem,Saba(1981)

* Limaye y Sarvate (1998)

•Yang y Liang (2003)

	$g = 5$	$g = 6$ *
$f(\{3, m\}; g)$	$3m + 1,\ m \geq 4$ *	$4m + 2,\ m \geq 4$
$f(\{4, m\}; g)$	$4m + 1,\ m \geq 6$ *	$6m + 2,\ m \geq 5$
$f(\{5, m\}; g)$	$5m + 1,\ m \geq 6$ *	$8m + 2,\ m \geq 6$
$f(\{r, m\}; g)$ $5 < r < m$		$2(rm - m + 1)$ $m - 1$ a prime power $2 \leq r \leq m$

Usando sus resultados y la cota inferior Yang y Liang conjecturaron que:

$$f(\{r,m\};6) = 2(rm - m + 1) \ for \ 2 \leq r < m$$

$$f(\{r,k(r-1)+1\};6) = 2k(r-1)^2 + 2r, \ r-1 \ being \ a \ prime \ power.$$

Lo cual comprueba la conjetura. Además para cuello par mayor que 8 mejoramos las cotas inferiores.

Resultados (A, Balbuena, García-Vázquez, Marcote, Valenzuela, 2007)

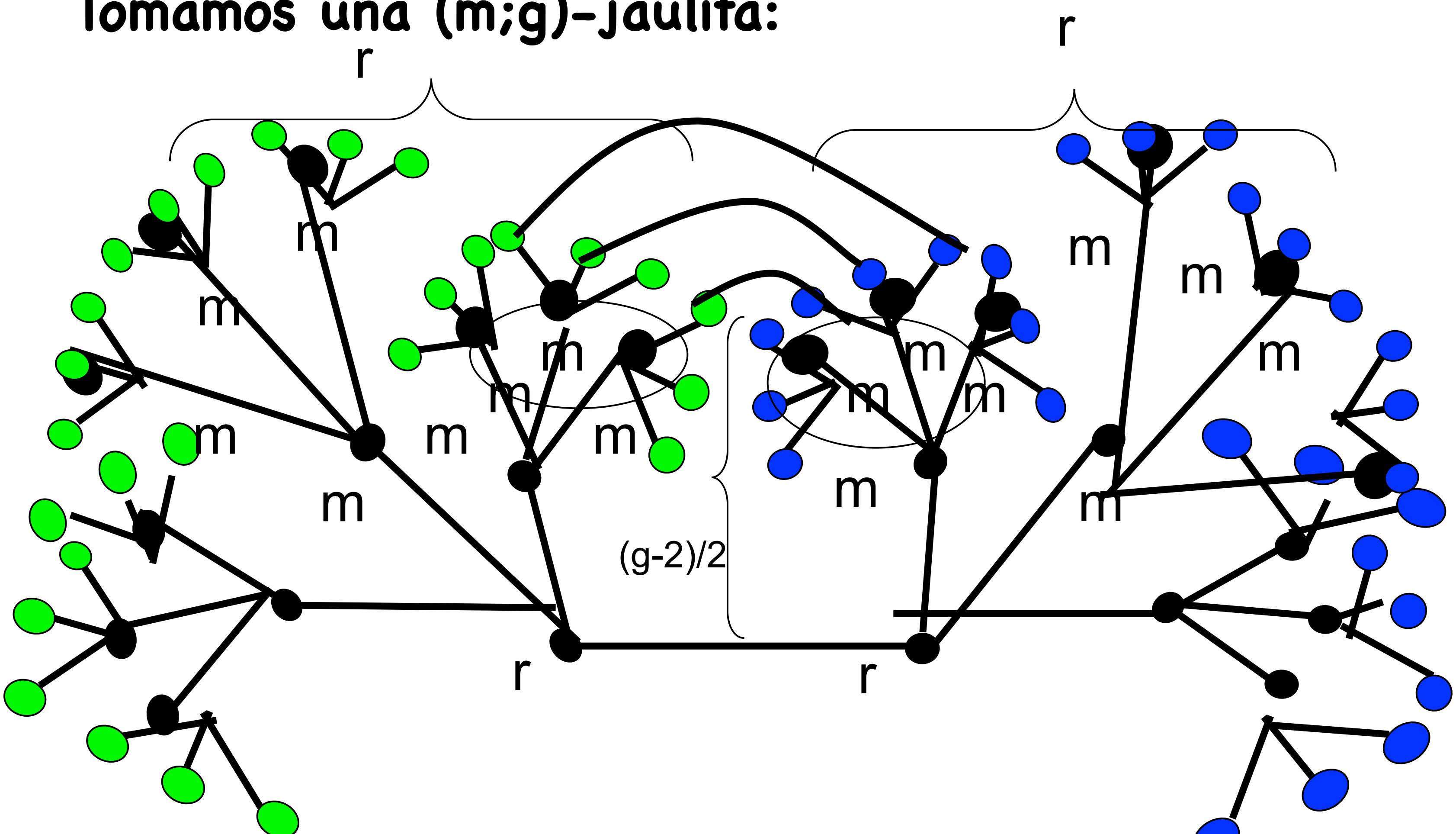
Usando la existencia y las propiedades de los polígonos generalizados o jaulitas de cuello 8 probamos:

Teorema 1: $If\ 2 \leq r < m\ where\ m-1\ is\ a\ prime\ power\ and\ g \in \{6, 8, 12\}\ then$

$$f(\{r, m\}; g) \leq 2 + 2(r-1)\frac{(m-1)^{g/2-1}}{m-2}$$

Idea de la prueba:

Tomamos una (m;g)-jaulita:



Borramos algunos vértices en la gráfica

Una consecuencia inmediata del teorema anterior y de la cota inferior de Yuang y Liang es la siguiente:

Corolario (Yuansheng and Liang) *Let* $2 \leq r < m$ *be two integers such that* $m - 1$ *is a prime power. Then* $f(\{r, m\}; 6) = 2(rm - m + 1)$

También se probó para m=k(r-1)+1 y k ≥ 2

Let r, k be integers with $r \geq 3$ and $k \geq 2$. Then for all even $g \geq 6$ we have

$$(i)\ f(\{r, k(r-1)+1\}; g) \leq kf(r; g) - 2(k-1) \sum_{i=0}^{\lfloor (g-2)/4 \rfloor} (r-1)^i.$$

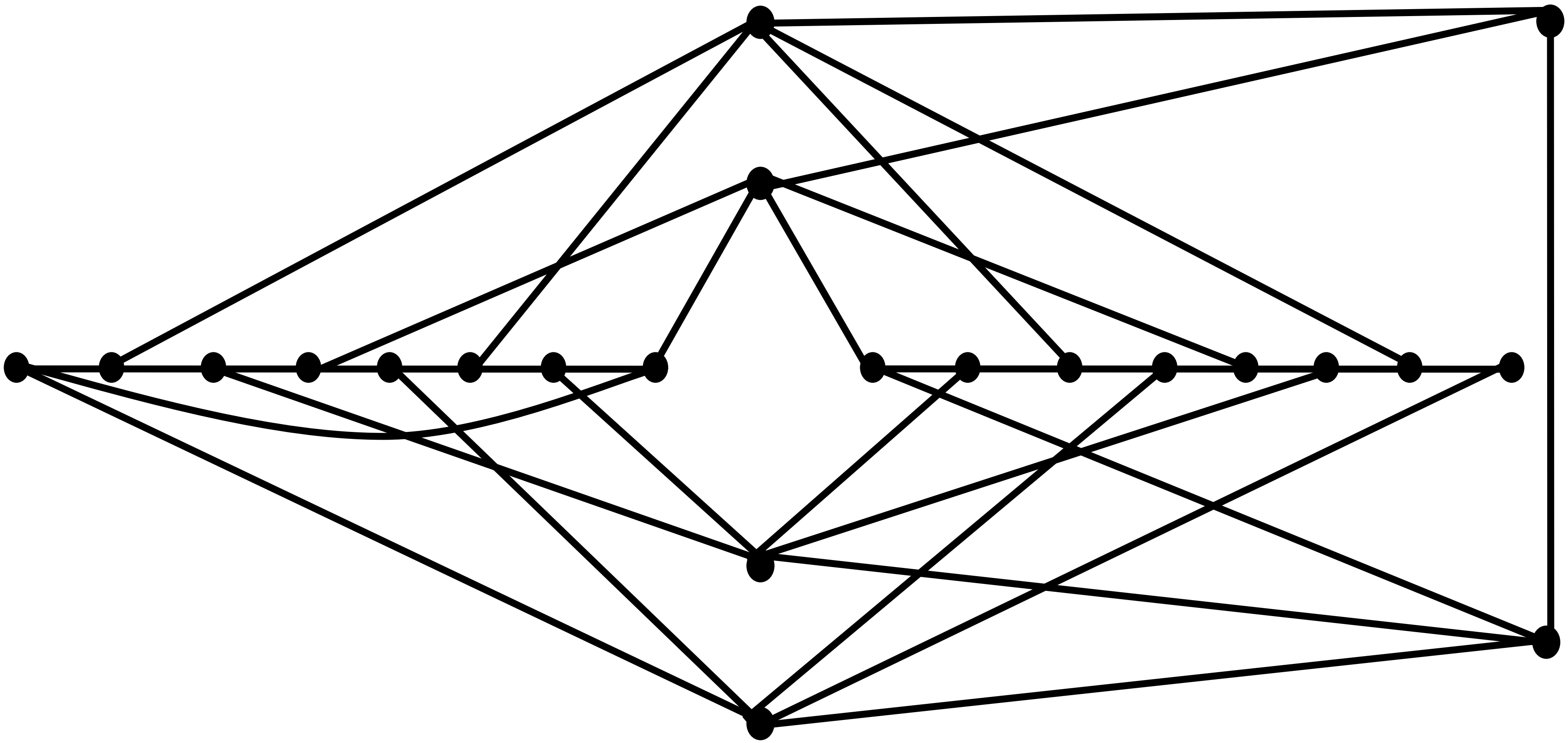
$$(ii)\ f(\{r, k(r-1)\}; g) \leq kf(r; g) + 2(r-1)^b - 2k \sum_{i=0}^{\lfloor (g-2)/4 \rfloor} (r-1)^i.$$

Corolario *Let $r, k \geq 2$ be two integers such that $r - 1$ is a prime power. Then*

$$f(\{r, k(r-1)+1\}; 6) = 2k(r-1)^2 + 2r$$

La prueba es constructiva "pegando jaulitas" .

La siguiente es una ({3,5},6)-jaulita obtenida mediante el razonamiento de este teorema.



A ({3,5};6)-cage.

Mejoras en las cotas inferiores salvo en el caso g=6

Teorema *Let G be a* $(\{r,m\};g)-$*cage of even girth* $g \geq 6$ *and* $3 \leq r < m$. *Then*

$$f(\{r,m\};g) \geq \begin{cases} m+2+(mr-2)\dfrac{(r-1)^{g/2-2}-1}{r-2}+(r-2)(r-1)^{g/2-2} \\ \quad if\ r \geq 4; \\ 1+\dfrac{(7m+3)2^{g/2-2}}{3}-m \\ \quad if\ r=3 \end{cases}$$

Cuello 8:

Corollary 6 *Let* $2 \leq r, k$ *be two integers such that* $r-1$ *is a prime power. Then*

$$(k+1)r^3 - (k+3)r^2 + (k+3)r - k + 3 \leq f(\{r, k(r-1)+1\}; 8) \leq 2kr(r-1)^2 + 2r, \ if\ r \geq 4;$$

and $(28k+8)/3 - 2k \leq f(\{3, 2k+1\}; 8) \leq 24k + 6.$

Para cuello impar:

Teorema (A,Balbuena,Valenzuela 2009): Sea G una (r;g+1)-jaula, de orden n(r;g+1), cuello par igual a g+1, entonces:

$$n(\{r, k(r-1)\}; g) \leq (k/2)(n(r, g+1) - 2) + 1$$

Para todo entero par k≥2.

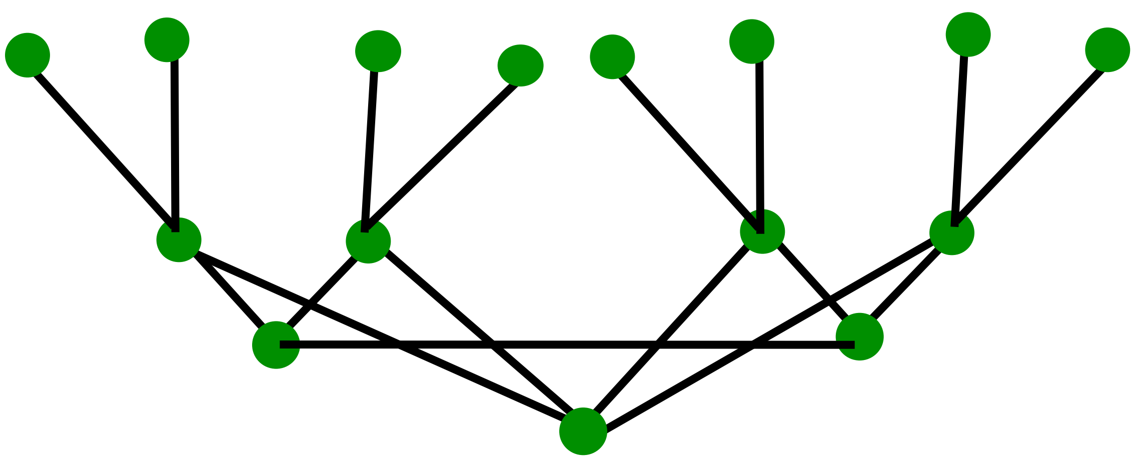
Si lo aplicamos a Jaulitas relacionadas con cuadrángulos generalizados tenemos que:

Corollary: **Existen $(\{r,m\};g)$-jaulitas de orden $n_0(\{r,m\};g)$ cuando r-1 es potencia de primo,**

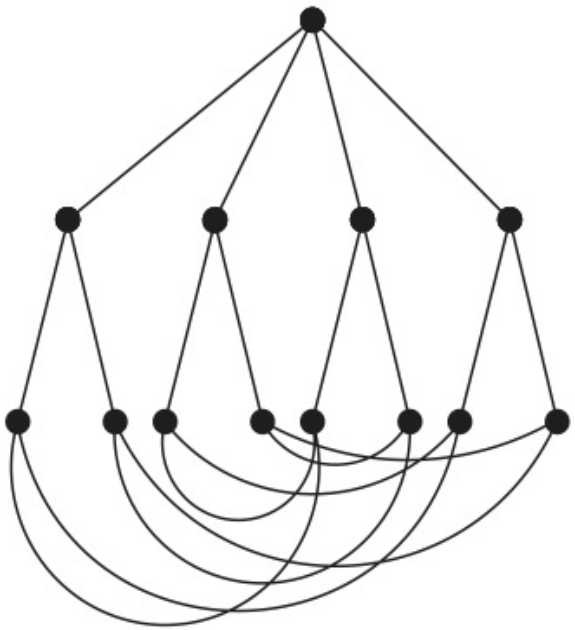
$m=k(r-1)$ para $k \geq 2$ (entero par) y $g=\{5,7,11\}$.

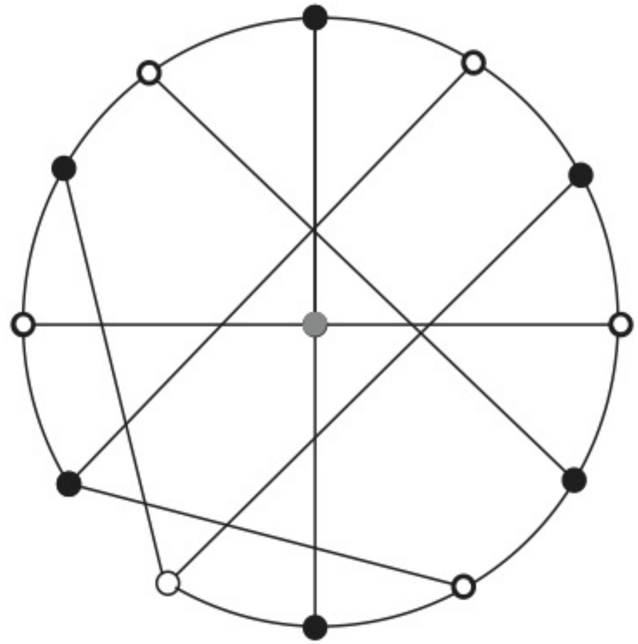
Idea de la prueba

Tomamos la (3;6)-jaulita o gráfica de Heawood para cosntruir una ({3,4};5)-jaulita



Dos dibujos de la ({3,4};5)-jaulita.





Nota: Existen al menos dos ({3,4};5)-jaulitas no isomorfas !!!

Theorema (ABV): Sean r y k enteros con r≥3 y k≥2:

(I) Si g es par, entonces:

$$n(\{r, rk\}; g) \leq k(n(r; g) - 2(k-1)).$$

(ii) Si g es impar y existe una (r;g)-jaulita entonces:

$$n(\{r, rk\}; g) \leq n_0(r; g) + (k-1)n(r; g+1) - 2(k-1).$$

(iii) Si g es impar y las (r;g)-jaulas no son jaulitas, entonces:

$$n(\{r, rk\}; g) \leq kn(r; g+1) - 2(k-1).$$

Corollary: Si k≥2 y r-1 es una potencia de primo, entonces:

$$n(\{r, rk\}; g) = 2(kr^2 - rk + 1)$$

Una ({r,kr};6)-jaulita se construye identificando k copias de las (r;6)-jaulitas en un par de vértices 3-remotos (la distancia entre ellos es igual a 3).

Este colorario da otro ejemplo de $(\{r,m\};6)$-jaulita que satisface la Conjetura de Yiang y Liang.

Un **m-agono generalizado de orden q** es una estructúra de incidencia punto-línea cuya gráfica de incidencia es bipartita **(q+1)**-regular, de cuello **2m** y diámetro **m.**

Para q potencia de primo:

- Las (q+1,8)-jaulitas son las gráficas de incidencia de los 4-gons de orden q.
- Las (q+1,12)-jaulitas son las gráficas de incidencia de los 6-gons de orden q.

Dos vértices en una (q;g)-jaulita se llaman **opuestos** o **g/2-remotos** si están a distancia máxima uno de (A distancia g/2)

Un **ovoide** es un conjunto de vértices g/2-remotes dos a dos.

Si el ovoide **existe tiene** cardinalidad igual a:

$$(q-1)^{g/4}+1$$

Proposición: Las (r;g)-jauitas con r-1 potencia de primo tienen exactamente $(r-1)^{g/4}+1$ vertices que están mutuamente a distancia al menos g/2 si:

- El cuello de g es 8
- El cuello de g es 12 y r-1 es un primo impar diferente de 5 y 7.

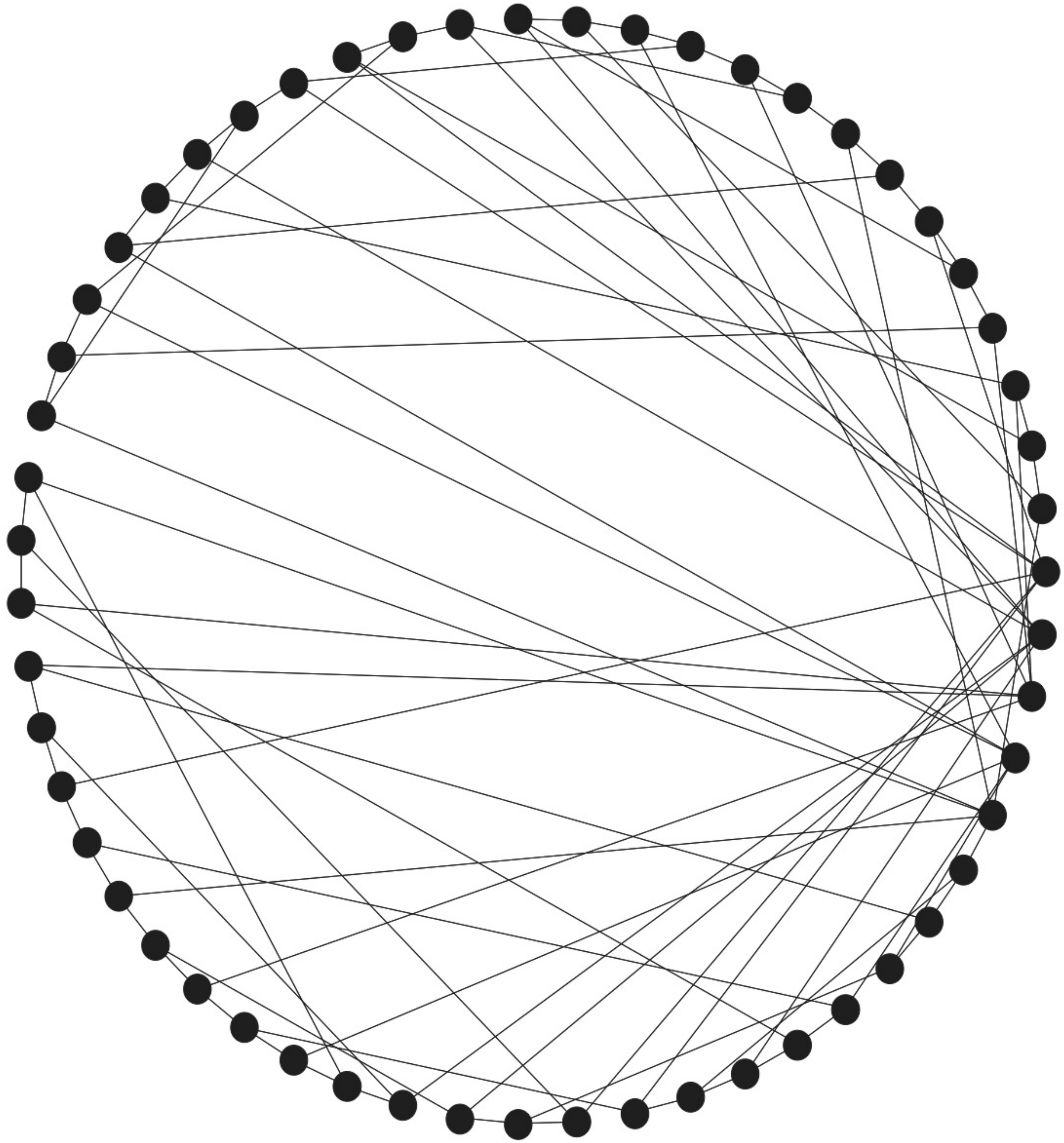
Corolario: Si k≥2 y la gráfica de incidencia tiene los parámetros requeridos entonces:

$$n(\{r,rk\};g) \leq 2k\frac{(r-1)^{g/2}-1}{r-2} - (k-1)((r-1)^{g/4}+1)$$

Valores exactos para r=3 y g=8:

Corolario: Si $k \geq 2$ entonces:

$$n(\{3, 3k\}; 8) = 25k + 5$$



A, Balbuena, García and Montejano (2012) estudiamos especificamente el caso $g=8$ y obtuvimos los siguiente:

Mejoramos la cota inferior para $(\{3,m\};8)$-jaulas para $m \geq 7$ y $m \not\equiv 0 \pmod 3$:

$$n(\{3,m\};8) \geq [25m/3]+7$$

Como consecuencia de eso $m=3k+t$, $k \geq 2$ y $t=1,2$ tenemos que:

$$25k+8t+8 \leq n(\{3,3k+t\};8) \leq 25k+3t+21$$

Cosntruimos ({3,m};8)-jaulitas para m=4,5,7 y ordenes 39, 48 y 66 respectivamente.

- Construimos ({3,m};8)-gráficas de orden 9m+3 que mejoran las cotas superiores que existen para m=8,10,13,16.

Para dichas construcciones también usamos el concepto de ovoide en cuadrángulos generalizados y la idea de remoticidad.

m	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
n({3,m};8)	39	48	55	66		80			105			130			155
Lower bound					74		91	100		116	124		141	150	
Upper bound					75		93	102		120	127		147	152	

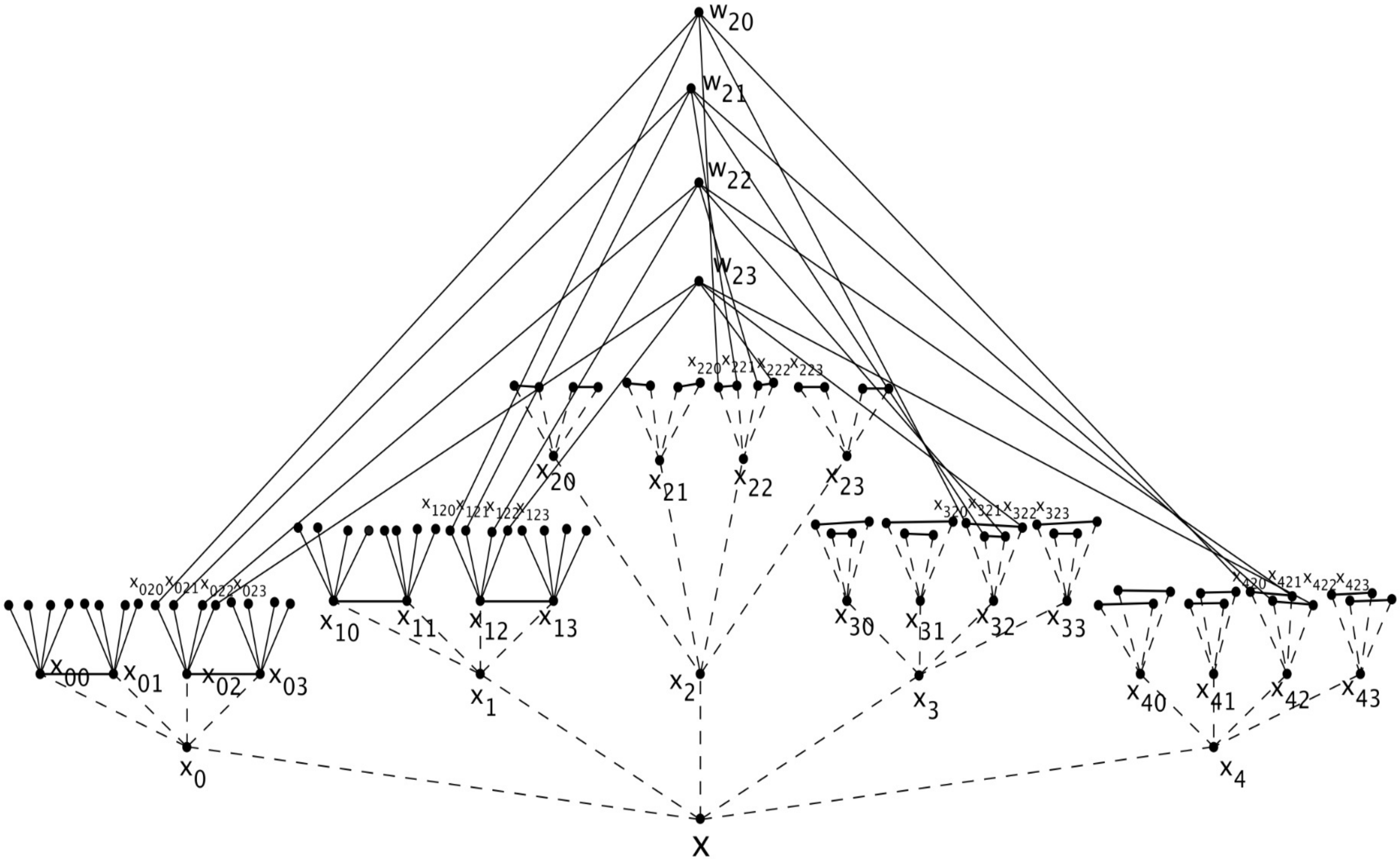
$n(\{r,m\};g)$	$g=5$	$g=6$	$g=7$	$g=8$	$g=9$	$g=11$
$r=3$ $m\geq 4$	$3m+1$	$4m+2$	$7m+1$	$25m/3+5$ $m=3k$	$15m+1$ $m\geq 6$	$31m+1$ $m=4k$
$r=4$ $m\geq 4$	$4m+1$	$6m+2$	$13m+1$ $m=6k$	$9m+3$ $m=4,5,7$		$121m$ $m=6k$
$5\leq r<m$ $m-1=p^{\alpha}$		$2(rm-m+1)$				
$5\leq r<m$ $m-1=p^{\alpha}$	$1+rm$ $m=2k(r-1)$	$2(rm-m+1)$ $m=k(r-1)+1$, or $m=kr$	$1+m(r^2+r+1)$ $m=2k(r-1)$			$1+\frac{m\ (r-1)-1}{r-2}$ $m=2k(r-1)$

Usando técnicas similares a las usadas con por Abreu, Balbuena y Labbate trabajamos en jaulitas biregulares de cuello 5.

En 2013, A. Abreu, Balbuena, Labbate y Salas trabajamos también para cuello 7 utilizando la descripción del cuadrángulo generalizado que se obtuvo para cuello 8 y las técnicas"combinatorias" usadas para la construcción en caso de cuello 5.

Se borraron conjuntos de vértices y aristas en la gráfica de incidencia se añadieron aristas.

Grado impar



w20
w21
w22
w23
x220 x221 x222 x223
x20
x21
x22
x23
x120 x121 x122 x123
x320 x321 x322 x323
x020 x021 x022 x023
x420 x421 x422 x423
x10
x11
x12
x13
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x31
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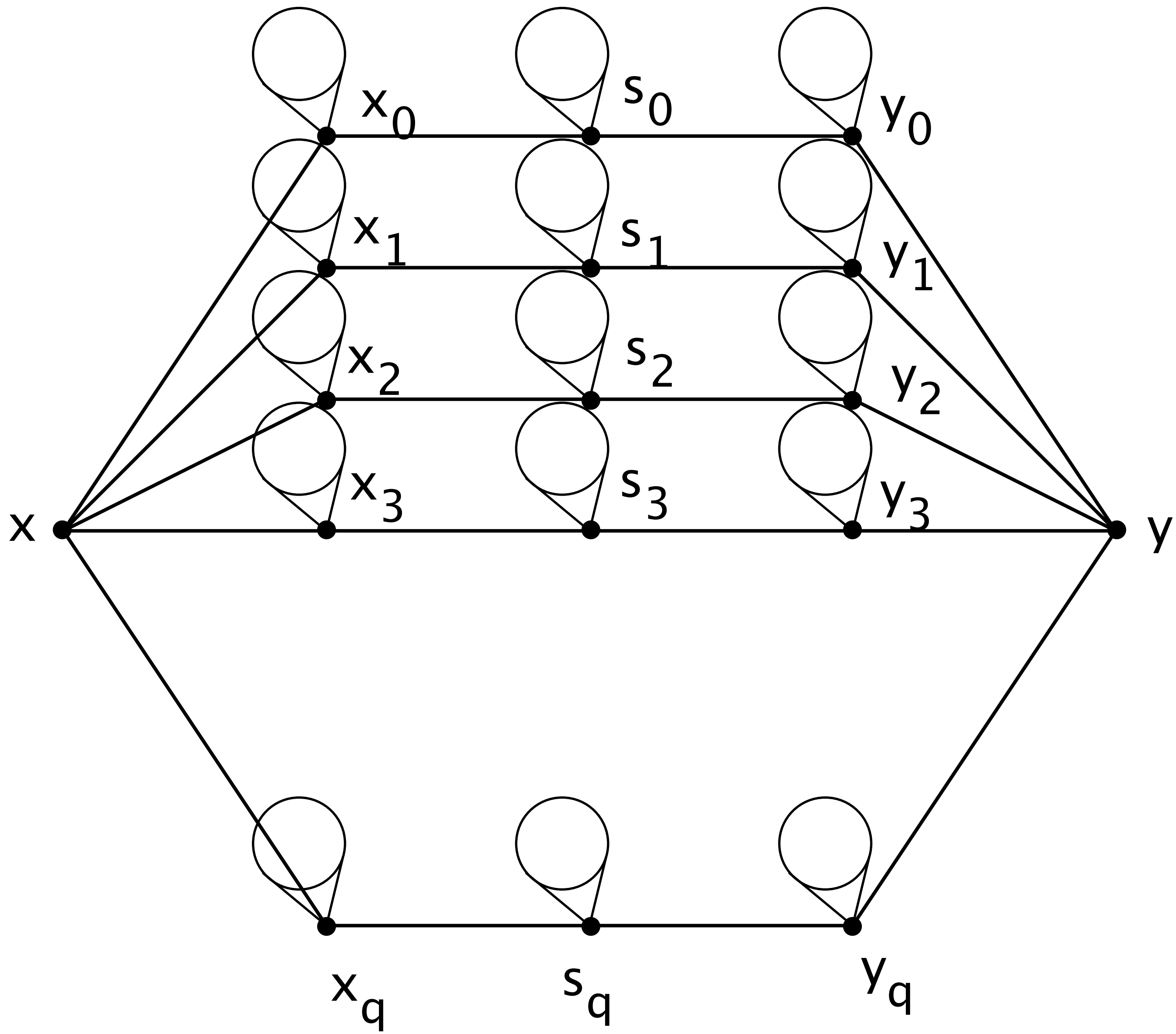
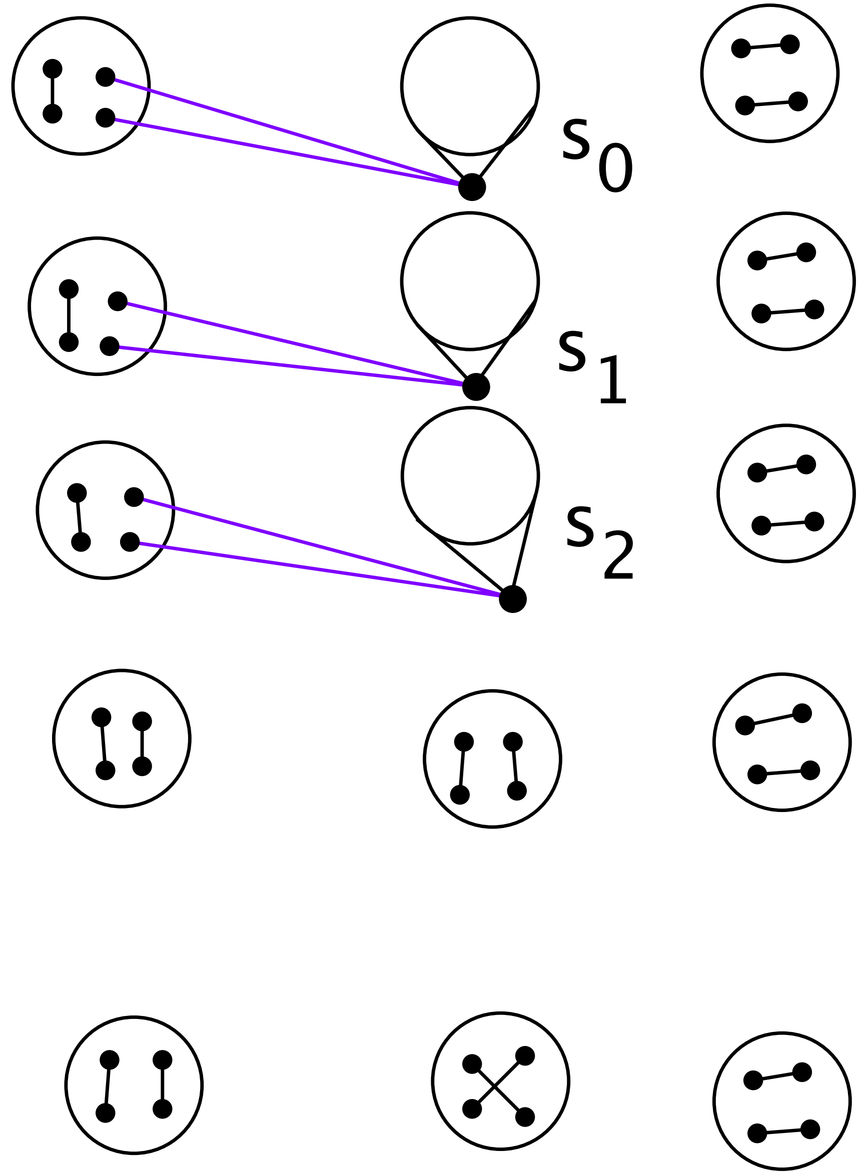
x4
x

Grado impar





s0
s1
s2

Valores anteriores (A, 2010)

$n(q+1;7) \leq 2(q^3+q^2)$ para $q \geq 4$ potencia de primo par.

(A,Abreu,Balbuena,Labbate,Salas, 2013)

$n(q+1;7) \leq 2q^3+q^2+2q$ para $q \geq 4$ potencia de primo par.

$n(q+1;7) \leq 2q^3+2q^2-q+1$ para $q \geq 5$ potencia de primo impar.

Jaulas bi-regulares

(D,g)-jaulas

- El conjunto de grados D de G es el conjunto de los grados de los vértices de G.

- n(D;g) denota el orden mínimo de una gráfica con conjunto de grados D y cuello g.
- Una gráfica con conjunto de grados D, cuello g y orden n(D;g) se llama una (D;g)-jaula